Mathematics Camp for Economists

Rice University

Summer 2016

Logistics

Instructor:	Metin Uyanık, muyanik1@jhu.edu
TA:	Atara Oliver, sao5@rice.edu
Schedule:	July 1 - July 29, MTWThF, 20 Lectures
Time:	2-hour Online Lecture MTWThF
Location:	Canvas Platform, https://canvas.rice.edu
Office Hours:	9am - 10am and 9pm - 10pm (Central Daylight Time), Canvas, MTWThF

Course Outline

The aim of this course is to introduce/remind you a basic level of mathematics which is required for the Ph.D. courses in economics. Specifically, in this course, we will learn/remember standard tools and cookbook procedures that are required for the first year Ph.D. courses.

I divide the lecture into five parts: Real Analysis, Linear Algebra, Calculus, Optimization and Difference & Differential Equations. There will be two homeworks for each part. I will ask questions to be answered during the online lectures to measure your attendance. You should expect to study three hours on average for reviewing the material and doing the homeworks. In addition, there will be daily office hours twice a day. The attendance to the office hours is not mandatory but highly encouraged. I will be available to answer your questions and you will have opportunity to interact with your classmates. There will be an exam at the end of the course. The weights of the attendance to the lectures, homeworks and the exam are 10%, 40% and 50%.

General Readings:

P. R. Halmos, 1970. *How to write mathematics*, L'Enseignement Mathématique, Vol.16, Zürich.

W. Thomson, 1999. *The Young Person's Guide to Writing Economic Theory*, Journal of Economic Literature, **36**, 157-183.

I. Real Analysis (Lectures 1-4)

(a) Sets:

- Algebra of Sets
- Families of Sets
- Cartesian Product
- Binary Relations and Ordered Sets, Supremum and Infimum
- Functions and Correspondences

Readings:

J. Dugundji, 1966. *Topology*, Allyn and Bacon, Inc., Boston. Sections 1.1-1.7 and 2.1-2.2.

D. H. Fremlin, 2001. *Measure Theory*, vol. 1, manuscript. Section 111C page 10 and Section 111X page 13.

(b) Metric Spaces:

- Metric Spaces, Euclidean Spaces
- Topological Properties of Sets: Open, Closed, Compact, Dense and Connected Sets; Interior, Closure and Boundary of Sets
- Topological Properties of Sequences: Convergence, Subsequences, Cauchy Sequences, Upper and Lower Limits, Complete Metric Space, Series and Absolute Convergence
- Topological Properties of Functions and Correspondences: Continuity, Upper and Lower Semicontinuity
- Continuity, Compactness and Connectedness

Readings:

W. Rudin, 1976. *Principles of Real Analysis*, McGraw-Hill, London. Sections 2.1-3, 3.1-3, and 4.1-3, and Chapter 5 Exercise 22.

C. P. Simon and L. Blume, 1994. *Mathematics for Economists*, Norton, New York. Chapters 12, 29.

II. Linear Algebra (Lectures 5-9)

(a) Linear Algebra:

- Vectors, Vector Operations
- Linear Dependence
- Basis, Vector Spaces and Subspaces
- Matrices and Matrix Algebra
- Inner Product and Projection
- Linear Transformations
- Rank and Determinant
- Solutions to Systems of Linear Equations: Gaussian Elimination and Cramer's Rule

Readings:

G. Hadley, 1987. *Linear Algebra*, Narosa Publishing House, New Delhi. Chapters 2-5.

C. P. Simon and L. Blume, 1994. *Mathematics for Economists*, Norton, New York. Chapters 7-11.

(b) Convexity:

- Convex Set, Convex Hull, Extreme Points and Convex Cone
- Carathedory's Theorem
- Convex, Concave, Quasiconvex and Quasiconcave Functions
- Separating Hyperplane Theorems

Readings:

G. Hadley, 1987. *Linear Algebra*, Narosa Publishing House, New Delhi. Chapter 6.

C. P. Simon and L. Blume, 1994. *Mathematics for Economists*, Norton, New York. Chapter 21.

III. Calculus: (Lectures 10-12)

Differentiation and Integration:

- The Derivative of a Real Function
- Mean Value Theorems
- L'Hospital's Rule and Taylor's Theorem
- Integral of a Real Valued Function
- Integration and Differentiation: The Fundamental Theorem of Calculus
- Integration by Parts and Leibniz Integral Rule
- The Inverse Function Theorem and The Implicit Function Theorem

Readings:

W. Rudin, 1976. *Principles of Real Analysis*, McGraw-Hill, London. Chapters 5-6 and Sections 9.4-6.

C. P. Simon and L. Blume, 1994. *Mathematics for Economists*, Norton, New York. Chapters 3-5, 14.

IV. Optimization (Lectures 13-17)

Static Optimization:

- Linear Programming, Duality Theorems and Simplex Method
- Weierstrass Theorem: Existence of a Maximizer
- Unconstrained Optimization: Fermat's Theorem, First and Second Order Conditions
- Constrained Optimization: Constraint Set, Lagrangean, KT-conditions
- Convexity and Optimization: Necessity and Sufficiency
- Saddle Point Theorem, The Envelope Theorem and the Theorem of the Maximum

Readings:

R. K. Sundaram, 1996. *A First Course in Optimization*, Cambridge University Press, New York. Chapters 3-9.

C. P. Simon and L. Blume, 1994. *Mathematics for Economists*, Norton, New York. Chapters 17-19.

V. Difference and Differential Equations (Lectures 18-20)

Difference and Differential Equations:

- Difference Equations
- Cobweb Diagram
- First and Second Order Linear Differential Equations
- Homogenous and Nonhomogenous Differential Equations
- Phase Diagram
- System of Differential Equations
- Existence and Stability of Rest Points

Readings:

M. W. Hirsch and S. Smale, 1974. *Differential Equations, Dynamical Systems, and Linear Algebra*, Academic Press, Inc., San Diego. Sections 3.1-3, 5.4-6, 8.2-3, 8.7, and 9.2-3.

C. P. Simon and L. Blume, 1994. *Mathematics for Economists*, Norton, New York. Chapters 24-25.

Mathematics Camp for Economists: Part II

Rice University

Summer 2016

Logistics

Instructor:	Metin Uyanık, muyanik1@jhu.edu
Schedule:	August 1 - 14, MTWTF, 10 Lectures
Time & Location:	TBA

Course Outline

The aim of this course is to introduce/remind you a basic level of mathematics which is required for the Ph.D. courses in economics. In this course, we take some simple examples and try out notions of closeness (topological and metric spaces), order (relations) and linearity (vector spaces) on them. Also we will learn, and practice, how to write a statement and to prove it by working on elementary theorems. This course is a complement to the online math camp course you already took. In the online part, we covered some basic cookbook procedures and tools that are useful for the Ph.D. courses. In a nutshell, the online course is mathematics as a science of measurement and this course is mathematics as language.

I organize this course around five mathematical structures: Order, Topological, Metric, Measurable and Linear. There will be one homework for each mathematical structure. You are highly encouraged to study together but should submit the homeworks separately. The attendance to the lectures are mandatory. You should expect to study three hours on average for reviewing the material and doing the homeworks. Moreover, there will be daily one hour office hour. The attendance to the office hours is not mandatory but highly encouraged. There will be an exam at the end of the course. The weights of the homeworks and the exam are 30% and 70%.

General Readings:

P. R. Halmos, 1970. *How to write mathematics*, L'Enseignement Mathématique, Vol.16, Zürich.

W. Thomson, 1999. *The Young Person's Guide to Writing Economic Theory*, Journal of Economic Literature, **36**, 157-183.

