# Legislative and Multilateral Bargaining

## Hülya Eraslan[1] and Kirill S. Evdokimov[2]

[1]Department of Economics, Rice University, Houston, Texas, 77251; email: eraslan@rice.edu
[2]Department of Economics, Rice University, Houston, Texas, 77251; email: kirill.evdokimov@rice.edu

### Keywords

legislative bargaining, multilateral bargaining, weighted voting, coalitional bargaining, spatial bargaining

### Abstract

This survey of the theoretical literature on legislative and multilateral bargaining begins with the seminal work of Baron & Ferejohn (1989). The survey then encompasses the extensions to bargaining among asymmetric players in terms of bargaining power, voting weights, and time and risk preferences; spatial bargaining; bargaining over a stochastic surplus; bargaining over public goods; legislative bargaining with alternative bargaining protocols in which players make demands, compete for recognition, or make counter-proposals; and legislative bargaining with cheap talk communication.

**Contents**

# 1. INTRODUCTION

This paper surveys the theoretical literature on legislative and multilateral bargaining. Our inability to locate a specific definition of the term "legislative bargaining," although prevalent in the literature, makes this task difficult. Based on our discussions with other researchers, we ultimately concluded that it is one of those phrases that fits Justice Potter Stewart's memorable "I know it when I see it" categorization. Although tempting, this approach would not have allowed us establish the scope of our survey. Therefore, we begin our paper by first defining what is it that we intend to survey.[1]

The word *bargaining* describes a process during which parties with conflicting interests attempt to reach a mutually beneficial agreement, and proposals on which agreement to reach arise endogenously. *Multilateral bargaining* describes bargaining situations involving three or more parties. By *legislative and multilateral bargaining* (henceforth legislative bargaining), we mean multilateral bargaining situations in which agreement requires less than unanimous consent and agreement on a proposal binds all parties.

The requirement of the endogeneity of the proposals emphasizes the difference between legislative bargaining and committee decision making.[2] As a result of our insistence that an agreement must bind all players, we exclude other important models of multilateral bargaining such as exit games,[3] decentralized trade,[4] models in which an active player

---

[1]To the best of our knowledge, this is the first survey of legislative bargaining literature. Austen-Smith & Banks (2005) (Chapter 6) provide a textbook treatment of some models we discuss below.

[2]See Li & Suen (2009) for a survey of decision-making in committees.

[3]In exit games, partial agreements bind a subset of players, who then exit the game after reaching agreement and the remaining players continue to bargain. See, for example, Krishna & Serrano (1996).

[4]In these models, many pairs of agents bargain simultaneously over gains from trade, and the bargaining outcome between any pair of agents may depend on outcomes of other bargaining encounters. See part 2 of Osborne & Rubinstein (1990) for an extensive discussion of this literature.

bargains with passive players one at a time,[5] and, most importantly, the literature on coalition formation. A subset of this literature is closely related to legislative bargaining,[6] but to elaborate on these connections would exceed the space limit allowed. Instead, we refer the readers to the excellent surveys of Ray (2007) and Ray & Vohra (2015). We also limit our attention to non-cooperative models of bargaining and refer to readers to Austen-Smith & Banks (1999, 2005) and Schofield (2007) for the cooperative approach.

Legislative bargaining literature, launched by the seminal work of Baron & Ferejohn (1989) (henceforth BF), is best viewed as a response to general inability of voting rules to produce *stable* outcomes.[7] Since Condorcet, it is well known that pairwise majority comparisons may produce cycles, and thus majority rule by itself is not able to offer predictions based solely on individual preferences.[8] One way to circumvent this problem is to impose restrictions on the set of feasible alternatives and on the preferences over the feasible alternatives. As shown by Black (1948), if the policies can be represented by points on a real line and the preferences of individuals are single-peaked, then there exists an alternative which can defeat any other by a simple majority rule, i.e. the majority rule core is nonempty. Unfortunately, this result does not generalize to the multi-dimensional policy spaces. A rich literature starting with the seminal work of Plott (1967) established that when the policy space is multi-dimensional, the majority rule core is empty except for a very special configuration of preferences.[9]

An alternative approach is to impose institutional structure on the process of decision making. One might conjecture that if the agenda is restricted, so that the voting includes only a finite number of steps, it is possible to make predictions about the policy outcomes. As shown by McKelvey (1976, 1979), this conjecture is wrong however: any alternative can be defeated by any other through a sequence of pairwise comparisons. An implication of these results is that agenda control is valuable. This observation resulted in the emergence of a literature on agenda control which focuses on sophisticated voting under various types of agendas.[10] A related idea was the agenda-setter model of Romer & Rosenthal (1978) in which a single player has the exclusive right to make a single, take-it-or-leave-it proposal to other players, who must then vote either to accept or reject her proposal. In the so-called structure-induced equilibrium approach, different subsets of players (interpreted as committees) control the agenda for different dimensions of the multi-dimensional policy.[11]

The BF model differed from the structure-induced equilibrium approach by considering

---

See also Iaryczower & Oliveros (2016) for a model of decentralized legislative bargaining building on Gül (1989).

[5]See, for example, Cai (2000, 2003).

[6]Major contributions with closer connections to legislative bargaining include Chatterjee et al. (1993), Okada (1996, 2000, 2011), Seidmann & Winter (1998), Ray & Vohra (1999), Konishi & Ray (2003), Gomes (2005, 2015), Gomes & Jehiel (2005) and Hyndman & Ray (2007).

[7]See the excellent survey by Inman (1987) on the earlier literature leading up to BF.

[8]In fact, the Arrow (1951) impossibility theorem implies that the lack of a voting equilibrium is not confined to pairwise majority comparisons only, and no democratic voting procedure can consistently combine individual preferences into a social preference.

[9]There is a vast literature on the existence of core policies. Austen-Smith & Banks (1999) provides a unified treatment of social choice theory including key theorems on the existence of core policies.

[10]See, for example, McKelvey & Niemi (1978), Miller (1980), Shepsle & Weingast (1984), Banks (1985) and McKelvey (1986).

[11]See, for example, Shepsle (1979) and Denzau & Mackay (1981).

a simpler institutional setting, and from the agenda control and agenda-setter models by allowing all players to make proposals. Extending the seminal Rubinstein (1982) bilateral bargaining model of dividing a dollar under unanimity rule to multilateral bargaining under majority rule, BF were able to establish not only the existence of equilibrium but also the uniqueness of equilibrium payoffs. The resultant ability to make sharp predictions about bargaining outcomes then paved the way for numerous extensions and applications to government formation and dissolution, bicameralism, judicial review, executive veto power in legislatures, lobbying, class structure, international relations and trade, federal and local public policy, bankruptcy, business partnership, and bargaining in appellate courts.[12] Due to space limitations, we omit discussions of these applications and instead focus our survey on the theoretical extensions of the BF model.

We begin our review in Section 2 with an analysis of the BF model of bargaining over the division of fixed surplus. Models of distributive politics without unanimity can lead to tension between a party's ability to extract surplus as a proposer and its attractiveness as a coalition partner. We show how this tension is resolved under various voting thresholds. In Section 3, we relax the restrictive symmetry assumptions pertaining to bargaining power and voting weights, as well as time and risk preferences. In legislative settings, it is common for policies to have an ideological component in place of or in addition to distributive policies and for the provision of public goods to accompany distributive decisions. These models which introduce new strategic trade-offs, are reviewed in Sections 4.1 and 4.3. As we shall see, agreement is typically immediate in these models due to losses associated with discounting. Section 4.2 introduces the possibility of a stochastic surplus, which can make waiting beneficial and thus can cause delays. Voting on a proposal made by a randomly recognized proposer is not the only way to approach legislative bargaining, and so, in Section 5, we consider the possibility of players making demands, competing for recognition, and making counter-proposals. In Section 6, we review the role of private information and communication in models of legislative bargaining and conclude in Section 7 with a brief discussion of important avenues for future research.

## 2. BARON-FEREJOHN MODEL

In their seminal contribution, Baron & Ferejohn (1989) present a sequential model of multilateral bargaining with majority rule. The game they consider is a standard "divide the dollar" game where an odd number of risk neutral players bargain over how to divide a cake of size one, and agreement requires the consent of a simple majority. In each period, one of the players in the set $N = \{1, \ldots, n\}$ is randomly recognized to make a proposal that specifies the division of the cake among $n$ players, so the feasible set $X$ is the $(n-1)$-dimensional

---

[12]See, for example, Austen-Smith & Banks (1988, 1990), Baron (1991, 1993, 1998, 2018), Lupia & Strøm (1995), Diermeier & Feddersen (1998), Diermeier & Merlo (2000), Baron & Diermeier (2001), Diermeier et al. (2002, 2003), Bassi (2013) on government formation and dissolution, Diermeier & Myerson (1999), McCarty (2000b), Ansolabehere et al. (2003) Kalandrakis (2004a), Diermeier et al. (2007), Parameswaran (2018) on bicameralism, Vanberg (1998) on judicial review, McCarty (2000a) on executive veto power in legislatures, Persson (1998), Helpman & Persson (2001), Bennedsen & Feldmann (2002), Levy & Razin (2013) on lobbying, Diermeier et al. (2017) on class structure, Tarar (2005), Çelik et al. (2013), Bils & Spaniel (2017) on international relations and trade, Adams et al. (1996), Persson et al. (1998), Persson et al. (2000), Woon & Anderson (2012), Albouy (2013) on federal and local public policy, Eraslan (2008) on bankruptcy, Baranski (2016a) on business partnerships and Parameswaran et al. (2017) on bargaining in appellate courts.

unit simplex. The probability that player $i$ is selected to be the proposer, referred to as her *recognition probability*, is $\frac{1}{n}$. If at least $\frac{n-1}{2}$ other players accept the proposal, then it is implemented; otherwise, the process is repeated in the next period. The utility of player $i$ if the proposal $x$ is implemented in period $t$ is $\delta^t x_i$, where $\delta$ is the common discount factor for all players. If agreement is never reached, then each player's utility is zero.

BF show that no meaningful predictions can be made without restricting the strategies followed by legislators. If there are at least five players, and they are allowed to take the history of play into account when making decisions, then anything can happen: any distribution of payoffs, including the ones with total benefits strictly less than one, can arise in equilibrium. The underlying intuition is that if history matters, one can induce a wide variety of behavior by threatening to punish deviators. Faced with this indeterminacy, they restrict attention to equilibria in which all players use stationary strategies, i.e. strategies that do not depend on the current date and past history.[13,14]

A stationary behavioral strategy for each player consists of a random offer to make as a *proposer*, and for each possible offer $x \in X$ by each other player, a probability of accepting the offer $x$ as a *responder*. Fix an equilibrium strategy profile $\sigma$, an let $v_i$ denote the continuation payoff of player $i$ under this strategy profile, that is, $v_i$ is the payoff $i$ will obtain at the end of any period discounted back to the beginning of that period if the players continue to play $\sigma$ in all future periods.

Now consider the optimal response under $\sigma$ by player $i$ when the proposal on the table is $x$. Assume, as BF do, that players do not use weakly dominated strategies. Player $i$ is better off accepting the proposal if $x_i \geq v_i$ and rejecting it if $x_i < v_i$.[15]

Thus, any proposal that player $j$ makes with positive probability in equilibrium can be written as $x^j = (x_1^j, \ldots, x_n^j)$ with $x_i^j = r_{ji} v_i$ for any player $i \neq j$, and $x_j^j = 1 - \sum_{i \neq j}^n r_{ji} v_i$, where $r_{ji} \in \{0,1\}$ for all $i$, and the vector $r^j = (r_{j1}, \ldots, r_{jn})$ solves $r^j \in \operatorname{argmin}_{r' \in \{0,1\}^n} \sum_{i \neq j}^n r_i' v_i$ subject to $\sum_{i \neq j}^n r_i' = \frac{n-1}{2}$. Given the payoff vector $v$, this nails down the support of any equilibrium proposal strategy by player $j$.

Consider now the following problem:

$$r^j \in \operatorname{argmin}_{r' \in [0,1]^n} \sum_{i \neq j}^n r_i' v_i \ \text{ subject to } \ \sum_{i \neq j}^n r_i' = \frac{n-1}{2}. \qquad\qquad 1.$$

This problem differs from the original problem because its minimizers correspond to mixed proposals rather than pure proposals. Clearly, any solution $r^j$ to problem (1) can be represented as a weighted average of the solutions to the original problem for some nonnegative weights that add up to 1. Using the one-shot deviation principle, one can then represent

---

[13]Baron & Kalai (1993) show that the stationary equilibrium characterized below is the unique simplest equilibrium of the corresponding automaton game. Measuring complexity in terms of past information used to determine actions, Chatterjee & Sabourian (2000) show that that noisy Nash equilibrium with complexity costs leads to the unique stationary equilibrium in bargaining games with unanimity rule.

[14]There is a strand of the literature that studies equilibria with history-dependent strategies. See Cho & Duggan (2009), Cho & Duggan (2015), Herings et al. (2017), Herings et al. (2018).

[15]Assuming players accept when indifferent is without loss of generality. See footnote 5 in Eraslan (2016).

the equilibrium payoffs as

$$v_j = \frac{\delta}{n}\left[(1 - \sum_{i \neq j}^{n} r_{ji}v_i) + \sum_{i \neq j}^{n} r_{ij}v_j\right] \qquad\qquad 2.$$

for all $j$, where $r^j = (r_{j1}, \ldots, r_{jn})$ solves (1). Thus $v = (v_1, \ldots, v_n)$ is an SSP equilibrium payoff vector iff (1) and (2) are satisfied for all $j$.

Let's assume, as BF do implicitly in their proof of Propositions 3 and 4, that equilibrium is symmetric, that is, $v_i = v_j = v$ for all $j$. Then (2) can be written as

$$v = \delta\left[\frac{1}{n}(1 - \frac{n-1}{2}v) + \mu_j v\right] \qquad\qquad 3.$$

where $\mu_j$ is the joint probability that player $j$ is not the proposer and is included in others' coalitions. Clearly, symmetric payoffs are possible only if $\mu_j$ is constant, that is, all players are included in winning coalitions by other players with equal probability. This, in turn, implies that $\mu_j = \frac{n-1}{2n}$ for all $j$. Intuitively, since each proposer includes only $\frac{n-1}{2}$ players among $n-1$ others in her coalition, the probability of being included in the coalition of any other proposer is $\frac{1}{2}$. Since the probability that player $j$ is not the proposer is $\frac{n-1}{n}$, it follows that $\mu_j = \frac{n-1}{2n}$. Plugging this in (3) and solving for $v$, we obtain $v = \frac{\delta}{n}$. Thus, conditional on being included in someone else's winning coalition, each player can expect to receive a larger piece of the cake if there are fewer players or if the players are more patient.

The fact that the proposer's share $1 - \frac{n-1}{2}v$ exceeds $v$ has two implications. First, since $v$ is the continuation payoff of all players, any proposer can receive $v$ if she passes the opportunity to make an offer or makes an offer that is unacceptable to at least $\frac{n-1}{2}$ others. But since $1 - \frac{n-1}{2}v > v$, these alternatives yield a lower payoff to the proposer. Hence agreement is immediate in equilibrium.[16] Second, there is a gain from proposing since $v$ is the maximum payoff a responder can receive while the proposer receives $1 - \frac{n-1}{2}v$. The difference between the proposer's cake share and the cake share of any winning coalition member, given by $1 - \frac{n-1}{2}v - v = 1 - \frac{n+1}{2}\frac{\delta}{n}$, is the *proposer's surplus*. The fact that proposer's surplus is positive is also known as *formateur advantage*. Note that the proposer's surplus is increasing in $n$ and decreasing in $\delta$. Clearly as the players become more patient, the proposer has to give up a larger share of the cake to have her proposal accepted. The fact that the proposer's surplus is increasing in the number of players is less obvious. On the one hand, when there are more players, each player expects to receive a smaller share conditional on not being the proposer. On the other hand, there are now more players to include in the winning coalition. The former effect dominates since the proposer is able to play off players against each other. In fact, as the number of players approaches infinity, the proposer captures $1 - \frac{\delta}{2}$, which is almost half of the cake.[17]

The equilibrium characterization established so far is for a bargaining game with simple majority rule. Simple majority is a special case of a $q$-quota rule where agreement on a

---

[16]In fact, in writing (2), we have already implicitly assumed that there is immediate agreement. It is straightforward to show that there is no equilibrium with delay. In a more general setting, Banks & Duggan (2000) show that when $\delta_i < 1$ for all $i$, agreement is reached immediately in every equilibrium. See section 4.1.

[17]More generally, when utilities are concave the proposer captures at least $1 - \alpha$ of the surplus for any $\alpha$-quota rule, irrespective of the number of players (see Harrington (1990a)).

proposal requires approval by $q$ players. As BF point out, the equilibrium characterization directly extends to any $q$-quota bargaining game. When selected as the proposer, any player offers $v$ to $q-1$ other players. By arguments similar to above, $\mu_j = \frac{n-1}{n}\frac{q-1}{n-1}$. Consequently, $v = \frac{\delta}{n}$ for all $q$. While the continuation payoffs do not depend on the agreement rule, the proposer's surplus $1 - q\frac{\delta}{n}$ does and is decreasing in $q$ as expected.

Notice that we defined $v$ to be the continuation payoff discounted back to the current period. Thus at the beginning of the first period, the expected payoff of each player is $\frac{1}{n}$. This makes sense: total size of the cake is 1, agreement is reached immediately, and by symmetry, each player receives a fraction $\frac{1}{n}$ of the cake in expected terms. Although the symmetric equilibrium payoff is unique, there are a continuum of equilibrium strategies, including asymmetric ones, that support it. As an example, consider a three player game. In one equilibrium, when selected as the proposer, player 1 includes player 2 in her coalition, player 2 includes player 3, and player 3 includes player 1. In another equilibrium, all players randomize and include each of the other players with equal probability. This means there is no hope of uniqueness of equilibrium strategies,[18] but perhaps one can hope for uniqueness of equilibrium payoffs without imposing the symmetry restriction. We now turn to this and other extensions.

## 3. ASYMMETRIC PLAYERS

### 3.1. Recognition Probabilities and Discount Factors

Suppose now players are heterogeneous with respect to their recognition probabilities and discount factors. Let $p_i \in [0,1]$ and $\delta_i \in [0,1)$ denote player $i$'s recognition probability and discount factor respectively, and assume $\sum_{i=1}^{n} p_i = 1$. Equations (1) and (2) corresponding to the $q$-quota game become

$$r^j \in \operatorname{argmin}_{r' \in [0,1]^n} \sum_{i \neq j}^{n} r_i' v_i \text{ subject to } \sum_{i \neq j} r_i' = q-1 \qquad 4.$$

and

$$v_j = \delta_j [p_j (1 - \sum_{i \neq j} r_{ji} v_i) + \sum_{i \neq j} p_i r_{ij} v_j] \qquad 5.$$

for all $j$, where $r^j = (r_{j1}, \ldots, r_{jn})$ solves (4) for all $j$. Analogously, $v = (v_1, \ldots, v_n)$ is an SSP equilibrium payoff vector if and only if it satisfies (5) for all $j$.

There are two special cases where the solution to (4) does not depend on $v$. When $q = 1$, i.e. under random dictatorship, each player includes every other player with probability 0. It immediately follows that for all $j$

$$v_j = \delta_j p_j. \qquad 6.$$

When $q = n$, i.e. under unanimity rule, each player includes every other player with probability 1. Substituting $r_{ij} = r_{ji} = 1$ for all $i,j$ in (5), we obtain

$$v_j = \frac{a_j}{1 + \sum_{i=1}^{n} a_i} \qquad 7.$$

for all $j$ where $a_j = \frac{\delta_j p_j}{1 - \delta_j}$.

---

[18]Çelik & Karabay (2016) add veto players to the BF model to get unique equilibrium strategies.