0. Mathematical Structures (Lecture 1)

Mathematical Structures: Order, Topological, Metric, Measure, Linear

I. Ordered Spaces (Lecture 2)

Order Structure:

- Binary Relations, Partially Ordered Sets, Zorn's Lemma
- Lattice Structure

Readings:

J. Dugundji, 1966. *Topology*, Allyn and Bacon, Inc., Boston. [(D), hereafter.] Sections 1.3, 1.6, and 1.7, 2.1 and 2.2.

II. Topological Spaces (Lectures 3-5)

Topology and Topological Spaces:

- Topology: Discrete, Indiscrete, Euclidean and Cofinite Topology
- Topological Properties of Sets: Open and Closed, Compact and Dense Set; Closure, Interior, Boundary, Limit Point and Isolated Point of a Set
- Topological Properties of Sequences: Convergence and Divergence; Existence and Uniqueness of a Limit of a Sequence
- Topological Properties of Functions and Correspondences: Continuity, Upper and Lower Semicontinuity
- Homeomorphism, Relative Topology, Subspaces, Bases, Obtaining Topological Spaces from Other Topological Spaces, Quotient Space

Readings:

M. Ali Khan, 1999. 'A result on large anonymous games: an elementary and self-contained exposition,' in J. R. Melvin, J. C. Moore and R. Riezman (eds.) *Trade Theory and Econometrics*, Roudledge, London, 206-236. Pages 209-2011.

(D) Sections 3.1-4, 3.8-9, 3.12, 4.1, 5.1-2, 7.1, 9.1-5, 11.1-2

L. A. Steen and J. A. Seebach, Jr., 1978. *Counterexamples in Topology*, Springer-Verlag, New York. Part II, Examples 3, 4 and 19.

III. Metric Spaces (Lectures 6-7)

Metric Spaces:

- Metric vs Topology
- Elementary properties of Metric Spaces, Continuity Properties of Sequences and Functions, Cauchy sequences, Complete Metric Space, Euclidean spaces. Continuity properties of correspondences.
- Weierstrass Maximum Theorem, Berge's Maximum Theorem, Intermediate Value Theorem, Brouwer, Kakutani and Browder Fixed Point Theorems, Applications in Economics

Readings:

W. Rudin, 1976. *Principles of Real Analysis*, McGraw-Hill, London. Sections 2.1-3, 3.1-3, and 4.1-3.

G. Debreu, 1959. *Theory of Value*, Manuscript, Cowles Foundation, Chapter 1.

IV. Measurable Spaces (Lecture 8)

Measurable and Measure Spaces:

- Algebra, σ-algebra, Measurable Sets, Measurable Space
- Measure, Measure Spaces, Random Variables, Induced Distribution
- Three Measure Spaces: Discrete, Indiscrete, Lebesgue
- Integration: Lebesgue vs Reimann

Readings:

D. H. Fremlin, 2001. *Measure Theory*, vol. 1, manuscript.

H. L. Royden, 1988. *Real Analysis*, Macmillan, New York. Chapters 11-13.

W. Rudin, 1970. *Real and Complex Analysis*, McGraw-Hill, New York. Chapter 1.

V. Linear/Vector Spaces (Lectures 9-10)

Linear/Vector Spaces:

- Linear Structure: Fields, Linear Spaces, Bases
- Linear Dependence, Dimension, Isomorphism, Subspaces
- Space of Functions, Space of Sequences, Obtaining Linear Spaces from Other Spaces, Quotient Spaces
- Riesz Representation Theorem
- Applications in Economics: Walrasian Equilibrium, Welfare Theorems, Separating Hyperplane Theorems

Readings:

P. R. Halmos, 1958. *Finite-dimensional Vector Spaces*, D. Van Nostrand Company, Inc., Princeton, New Jersey. Sections 1.1-12 and 1.21.

P. R. Halmos, 1995. *Linear Algebra Problem Book*, Mathematical Association of America, Washington. Chapters 1-3.