Matters become complicated for all other $q$. This is because when a proposer has multiple equal cost coalitions she can choose from, the choice she makes does not affect her payoff but may affect the payoffs of others. As a result, contraction mapping theorem is not applicable since the payoff vector $v$ is a fixed point of a correspondence, not a function.[19]

Eraslan (2002) establishes the uniqueness of the SSP payoff vector by first establishing certain monotonicity properties of the equilibrium payoffs,[20] and then using these monotonicity properties to rule out the existence of two different equilibrium payoffs. Intuitively, the idea is as follows. Since each proposer needs at least one other vote but not all other votes, if player $i$ included less often in others' winning coalitions in the equilibrium with payoff vector $v$ compared to the equilibrium with payoff vector $\bar{v}$, then another player $j$ must be included more often. The fact that player $i$ is included less often and player $j$ more often makes $i$'s vote relatively cheaper and $j$'s vote relatively more expensive. But then it is optimal for others to include player $i$ more often, not less often. This argument is not complete because $i$'s and $j$'s payoffs also depend on the costs of their coalitions when they are the proposers. The monotonicity properties allow us to enumerate players in the order of payoffs and use this enumeration to establish a link between the cost of coalitions for player $i$ and player $j$ for a given equilibrium. That link, in turn, allows us to limit the changes in the equilibrium payoffs across equilibria arising from the changes in the cost of coalition partners. Together with the fact that the probability of being included in others' coalitions move in the opposite direction of the changes in payoffs, this limit implies a contradiction to the possibility of two different equilibrium payoff vectors.

Importantly, the payoff $v_q$ of the $q$-th cheapest player and the cost of her coalition, denoted by $w_q = \sum_{i \neq q} r_{qi} v_i$, play an important role in establishing the links between the costs of coalitions of any two players in the proof above. These two variables also play a crucial role in the alternative proof of uniqueness due to Kalandrakis (2015). Specifically, he shows that the knowledge of the $(w_q, v_q)$ pair together with the recognition probabilities and the discount factors is sufficient to recover a payoff vector $v'(w_q, v_q)$ as a linear function of $w_q$ and $v_q$.[21] The coefficients of $w_q$ and $v_q$ in player $i$'s payoff $v_i'(w_q, v_q)$ depend on whether $i$ can be classified as a low cost player who is always included in others' coalitions (i.e. $v_i'(w_q, v_q) < v_q$), or a high cost player who is never included in others' coalitions (i.e. $v_i'(w_q, v_q) > v_q$). If neither is true, then $i$ is classified as a medium cost player (i.e. $v_i'(w_q, v_q) = v_q$). This payoff vector is an equilibrium payoff vector if and only if $\sum_i v_i'(w_q, v_q) = 1$ and

$$w_q = \sum_{i \in L(w_q, v_q)} v_i'(w_q, v_q) + (q - 1 - \#L(w_q, v_q))v_q \qquad 8.$$

where $L(w_q, v_q) = \{i \in N : v_i'(w_q, v_q) < v_q\}$ is the set of players classified as low cost. In other words, the $(w_q', v_q')$ pair corresponding to $v'(w_q, v_q)$ must be equal to $(w_q, v_q)$. Thus, to show that the equilibrium payoff vector is unique boils down to showing the existence of a unique solution to a piecewise linear bivariate system of equations. Kalandrakis (2015) shows that this is indeed the case.

---

[19]This is because the probability $r_{ij}$ of $j$ being included in $i$'s coalition can take a continuum of values for a given $v$ when $q \in \{2, \ldots, n-1\}$, all of which yield a different payoff to player $j$.

[20]See Theorem 4 in Eraslan (2002) and Proposition 1 in Eraslan (2016). The latter extends the result to allow risk averse players, and adds the proof of a missing step.

[21]There is a one-to-one mapping between the variables $S$ and $r$ used in Lemma 1 of Kalandrakis (2015) and the $(w_q, v_q)$ pair. Specifically, $S = 1 - w_q - v_q$ and $r = v_q$.

Having established uniqueness of the equilibrium payoffs, a natural question to ask is: how does the payoff a player vary with her recognition probability or her discount factor? One implication of the monotonicity properties established in Eraslan (2002) is that when players have identical discount factors, their payoffs are nondecreasing in the recognition probabilities, i.e. if $\delta_i = \delta$ for all $i$, then $p_i \leq p_j$ implies $v_i \leq v_j$. This property, in turn, implies that the assumption in BF that equilibrium is symmetric is not restrictive since the model they consider involves symmetric players.

Another implication of the monotonicity properties is that when players have identical recognition probabilities, their payoffs are nondecreasing in the discount factors, i.e. if $p_i = \frac{1}{n}$ for all $i$, then $\delta_i \leq \delta_j$ implies $v_i \leq v_j$.[22] This immediately implies that conditional on being included in someone else's winning coalition, a player's cake share (weakly) increases as she becomes more patient. Not so obvious is the implication that a player's cake share (weakly) increases as she becomes more patient conditional on being the proposer as well. To see, enumerate the players so that $\delta_1 \leq \ldots \leq \delta_n$. Then, the cost of coalition for any player $i \geq q$ is equal to $w_q$, and therefore conditional on being the proposer, all players $\{q, \ldots, n\}$ receive the same cake share. In addition, for any $i, j \leq q$, we have $w_i + v_i = w_j + v_j$. Thus, for any $i < j \leq q$, the fact that $v_i \leq v_j$ implies that $w_i \geq w_j$. This, in turn, implies that conditional on being the proposer, $i$'s cake share is smaller than that of $j$.

But these do not tell the whole story. The ex-ante expected payoffs of a player at the beginning of the game could be decreasing as she becomes more patient. To see, consider the following example. There are 3 players with linear preferences, equal recognition probabilities, and the discount factors are $\delta_1 = 0.5$, $\delta_2 = 0.65$ and $\delta_3 = 0.95$. Under majority rule, the equilibrium payoffs are $v_1 = 0.194$, $v_2 = 0.223$ and $v_3 = 0.255$. Let $\tilde{v} = (\tilde{v}_1, \tilde{v}_2, \tilde{v}_3)$ denote the undiscounted payoff vector, i.e. the ex-ante expected payoffs at the beginning of the game. Then $v_i = \delta_i \tilde{v}_i$ for all $i$ and we have $\tilde{v}_1 > \tilde{v}_2 > \tilde{v}_3$.

Kawamori (2005) shows that this is more general: if sufficiently patient players with linear preferences have similar recognition probabilities, then player $i$'s payoff is given by

$$v_i = \big(\sum_{j=1}^{n} \delta_j^{-1}\big)^{-1} \qquad\qquad 9.$$

in any $q$-quota game with $q \in \{2, \ldots, n-1\}$. Consequently, all players receive the same payoff which does not depend on $q$ or the recognition probabilities. This, in turn, implies that the ex-ante expected payoffs are decreasing in discount factors in any non-unanimity game with sufficiently patient players who have similar recognition probabilities.[23]

From the discussion above, we know that a more patient player is better off conditional on being the proposer, and conditional on being included in someone else's coalition. In light of this, it might be puzzling that a more patient player could be worse off ex-ante.

---

[22]Note that here we are talking about discounted payoffs consistent with the characterization above. By contrast, the payoffs in Eraslan (2002) are undiscounted, and consequently, Corollary 2 in that paper is not correct. See Corollary 2 in Eraslan (2016) for a corrected statement and footnote 3 in the same paper for further details and acknowledgments.

[23]It is important to notice the necessity of non-unanimity rule and sufficiently patient players for this result. Indeed, iis straightforward from (6) and (7) that $v_j/\delta_j$ is increasing in $\delta_j$ for when $q = 1$ or $q = n$. Kawamori (2004) fully characterizes the equilibrium payoffs of a three player game with equal recognition probabilities for all triplets of discount factors. He also provides numerical examples which illustrate that ex-ante payoff of a player could be increasing in her discount factor under non-unanimous rules in line with what intuition would suggest.

But notice that a player's equilibrium payoff is determined by two endogenous factors: the cost of her coalition when she is the proposer and the probability of being included in others' coalitions. It turns out that being more patient could make a player a less attractive coalition partner, and hence decrease the probability of being included in the coalition of others. This decrease in the probability of being invited to join others' coalitions could then make a player worse off ex-ante as she becomes more patient.

## 3.2. Weighted Voting and Coalitional Bargaining

The analysis up to this point assumed that players have equal voting weights, but in legislative settings it is also natural to allow different players to have different weights in the voting over approval of a proposal. This generalization is of interest from the point of view of other applications as well. In corporate bankruptcies governed by Chapter 11 in the U.S., creditors who are owed more have larger voting weights. One share, one vote is standard in corporate law and corporate governance, which translates into larger shareholders having larger vote shares. Other examples include the European Union Council of Ministers and the International Monetary Fund.

One way to represent heterogeneous voting weights under $q$-quota bargaining game is by introducing a new variable $\omega_i$ to capture voting weight of player $i$, and require that proposal $x$ is accepted if $\sum_{i \in A(x)} \omega_i \geq q$ where $A(x)$ is the set of players who vote to accept $x$.[24] The combination $[q; \omega_1, \ldots, \omega_n]$ is a *representation* of the voting stage within the bargaining game. Given representation $[q; \omega_1, \ldots, \omega_n]$, a coalition $C \subset N$ is a *winning coalition* if and only if $\sum_{i \in C} \omega_i \geq q$.

One complication with weighted voting games is that there may be multiple ways to represent the same bargaining situation. For example, the representations $[4; 3, 2, 2]$ and $[2; 1, 1, 1]$ induce the same set of winning coalitions. A representation is *homogenous* if all winning coalitions have the same total voting weight. Homogeneous representations are preferred since they provide a more accurate representation of the voting power of the players. In the example above, all three players are in a symmetric position since any two of them can form a winning coalition but the non-homogeneous representation $[4; 3, 2, 2]$ is not symmetric. Unfortunately, not all voting games have a homogenous representation,[25] and some games have multiple homogenous representations.[26]

Because of these complications, it is convenient to describe heterogeneous voting weights via the (unique) collection of winning coalitions they induce. Eraslan & McLennan (2013) (EM1) follow this approach. For each $i$ there is a collection $\mathcal{S}_i = \{S_{i1}, \ldots, S_{iK_i}\}$ of subsets of $N$, called *winning coalitions for $i$*. When $i$ is the proposer, agreement is reached if the set of agents voting in favor is an element of $\mathcal{S}_i$. As a matter of convention assume that $i$ is a member of every coalition in $\mathcal{S}_i$. For each $i$ and $S_{ik} \in \mathcal{S}_i$ let $\tilde{S}_{ik} \in \{0, 1\}^n$ be the vector whose $j^{\text{th}}$ component is 1 if $j \in S_{ik}$ and 0 otherwise. Let $\mathcal{Y}_i \subset [0, 1]^n$ be the convex hull of $\{\tilde{S}_{ik} : S_{ik} \in \mathcal{S}_i\}$. Equation (4) describing optimal proposals now becomes

$$r^j \in \operatorname{argmin}_{r' \in [0,1]^n} \sum_{\substack{i=1 \\ i \neq j}}^n r'_i v_i \ \text{ subject to } \ r' \in \mathcal{Y}_i. \qquad\qquad 10.$$

---

[24] See Snyder et al. (2005) and Montero (2017).

[25] See pages 1291-1292 in Weber (1994) for an example.

[26] For example, [5;3,2,2,1] and [7;4,3,3,1] are homogeneous representations of the same game.

EM1 call any pair $(v_1, \ldots, v_n)$ and $(r^1, \ldots, r^n)$ satisfying (10) and (5) a *reduced equilibrium*[27] and show that if each $\mathcal{S}_i$ is *monotonic*,[28] then each SSP equilibrium induces a reduced equilibrium, and each reduced equilibrium is induced by at least one SSP equilibrium. EM1 then use the theory of the fixed point index to show that SSP equilibrium payoffs are unique. The proof involves showing that there is a unique connected component of equilibria all sharing the same vector of equilibrium payoffs.

An interesting question to ask is in this framework is: how does the payoff of a player vary with her voting power? The coalitional approach is not amenable to answer or even ask this question. This is because the voting power is measured by the collection $\mathcal{S}_i$ and it is not clear whether any two arbitrary collections can be ranked in terms of the voting powers they imply. Because of that, it is useful to go back to the weighted voting approach to tackle this question despite the challenges discussed earlier.

Snyder et al. (2005) (STA) resolve the problems created by nonhomogeneity by analyzing a "replicated" game with no discounting. Specifically, in the weighted voting game with representation $[q; \omega_1, \ldots, \omega_n]$ is replicated $r$ times there are $rn$ players $r$ of which have voting weight $\omega_i$ and recognition probability $p_i$ for each $i = 1, \ldots, n$. STA consider two possibilities for the recognition probabilities. In the first case, the recognition probabilities are proportional to the voting weights which is a realistic assumption given the empirical patterns.[29] In this case, STA show that, for a sufficiently large $r$, each player's payoff is proportional to her voting weight, i.e. it equals her recognition probability. In the second case, each player has the same recognition probability. This is a more useful assumption for answering how a player's payoff varies with her voting power holding fixed her proposal making power. STA show that there are two types of equilibria in this case. When the voting weights are not too skewed, each player's payoff is proportional to her voting weight as before. But when the voting weights are highly skewed, it is possible that voters with low voting weight can have a disproportionately high payoff due to their proposal powers. This is reminiscent of Kalandrakis (2006) who shows that proposal power is in general much more significant in determining a player's payoff than her voting power.[30] At the extreme, when one player has veto power, that player extracts all the surplus as the players become perfectly patient.[31]

---

[27]Eraslan & McLennan (2013) also allow that no proposer is selected in any period with probability $p_0 \geq 0$. That generalization would modify (5) by adding $\delta_j p_0 v_j$ to the right hand side.

[28]$\mathcal{S}_i$ is *monotonic* if $C \in \mathcal{S}_i$ and $C \subset C'$ implies $C' \in \mathcal{S}_i$. Any $q$-majority rule induces monotonic collections of winning coalitions which in addition satisfy for any $i, j \in N$ and $S \in \mathcal{S}_i$, either $S \in \mathcal{S}_j$ or $S \cup \{j\} \setminus \{i\} \in \mathcal{S}_j$.

[29]See, for example, Diermeier & Merlo (2004).

[30]Ali et al. (2018) argue that the predictability of recognition process is the main source of bargaining power. In particular, under a $q$-quota rule the current proposer can extract all surplus if $q - 1$ players can be ruled out as the next period proposers. Related observations are made in Breitmoser (2011) and Kim (2018a,b). The former assumes that the order of several first proposers is known in advance, while in the latter the players whose proposals have been previously rejected cannot propose again until all other players have also proposed. Bernheim et al. (2006) consider a game in which the policy is implemented at a known future date and the order of proposers is known. In their model, the last player to propose has an almost dictatorial power.

[31]Winter (1996) establishes this result in a legislative bargaining model in which the winning coalitions are exogenously and randomly chosen and the division of the surplus within the coalition is endogenous. He states that all the results hold true a BF style legislative bargaining model with endogenous winning coalitions when the players have equal recognition probabilities.

In summary, STA find support for "Gamson's Law"[32] in games replicated a finite (but potentially large) number of times when the recognition probabilities are proportional to the voting weights, but not when the recognition probabilities are equal. This raises the question: does Gamson's Law continue to hold with proportional recognition probabilities in small (non-replicated) games? Montero (2017) answers this question, strengthening an earlier result of Montero (2006).[33]

Given a weighted voting game $[q; \omega_1, \ldots, \omega_n]$, let $W^* = \text{argmin}_{S \in \mathcal{S}} \sum_{i \in S} w_i$ denote the collection of winning coalitions with minimum total weight. The collection $W^*$ is *weakly balanced* if there exists $\lambda_S \geq 0$ for each $S \in W^*$ such that $\sum_{\{S \in W^* : i \in S\}} \lambda_S = 1$ for all $i \in N$.[34] If the equilibrium payoffs of the players are proportional to their voting weights, then the set of coalitions that are likely to form belongs to $W^*$ since they are the cheapest. Balancedness condition captures the idea that no player should be systematically overrepresented or underrepresented in the set of cheapest coalitions. An implication of Montero (2006) is that weak balancedness of $W^*$ is sufficient for the existence of an equilibrium with proportional payoffs. Montero (2017) shows that it is also necessary.[35]

It is worth noting that the main result of Montero (2017) holds irrespective of the discount factor as long as it is the same for all players. As noted earlier, heterogeneous discount factors could result in more patient players being worse off ex-ante. A similar result arises when the players are asymmetric with respect to their utility functions: unlike in bilateral bargaining,[36] a less risk averse player could be worse off ex-ante in a multilateral bargaining with unanimity rule. We now turn to discussing this result.

### 3.3. Preferences

So far we assumed that players have identical and risk neutral preferences. Harrington (1990b) considers a legislative bargaining model in which players are risk averse and heterogeneous with respect to risk aversion. Specifically, the utility of player $i$ if proposal $x$ is implemented in period $t$ is now given by $\delta^t u_i(x_i)$ where $u_i$ is increasing, twice differentiable, concave and normalized so that $u_i(0) = 0$ and $u_i(1) = 1$. In order to rank players according to their risk aversion, assume that $u_{i+1}(\cdot) = h_i(u_i(\cdot))$ where $h_i : [0, 1]$ is increasing,

---

[32]Gamson's Law is a hypothesis that was first proposed in Gamson (1961) and states that "Any participant will expect others to demand from a coalition a share of the payoff proportional to the amount of resources which they contribute to a coalition" without specifying what "payoff" or "resource" mean. Following Ansolabehere et al. (2005), here we take "payoff" to mean ex-ante equilibrium payoff and "resource" to mean "voting weight".

[33]See also Kurz et al. (2014). These three papers show a connection between equilibrium payoffs and nucleolus – a solution concept in cooperative game theory introduced by Schmeidler (1969). Some other cooperative solution concepts include the core, stable set, bargaining set, and Shapley value. See Osborne & Rubinstein (1994) and references therein.

[34]Bondareva (1963), Shapley (1967) and Scarf (1967) show that balancedness plays an important role in the existence of core points. A collection of weights $\{\lambda_S\}_{S \subseteq N}$ is *balanced* if $\lambda_S \geq 0$ for all $S \subseteq N$ and for each $i \in N$ holds $\sum_{\{S \subseteq N : i \in S\}} \lambda_S = 1$. If $W^*$ is weakly balanced, then the collection of all winning coalitions $\mathcal{S}$ is also weakly balanced. Therefore, Montero's condition implies that $\sum_{S \in \mathcal{S}} \lambda_S \leq 1$ for *some* balanced collection of weights, while the Bondareva-Shapley condition for non-empty core requires this to hold for *any* balanced collection of weights.

[35]Note that this condition is logically independent of homogeneity: it fails for some homogeneous representations, such as $[5; 3, 2, 2, 1]$, and holds for some games that have no homogeneous representation, such as $[5; 2, 2, 2, 1, 1, 1]$.

[36]See Roth (1985).

twice-differentiable and concave function for all $i$ and satisfies $h_i(0) = 0$ and $h_i(1) = 1$. The assumption that player $i+1$'s utility function is a concave transformation of player $i$'s utility function ensures that player $i + 1$ more risk-averse than player $i$. Players are symmetric with respect to their discount factors, recognition probabilities and voting weights.[37]

Equations (1) and (2) characterizing SSP equilibrium payoff vector $v = (v_1, \ldots, v_n)$ of the $q$-quota game become

$$\min_{r' \in [0,1]^n} \sum_{i \neq j}^{n} r_i' u_i^{-1}(v_i) \text{ subject to } \sum_{i \neq j} r_i' = q - 1 \qquad 11.$$

and

$$v_j = \delta[\frac{1}{n} u(1 - \sum_{i \neq j} r_{ji} u_i^{-1}(v_i)) + \sum_{i \neq j} \frac{n-1}{n} r_{ij} u_j^{-1}(v_j)] \qquad 12.$$

for all $j$, where $r^j = (r_{j1}, \ldots, r_{jn})$ solves (11) for all $j$.

Comparing players' payoffs is subtle because they have different preferences. Harrington (1990b) first considers unanimity rule and shows that $u_1^{-1}(v_1) > \ldots > u_n^{-1}(v_n)$, that is, less risk-averse players receive higher cake shares conditional on being included in others' coalitions.[38] He then shows that the ex-ante equilibrium probability distributions over cake shares can be ranked by first-order stochastic dominance. This allows him to show that less risk-averse players receive a higher cake share in expected terms as well, consistent with the results from alternating-offer bilateral bargaining. Intuitively, when player $i$ is not the proposer, she faces the choice between a certain outcome and a non-degenerate lottery which she can obtain by accepting the proposal and rejecting it respectively. More risk-averse players require a lower cake share to prefer acceptance to rejection, and therefore, being relatively risk-averse is disadvantageous under unanimity rule.

For $q \in \{2, \ldots, n - 1\}$, results are restricted to the case when preferences are not too diverse. Harrington (1990b) shows that for each $q$ there exists an SSP equilibrium, and each player has identical reservation price, i.e. $u_i^{-1}(v_i) = u_j^{-1}(v_j)$ for all $i, j$. Thus, the degree of risk aversion does not affect a player's cake share conditional on being included in someone else's coalition. Consequently, a more risk-averse player has a higher chance of being included in others' coalitions. This is because players have different degrees of risk aversion and face lotteries with the same outcomes, and so it must be that the probabilities of these outcomes vary. Since the probability of being a proposer is fixed, then what must be different is the probability of being included in others' coalitions. The lotteries faced by players are again ranked in terms of first order stochastic dominance, but now more risk-averse players face better lotteries. It is worth noting that analogous to Kawamori (2005) results associated with discounting discussed in section 3.1, these results are established only when then the preferences are heterogeneous but not too heterogeneous.

An alternative way to introduce heterogeneity in preferences is through externalities. Calvert & Dietz (2005) consider a variant of the BF model in which player $i$'s payoff from proposal $x = (x_1, \ldots, x_n)$ is given by $\sum_{j \in N} a_{ij} x_j$ where $a_{ii} = 1$ and $a_{ij} \leq 1$ for all $i, j$. Thus each player places the highest value in her own cake share, but she might also value

---

[37]Technically, Harrington (1990b) does not have discounting. Instead, $\delta$ is the exogenous probability of breakdown. He motivates this interpretation as a motivation for having a role for risk preferences. But as in bilateral bargaining, the two interpretations are strategically equivalent. See, for example, Osborne & Rubinstein (1990, Section 4.2).

[38]Though Harrington (1990b) does not show it, this result would also hold for $q = 1$

the shares received by other players to varying degrees.[39] They use this model to show that party-like behavior is possible even when two players are not in the same party.

## 4. ALTERNATIVE FEASIBLE POLICIES

Common with the BF model, the literature discussed in Section 3 focuses on legislative bargaining where the decision is entirely *distributive* and players bargain over a *fixed* amount of a *private* good. In this section, we discuss the way the literature has extended the BF model by relaxing the three assumptions emphasized in the previous sentence.

### 4.1. Spatial Bargaining

The main limitation of the distributive approach is that it does not inform us about the relationship between legislative behavior and the ideological positions of the legislators. With distributive policies, one legislator's gain is another legislator's loss, which allows proposers to target other players and make the members of the winning coalition exactly indifferent between accepting and rejecting a proposal. This is no longer possible when bargaining over ideological policy as players can have preferences that are aligned at least on some subset of the policy space. The relative position of legislators in the ideological space becomes one of the main determinants of equilibrium outcomes.[40]

The first paper to extend the BF model to spatial bargaining is Baron (1991), which examines the case of a two-dimensional set of alternatives, three voters with quadratic preferences with an application to government formation in a parliamentary system. Richer policy spaces are introduced by Banks & Duggan (2000) and Jackson & Moselle (2002). The bargaining process is the same as in the BF model. The main difference is that the feasible set $X$ is now assumed to be a non-empty, compact, and convex subset of $\mathbb{R}^d$ with $d \geq 1$.[41] Player $i$'s utility from implementing policy $x$ in period $t$ is given by $(1 - \delta_i^{t-1})\bar{u}_i + \delta_i^t u_i(x)$ where $\bar{u}_i$ is the payoff from the status quo policy $\bar{x}$.[42]

Letting $\Delta(X)$ denote the set of all probability measures on $X$, a stationary behavioral strategy for each player consists of a pair $(\pi_i, A_i)$, where $\pi_i \in \Delta(X)$ is a random offer to make as a proposer, and $A_i$ is the set of proposals that $i$ votes to accept as a responder. Fix an equilibrium strategy profile $\sigma$, an let $v_i$ denote the continuation payoff of player $i$ under this strategy profile. Assuming player $i$ accepts an offer when indifferent, we have $A_i = \{x \in X : u_i(x) \geq v_i\}$. Recall from section 3.2 that $\mathcal{S}_i$ is the collection of winning coalitions for player $i$. Thus an offer $x$ is *acceptable when $i$ is the proposer* if and only if $x \in A_j$ for all $j \in C$ for some $C \in \mathcal{S}_i$. Let $A^i$ denote the set of all offers acceptable when $i$ is the proposer. When selected as the proposer, player $i$ either offers the best alternative from $A^i$, or delays by proposing a policy in $X \setminus A^i$. Thus the equilibrium condition on the optimal proposals analogous to (10) is

$$\pi_i(\mathrm{argmax}_{x \in A^i} u_i(x)) = 1 \qquad\qquad 13.$$

---

[39]Choate et al. (2018a,b) use this idea to study the role of partisanship and empowerment of leaders respectively.

[40]Distributive models in which legislators cannot be targeted due to the availability of a limited number of policy levers can be viewed as spatial models. See, for example, Martin (2018).

[41]See Herings & Houba (2016) for a model without convexity.

[42]By setting $\bar{u}_i = 0$, $u_i(x) = x_i$ for all $i$, $d = n$ and $X$ to be the $(n-1)$-dimensional unit simplex, we obtain the BF model. One-dimensional models of bargaining are obtained by setting $d = 1$.

if $\sup_{x \in A^i} u_i(x) > v_i$ resulting in agreement; $\pi_i(X \setminus A^i) = 1$ when $\sup_{x \in A^i} u_i(x) < v_i(\sigma)$ resulting in delay; and $\pi_i(\text{argmax}_{x \in A^i} u_i(x)) + \pi_i(X \setminus A^i) = 1$ otherwise. Of course, in equilibrium, the continuation payoffs must be consistent with the equilibrium strategies:

$$v_i = \delta_i \sum_j p_j \left[ \int_{A^j} ((1 - \delta_i)\bar{u}_i + \delta_i u_i(x))\pi_j(\mathrm{d}x) + \pi_j(X \setminus X_j)((1 - \delta_i)\bar{u}_i + v_i) \right]. \quad 14.$$

Banks & Duggan (2000) (BD) assume that $\bar{u}_i = 0$ and $u_i(x) \geq 0$ for all $i$ and $x \in X$, and show the existence of an SSP equilibrium with no delay. They also provide conditions under which pure strategy equilibria exist. Unlike in distributive models, it is no longer natural to assume that no agreement is the worst outcome in spatial models. Banks & Duggan (2006) show that if some players favor the status quo, then it is possible to have delays in equilibrium under very specific circumstances.[43] In that case, the only possible outcome is the status quo itself. Since such delays can be avoided by proposing the status quo, there always exists a stationary equilibrium in which every proposal made passes as well. Thus there is typically immediate agreement in this framework.

It is possible to establish the uniqueness of SSP equilibria in one-dimensional models under certain conditions.[44] It is not surprising that equilibria in multi-dimensional models are not generally unique even when utility functions are quadratic. Using the model of Baron (1991), BD provide an example that illustrates this point. Not so obvious is the observation that the equilibrium payoffs need not be unique either. BD provide a two-dimensional, three-player, majority-rule example in which multiple equilibrium payoffs arise.[45]

An important focus of the spatial bargaining literature is the equivalence of the equilibrium outcomes and the core. A policy $x$ is in the *core* if there is no winning coalition $C$ and another policy $x'$ with $u_i(x') > u_i(x)$ for all players $i \in C$. BD show that in certain environments a policy $x$ is the unique outcome of some no-delay SSP equilibrium if and only if $x$ belongs to the core.[46] Specifically, when players are perfectly patient and utility functions are strictly quasiconcave, core equivalence holds if the policy space is one-dimensional or if some individual has a veto power.[47] Intuitively, if policy $x$ is not in the core, then there exists a winning coalition $C$ and alternative policy $y$ such that $u_i(y) > u_i(x)$ for all $i \in C$. If players in $C$ are sufficiently patient, they prefer to veto $x$ in anticipation that $y$ will be eventually proposed. When the collection of winning coalitions is proper,[48] coalition $C$ can

---

[43]If all players are risk averse or if limited transfers are possible, then delay is only possible if the status quo lies in the core.

[44]Cho & Duggan (2003) show that the no-delay SSP equilibrium is unique for all discount factors when utility functions are quadratic. With fixed orders of proposals, Cardona & Ponsatí (2007) show uniqueness when $\delta < 1$ and utility functions are symmetric around the ideal points, or when players are sufficiently patient. They also prove that equilibrium outcomes converge to a unique limit as $\delta \to 1$. With random recognition, Cardona & Ponsatí (2011) achieve uniqueness when utility functions have the same shape but different ideal points, and Predtetchinski (2011) shows that the set of accepted proposals converges to a single policy with arbitrarily patient players.

[45]See examples 2 and 3 in BD respectively.

[46]As mentioned in the introduction, the existence of core points typically requires placing restrictions on policy space and/or preferences. Nakamura number (Nakamura (1979)) plays an important role in many results of this kind. Again, we refer the reader to Austen-Smith & Banks (1999) for a detailed account.

[47]Core equivalence fails in one-dimensional models when recognition probabilities follow a Markov process. See Herings & Predtetchinski (2010).

[48]A collection $\mathcal{S}$ of winning coalitions is *proper* if $C \in \mathcal{S}$ implies $N \setminus C \notin \mathcal{S}$. Any $q$-quota rule with $q \geq \frac{n+1}{2}$ induces a proper $\mathcal{S}$.

indeed prevent $x$ to pass. With quadratic utility functions, if the voting rule is "strong" (e.g., majority rule with an odd number of individuals), and if a core point exists, then core equivalence once again obtains.

Another early and influential paper extending BF model to allow spatial bargaining is Jackson & Moselle (2002) (JM). They study a model in which legislators bargain over both distributive and ideological issues.[49] Their setup allows the distributive issue to serve as a compromise tool in choosing an ideological policy, similar to the role that transfers play in other social choice settings. This is especially important when the players' preferences vary in the intensities over the ideological issue, increasing the opportunities for compromise.

Players have separable preferences over the two dimensions which is single peaked over the ideological dimension and is strictly increasing in her share on the distributive dimension. As in BF, players are randomly chosen to make offers, and a proposal is accepted if a majority votes for it, otherwise the process is repeated. A novel feature of the JM model is that players can have partial agreements. In particular, the proposer can make offers on either issue or both issues. If agreement is reached on both issues, the game ends. If an agreement is reached on one issue only, the process is repeated with the restriction that the remaining proposals can consider the remaining issue only.

Focusing on a class of SSP equilibrium,[50] JM show that an accepted proposal distributes the private good among an exact majority in any equilibrium. Agreement is reached immediately on both issues if players have concave utilities and $\delta < 1$, or have strictly concave utilities.[51] Intuitively, it is not optimal to separate the decisions on the two issues even if the preferences are separable in order to take advantage of the distributive issue as a compromise tool. When players are sufficiently patient and utility functions are concave, each player $i$ is *excluded from a proposal* with positive probability, in a sense that following such proposal player $i$ would strictly prefer to delay if she could. This means that with positive probability an equilibrium proposal wins the approval of ideologically disjoint legislators.[52]

JM show that the equilibrium outcomes are Pareto-inefficient when utility functions over ideology are strictly concave and players are sufficiently patient. This suggests that political institutions may play a role of weeding out such inefficiencies. In particular, a political party that has a simple majority can provide a strict improvement for the party members over the non-party equilibrium outcomes. JM consider examples where predictions can be made concerning the structure of stable parties and the equilibrium policies that would emerge in the presence of parties.

---

[49]Austen-Smith & Banks (1988) and Diermeier & Merlo (2000) are two earlier papers in which players bargaining over ideological policy can make transfers.

[50]They extend the definition of stationary strategies to allow strategies to depend on the previously approved proposal following an agreement on one of the issues.

[51]If the players are perfectly patient, then the distributive policy is accepted immediately in every equilibrium but delays are possible on the ideological issue. Immediate agreement on the distributive issue arises due to the risk of being excluded from the distributive proposal. Median policy is accepted at some point later on.

[52]Christiansen et al. (2014) call such subsets of players *strange bedfellow coalitions* and provide an experimental evidence that the median player is often excluded from a proposal in a version of JM game with three players.

## 4.2. Stochastic Bargaining

An important aspect of the preceding analysis is that the set of agreements available to players does not change over time. As a consequence, delay is costly due to discounting. By contrast, when bargaining is over complex environments and a complete set of contingent contracts are not available, the set of agreements may change during negotiations. As a result, the players may find it optimal to delay agreement until uncertainty is resolved and more efficient contracts can be implemented. To allow for the possibility of such delays, we now relax the assumption that the cake to be divided is fixed and consider a stochastic bargaining framework introduced by Merlo & Wilson (1995) (MW1).[53]

At the beginning of each period, the size of the cake $s$ to be divided is determined according to an identically and independently distributed distribution $F$ with support $[0, \bar{s}]$. In all other aspects, the game is the same as the BF model. An important consequence of a stochastic cake process is the possibility of delays in agreement in equilibrium. This happens whenever some player perceives that a better agreement may be achieved by waiting. To allow for this possibility, let $\alpha(s)$ denote the endogenous probability that agreement is reached in a symmetric SSP equilibrium when the cake to be divided is of size $s$. Let $v$ denote the corresponding SSP payoff which is identical for all players by the symmetry assumption. In equilibrium, $\alpha(s) = 1$ if $s > qv$, $\alpha(s) = 0$ if $s < qv$ and $v$ must satisfy

$$v = \delta \int [\alpha(s) \frac{s}{n} + (1 - \alpha(s))v] \mathrm{d}F(s). \qquad 15.$$

Intuitively, conditional on agreement when the cake size is $s$, the expected payoff of any player is $\frac{s}{n}$ by symmetry, and in the event of a disagreement, each player receives her continuation payoff.

This is a special case of Eraslan & Merlo (2002) (EM2) who allow general recognition probabilities. When $q = n$, it is also a special case of MW1 and Merlo & Wilson (1998) (MW2). MW1 characterize both the set of subgame perfect (SP) payoffs and the set of stationary subgame perfect (SSP) payoffs for a general class of games in which the shape of the cake is relatively unrestricted. They give an example in which the unique SSP outcome is Pareto-inefficient. In this example, players reach immediate agreement even though their expected payoffs could be increased by waiting. MW2 restrict attention to case where the cake is a simplex of random size as in the model we consider in this section. They establish the uniqueness of SSP equilibrium and show that equilibrium is efficient even though it may involve delays. An important feature of their model is that the set of states in which agreement is reached doesn't depend on the proposer: when all the players have veto power, the interests of all players are aligned in their pursuit of the optimal time to agree.

By contrast, EM2 show that equilibrium need not be unique under majority rule. Furthermore, even when the equilibrium is unique, it need not be efficient. To illustrate this point consider the following example. There are 3 players, $\delta = 0.99$ and $F$ is uniform on $\{1, 2\}$. In the unique equilibrium under unanimity rule, agreement is reached only when

---

[53]Another way to relax the fixed cake size assumption is by endogenizing it. Baranski (2016b) assumes that players make costly contributions to the common surplus before bargaining over its division. In this model, the contributions are sunk costs and no player contributes in any SSP equilibrium. In Baranski (2016a), contributions are made after bargaining over the property rights on eventual surplus. In this case, some of the players included in the winning coalition make positive contributions while others do not. Therefore, timing plays a crucial role in providing incentives to make individual contributions to the cake size.

$s = 2$. By MW2, this is the only efficient outcome.[54] However, other outcomes are possible under other agreement rules. In particular, under majority rule, there are two other equilibria, one in which agreement is always reached, and another in which $\alpha(2) = 1$ and $\alpha(1) = 0.94$. Intuitively, when agreement only requires approval of $q < n$ players, in any equilibrium only $q$ players are included in a proposal. This creates an asymmetry between the players who are included and the players who are excluded. This asymmetry generates an incentive for players who are offered a positive share of the surplus today to agree even if the current level of surplus is small, since some of them (perhaps all) may be excluded from future agreements. Knowing that her proposal would be accepted, the same argument may then induce a proposer to make a proposal when the level of the realized surplus is small. In fact, the risk of being excluded gets larger as $q$ goes down resulting larger incentives to agree too soon. In the example above, when $q = 1$, there is a unique equilibrium in which agreement is always reached which is inefficient.

Stochastic bargaining has also been used to analyze fairness issues. Eraslan & Merlo (2017) study a model where players are heterogeneous with respect to the potential surplus they bring to the bargaining table. They show when the players are sufficiently patient, the equilibrium allocations are less equitable for more inclusive voting rules.

## 4.3. Bargaining Over Public Goods

When bargaining over distributive policies, legislators not only decide the allocation of private goods but can often allocate some amount to a public good. Volden & Wiseman (2007)[55] (VW) study an extension of the BF model in which proposals specify allocation $y$ to a single public good and private spending $x_i$ for each legislator $i$ such that $y + \sum_i x_i \leq 1$. Player $i$'s utility is $u_i(y, x) = \alpha x_i + \beta y$ where $\alpha > 0$ and $\beta > 0$ so the marginal rate of substitution between collective and particularistic goods is identical and constant for all players. The fact that both the private goods and the public good comes from the same finite resource makes the model distinct from that of Jackson & Moselle (2002).

VW show that there is a nonmonotonic relationships between preferences for public good spending and SSP equilibrium allocation to the public goods. In particular, a lower relative marginal value of the public good might result in higher equilibrium spending on the public good. The assumption of a constant marginal rate of substitution leads to two extreme results. First, when collective spending is positive, at most one player (the proposer) receives a private allocation; and second, when players become perfectly patient, either all or none of the budget is allocated to public spending. With these observations, Cardona & Rubí-Barceló (2014) extend VW model to quasi-linear preferences. They also study general $q$-quota rules.[56]

---

[54]Yıldırım (2018) shows that there can be inefficient early agreement even under unanimity rule when the recognition probabilities are endogenous.

[55]See also Volden & Wiseman (2008) correcting an error.

[56]Other papers that consider provision of public goods in legislative bargaining include Baron (1996), Besley & Coate (1998), LeBlanc et al. (2000), Battaglini & Coate (2007, 2008), Battaglini et al. (2012), Christiansen (2013), Bowen et al. (2014, 2017). Most of these papers study environments with endogenous status quo and we defer our discussion of them to our companion survey paper Eraslan et al. (2019).

## 5. ALTERNATIVE BARGAINING PROTOCOLS

In this section we discuss models with alternative bargaining protocols which allow players to compete for making proposals, make demands or make counterproposals. Before moving on to discussing these models, it is worth mentioning that in their seminal work, Baron & Ferejohn (1989) themselves consider a variant in which amendments to the current proposal can be made before voting.[57] This *open rule* version of their model provides a more equitable division of surplus among legislators compared to the usual *closed rule* which we focus on throughout this survey. Bargaining models with open rule are significantly more difficult to analyze and share similarities with the models featuring endogenous status quo. We postpone the discussion of the open rule legislative bargaining model to our companion survey paper focusing on legislative bargaining with endogenous status quo.

### 5.1. Endogeneous Proposer Selection Rules

As discussed earlier, (i) the BF model predicts that there is a positive proposer's surplus resulting in gains to being proposer; (ii) expected payoff of a player is increasing in her recognition probability when players have identical discount factors; and (iii) proposal rights play a significant role in determining political power. As a consequence, players have incentives to influence the selection of the proposer. This section is based on Yıldırım (2007) who incorporates an endogenous proposer selection process in legislative bargaining.[58]

The model is an extension of the $n$-player BF model with heterogeneous discount factors and $q$-quota rule. At the beginning of each period, players simultaneously exert costly efforts in a contest which determines the recognition probabilities for that period. Specifically, given an effort profile $e^t = (e_{1,t}, \ldots, e_{n,t})$ in period $t$, player $i$ incurs a cost of $c_i e_{i,t}$ in period $t$ where $c_i > 0$, and her recognition probability in that period is given by[59]

$$p_{i,t}(e^t) = \begin{cases} \frac{e_{i,t}}{\sum_{j \in N} e_{j,t}} & \text{if } \sum_{j \in N} e_{j,t} > 0, \\ \frac{1}{n} & \text{otherwise.} \end{cases}$$

When the history of efforts is $(e^0, \ldots, e^t)$, player $i$'s payoff is given by $\delta_i^t x_i^t - c_i \sum_{\tau=0}^t \delta_i^\tau e_{i,\tau}$ if agreement is reached on allocation $x^t$ in period $t$, and it is given by $-c_i \sum_{\tau=0}^t \delta_i^\tau e_{i,\tau}$ agreement is never reached. As usual, attention is restricted to SSP equilibria. The SSP

---

[57]Banks (1985) considers an alternative framework of amendment procedure via a binary agenda. In his setting amendments are collected in advance and voted on in a sequential order under majority rule. The first alternative in the order is viewed as the status quo and the second alternative is an amendment. The winner between the first two alternatives becomes the next status quo and is pitted against the next amendment, and so on. By contrast, in the BF model, amendments arise endogenously and depend on the status quo.

[58]There are other legislative bargaining models with endogenous proposer selection rules. In Ali (2015), the right to be a proposer is sold in an all-pay auction. In Diermeier et al. (2015) and Diermeier et al. (2016), the recognition probabilities are determined through bargaining before bargaining over policy takes place. In McKelvey & Riezman (1992), McKelvey & Riezman (1993), Muthoo & Shepsle (2014) and Eguia & Shepsle (2015), recognition probabilities are determined through seniority which is endogenously determined through elections at the end of each legislative session. Cardona & Polanski (2013) study the effects of the voting quota on the social cost of rent-seeking in a spatial bargaining model. Gomes (2017) allows all players to simultaneously propose which coalitions they want to join.

[59]Yıldırım (2007) allows more general functions but we focus on the simplest case here.

payoffs are now given by:

$$v_i = \max_{e_i \in [0, 1/c_i]} \delta_i [p_i (1 - \sum_{j \neq i}^{n} r_{ij} v_j) + \sum_{j \neq i}^{n} p_j r_{ji} v_i - c_i e_i] \qquad 16.$$

where $r^i = (r_{i1}, \ldots, r_{in})$ solves

$$\min_{r' \in [0,1]^n} \sum_{j \neq i}^{n} r'_j v_j \ \text{ subject to } \ \sum_{j \neq i}^{n} r'_j = q - 1. \qquad 17.$$

An important consequence of endogenous proposer selection rules is that inefficiencies arise in equilibrium due to socially wasteful activities. The size of the inefficiency depends on the net prize each player expects to win by being the proposer and marginal cost of effort. Under $q$-quota rule, the net prize for agent $i$ is given by

$$\pi_i = 1 - \sum_{j \neq i}^{n} r_{ij} v_j - \frac{\sum_{j \neq i}^{n} p_j r_{ji}}{1 - p_i} v_i. \qquad 18.$$

It is straightforward to see that the net prize for player $i$ is given by $1 - \sum_j v_j$ under unanimity rule. Since this is identical for all players, two players with the same marginal cost exert the same effort in equilibrium regardless of their discount factors. The discount factors affect the size of the inefficiency, however. In particular, when the marginal cost of effort is the same for all players, the social cost decreases as the group becomes more heterogeneous in their discount factors in the sense of mean preserving spread. Intuitively, such an increase necessarily introduces tougher bargainers to the mix, which, in turn, decreases prize from proposing. Likewise, when all players have the same discount factor, the social cost decreases as the group becomes more heterogeneous in their marginal costs of effort.

The net prize for agent $i$ under the $q$-quota rule with $q < n$ depends the discount factors of the players. All else equal, more patient players have a higher expected payoff and have higher benefits from proposing. Consequently, more patient players exert higher effort. Unlike the unanimity rule, the most expensive players need not be included in the winning coalition. This means the social cost can increase as the group becomes more heterogeneous. When the players have identical discount factors and marginal costs, the social cost becomes smaller as the voting rule becomes more inclusive.

In the model discussed above effort is *transitory* and has no effect on future recognition probability. By contrast, when effort is *persistent*, it is exerted once and for all determining the recognition probabilities for all future periods. Yıldırım (2010) shows that the distribution of surplus is more unequal with persistent recognition under unanimity rule.

## 5.2. Demand Bargaining

The models discussed up to this point are based on complete proposals which specify the payoff each player would receive if the proposal is accepted. Nash (1953) considered an alternative approach to bargaining that is based on demands rather than proposals, and Binmore (1985) was first to study a three-player case. In demand bargaining over distributive policies, players sequentially announce the share of the surplus they demand in order to participate in a coalition. When the bargaining is over both distributive and ideological policies, the players also announce the ideological policy they demand when it is their turn. If a winning coalition of players makes compatible demands, then the game ends. If no

winning coalition with compatible demands emerges, then the process is repeated in the following period.[60]

Morelli (1999) introduces a demand model of legislative bargaining with weighted majority.[61] In this model, the Head of State chooses the first mover, and the latter chooses the order in which the parties formulate demands. If agreement is not reached after $T$ periods, the game ends with no player receiving any surplus and median voter's ideal policy is implemented.

Morelli shows that when there are three players and any two players comprise a minimal winning coalition, the first proposer is indifferent between the two possible orders in any subgame perfect equilibrium, and the surplus is split evenly between the two players who make the first and second demand in the first period. This implies that, unlike in the BF model, there is no formateur advantage in this case.[62,63] It was initially thought that this result generalized to more than three players. However, Montero & Vidal-Puga (2007) give an example with five players in which one of the players can extract all surplus when chosen as the first proposer. Thus, the ability to choose the order of play results in extreme formateur advantage.[64] Proportionality is restored when players are restricted to make demands in order of decreasing voting weight, as shown in Montero & Vidal-Puga (2011).

## 5.3. Rejecter-Friendly Protocols

Another approach to reducing the formateur advantage is to give the potential coalition partners an ability to make counter-proposals. This leads to the rejecter-proposer model of legislative bargaining, first considered by Kawamori (2013),[65] in which the player who first rejects the current proposal becomes the next proposer. There is a predetermined initial proposer and a predetermined order on each possible winning coalition specifying the order in which players who are offered positive allocations vote in any given period. In all other respects, the model is identical to Eraslan (2002) and Kawamori (2005). As in these papers with uniform recognition, any proposer chooses $q - 1$ other least patient players as

---

[60]In Binmore's model, if there are no compatible demands after all players make demands, the process is repeated without voiding the existing demands. In the legislative bargaining papers discussed in this section, the existing demands are voided at the beginning of each period.

[61]Although Morelli allows both distributive and spatial bargaining, our discussion in the section focuses on the distributive bargaining case.

[62]In BF model with $T$ periods and a common discount factor the proposer's share equals $1 - \frac{\delta}{3}$ in the symmetric subgame perfect equilibrium. Symmetry implies that each responder is included in the winning coalition with $\frac{1}{2}$ probability in each period, and therefore the continuation values equal $\frac{\delta}{n}$ in period $t \leq T - 1$. Norman (2002) shows that infinitely many allocations can be supported in asymmetric SP equilibria in BF model when $T$ is finite and players have the same discount factor. He also shows that the equilibrium is typically unique with heterogeneous discount factors.

[63]Using experimental data, Fréchette et al. (2005a) and Fréchette et al. (2005b) found that the formateur advantage is usually present in three players divide the dollar games in which no player has enough votes to form a winning coalition on their own, but this advantage is less strong than predicted by the BF model.

[64]Breitmoser (2009) shows that formateur advantage exists in a demand bargaining model when the proposer reacts to the demands of other players, rather than making the first demand.

[65]Rejecter-proposer protocols are often used in the bargaining models of coalition formation, e.g. see the seminal works by Chatterjee et al. (1993) and Ray & Vohra (1999). Kalandrakis (2004b) and Britz et al. (2010, 2014) study models in which the identity of the previous rejecter determines the distribution from which the new proposer is drawn. In Britz et al. (2015) this distribution depends on the current state, which, in turn, depends on the previous state and rejecter.

coalition partners in any SSP equilibrium. When players have the same discount factor, each proposer's payoff is smaller in the rejecter-proposer model, reflecting a stronger bargaining position of potential rejecters. Kawamori (2013) shows that, somewhat surprisingly, when the discount factors are sufficiently high and heterogeneous, a patient player can be better off under the rejecter-proposer model.

## 6. ALTERNATIVE INFORMATION STRUCTURES

Legislative policy-making typically involves speeches and demands by legislators that may shape the proposals made by the leadership. Absent commitment, such demands are not compatible with complete information, which was the maintained assumption in the literature discussed so far. Chen & Eraslan (2013, 2014) introduce cheap talk communication into legislative bargaining with privately informed individual legislators.[66]

In both papers, three players bargain over an ideological and a distributive decision. Player 0, called the chair, is in charge of formulating the proposal $z = (y, x)$, where $y \in \mathbb{R}$ is a one-dimensional ideological policy and $x = (x_0, x_1, x_2)$ is a distributive policy. The set of feasible distributions is $X = \{x \in \mathbb{R}^3 : \sum_{i=0}^2 x_i \leq c, x_1 \geq 0, x_2 \geq 0\}$ where $x_i$ denotes the private benefit of player $i$ and $c \geq 0$ is the size of the surplus available for division.[67] Player $i$'s payoff from policy $z = (y, x)$ is given by

$$u_i(z, \theta_i, \hat{y}_i) = x_i + \theta_i v(y, \hat{y}_i),$$

where $\hat{y}_i \in \mathbb{R}$ is player $i$'s ideal point (ideological position), and $\theta_i > 0$ is the weight that player $i$ places on her payoff from the ideological decision relative to the distributive decision. In Chen & Eraslan (2013) (CE1), private information pertains to ideological positions. In Chen & Eraslan (2014) (CE2), positions are publicly known, but ideological intensities, captured by the weights, are private information.

CE1 distinguishes between *bundled bargaining* and *separate bargaining*. In the bundled game, the chair makes a proposal on the ideological dimension and the distributive dimension simultaneously, and the two dimensions are accepted or rejected together. By contrast, in the separate game, the chair makes one proposal only on the ideological dimension and another only on the distributive dimension, and each proposal is voted upon separately. Unlike in the bundled game, it is possible that a proposal on one dimension passes while the proposal on the other dimension does not. CE2 restrict attention to the bundled bargaining game.

Communication from the players 1 and 2 can be helpful since the chair is unsure how much of the surplus he has to offer to a player to gain support for an ideological policy decision. This communication takes the form of cheap talk messages, and each player announces her type before the chair formulates his proposal. Agreement rule is majority rule. Assuming, without loss of generality, that the chair votes for his own proposal, this means that at least one of players other than the chair needs to vote for the proposal for it to pass. In the bundled game, if the proposal does not pass, status quo $s = (\tilde{y}, \tilde{x})$ prevails

---

[66]In earlier work, Matthews (1989) models presidential veto threats as cheap talk in a bilateral bargaining game over a one-dimensional policy and assumes that the president's position is her private information.

[67]Notice that the chair is able to give up some surplus from his own resources.

where $\tilde{y} \in \mathbb{R}$ and $\tilde{x} = (0, 0, 0)$. In the separate game, voting over each policy is separate, and the relevant status quo policy prevails if a policy does not pass.

Both papers focus on the class of *simple equilibria*.[68] When the uncertainty is over ideological weights, the player whose ideological position is furthest from the chair's cannot be informative. The player whose ideological position is closer to the chair's can convey only limited information, no matter how close she is. In particular, a "compromise" message is sent when the closest player puts relatively small weight on the ideological dimension. In this case, the chair offers her a private transfer in return for supporting a policy improvement. However, when the closest player puts a relatively large weight on ideology, then no compromise is possible. Then a "fight" message is sent and a status quo is implemented. In contrast to complete information models, bargaining with two players may result in lower ex-ante expected payoff to the chair than bargaining with a single player who has veto power. This is because the competition between the players results in less information being transmitted in equilibrium. Despite the separable preferences, bundling is always beneficial as in Jackson & Moselle (2002). By contrast, when the uncertainty is over ideological positions, bundling the issues may result in lost surplus and less information being transmitted.

# 7. CONCLUDING REMARKS

In this article, we reviewed the legislative bargaining literature starting with the seminal work of Baron & Ferejohn (1989). Throughout, we focused exclusively on research where bargaining is once and for all, whereas many real life legislative bargaining situations involve recurrent, often perpetual, negotiations. Among many possible applications, budget negotiations is a prime example of such perpetual negotiations. Starting with Baron (1996) for the spatial case, and Epple & Riordan (1987) and Kalandrakis (2004c) for the distributive case, an active and growing body literature focuses on this recurrent nature of legislative bargaining.[69] In these models, there is a new negotiation in each period, and, if agreement is not reached in any given period, then the status quo prevails in that period. This status quo depends endogenously on the outcome of the negotiations in the previous period. Because of space limitations, we reserve the survey the literature on legislative bargaining with endogenous status quo for our companion paper Eraslan et al. (2019).

A considerable body of research recognizes the importance of information on politics in particular[70] and on bargaining in general.[71] Information can affect legislative and multilateral bargaining in three important ways. First, the players may be uninformed about each other's preferences, and such incomplete information has implications for the timing and terms of the agreement.[72] Second, when bargaining over ideological policies, the play-

---

[68]An equilibrium is *simple* if the proposal does not depend on the message of a player who receives no transfer. CE2 further restricts attention to equilibria in which types who send the same message form an interval.

[69]Among others, contributions include Baron (1996, 2017, 2018), Kalandrakis (2004c, 2010, 2016), Bowen et al. (2014, 2017), Anesi & Seidmann (2015), Baron & Bowen (2015), Dziuda & Loeper (2016), Chen & Eraslan (2017), and Zápal (2016, 2018).

[70]For example, see Gilligan & Krehbiel (1987, 1990) and Krishna & Morgan (2001) on legislative organizations, and Feddersen & Pesendorfer (1996, 1997, 1998, 1999) and Austen-Smith & Feddersen (2005, 2006) on voting.

[71]See, for example, Kennan & Wilson (1993).

[72]Tsai & Yang (2010a,b) show that both delays and surplus coalitions are possible when players

ers may be uncertain about the effects of chosen policies on final outcomes, and therefore about their own preferences over policies.[73] Third, the proposers may strategically leave some players "in the dark", thereby endogenously creating an imperfect information environment.[74] Incorporating asymmetric information into legislative and multilateral bargaining is a challenging and important avenue for future research.

In cooperative game theory, the influence of players in a simple game is measured using power indices.[75] Legislative bargaining games are simple games with additional parameters, namely recognition probabilities and discount factors. To obtain a power index for the models surveyed in this survey, one can take a limit of equilibrium payoffs (for recognition probabilities that are uniform or proportional to voting weights) as the common discount factor converges to one. One can also consider the inverse problem of designing a weighted voting game to achieve a given vector of power. This inverse problem for cooperative power indices has recently attracted attention in the economics literature and in the algorithmic game theory literature.[76]

Our survey focused on the theoretical literature on legislative bargaining. There is now a sizable literature on the experimental and empirical research on legislative bargaining, each deserving its own distinct surveys.[77] One important challenge to additional research on estimation of legislative bargaining models is computational. Kalandrakis (2015) and Uyanık (2017) provide algorithms for computing the equilibrium payoffs for the Eraslan (2002) and Eraslan & McLennan (2013) models, respectively. The development of algorithms for the other models discussed in this survey would pave the way for additional empirical research.

## DISCLOSURE STATEMENT

The authors are not aware of any affiliations, memberships, funding, or financial holdings that might be perceived as affecting the objectivity of this review.

---

are privately informed about their discount factors.

[73]Bond & Eraslan (2010) study a model in which a proposer makes a take-it-or-leave-it offer to a group, and the group members are uncertain about the value of the offer relative to the status quo. They show that unlike in the strategic voting literature, the group members can be better off under unanimity rule. Moreover, unanimity rule can be the Pareto-dominant voting rule.

[74]See, for example, Baliga & Serrano (1995) and Noe & Wang (2004).

[75]For example, the Shapley-Shubik power index (Shapley & Shubik (1954)) and the power indices of Banzhaf (1965), Deegan & Packel (1978), and Johnston (1978).

[76]See, for example, Alon & Edelman (2010), Kurz (2012) and Kurz (2016). Its analogue for a non-cooperative power index described above is an open problem.

[77]For experimental research, in addition to the papers we have already cited, see, for example, Drouvelis et al. (2010), Agranov & Tergiman (2014), Baron et al. (2017), Baranski (2018), Agranov et al. (2018). This literature demonstrates a gap between theoretical predictions and the behavior of subjects in the lab. To explain this gap, Nunnari & Zápal (2016) modify BF model by introducing erroneous beliefs about the recognition process and assuming that best responses are imperfect. They structurally estimate the resulting model using experimental data. For empirical papers using field data with close ties to standard theory reviewed in this survey, see, for example, Merlo (1997), Diermeier et al. (2003), Knight (2004, 2005, 2008); Diermeier & Morton (2005), Ansolabehere et al. (2005), Adachi & Watanabe (2007), Eraslan (2008), Le Breton et al. (2012), Proost & Zaporozhets (2013), Thomas & Zaporozhets (2017) and Koh (2018).

## ACKNOWLEDGMENTS

## LITERATURE CITED

Adachi T, Watanabe Y. 2007. Ministerial weights and government formation: Estimation using a bargaining model. *The Journal of Law, Economics, & Organization* 24:95–119

Adams G, Rausser G, Simon L. 1996. Modelling multilateral negotiations: an application to california water policy. *Journal of Economic Behavior & Organization* 30:97–111

Agranov M, Eraslan H, Tergiman C. 2018. Multilateral bargaining in the shadow of uncertainty. California Institute of Technology

Agranov M, Tergiman C. 2014. Communication in multilateral bargaining. *Journal of Public Economics* 118:75–85

Albouy D. 2013. Partisan representation in congress and the geographic distribution of federal funds. *Review of Economics and Statistics* 95:127–141

Ali SN. 2015. Recognition for sale. *Journal of Economic Theory* 155:16–29

Ali SN, Bernheim BD, Fan X. 2018. Predictability and power in legislative bargaining. *The Review of Economic Studies* :rdy013

Alon N, Edelman PH. 2010. The inverse banzhaf problem. *Social Choice and Welfare* 34:371–377

Anesi V, Seidmann DJ. 2015. Bargaining in standing committees with an endogenous default. *The Review of Economic Studies* 82:825–867

Ansolabehere S, Snyder JMJ, Strauss AB, Ting MM. 2005. Voting weights and formateur advantages in the formation of coalition governments. *American Journal of Political Science* 49:550–563

Ansolabehere S, Snyder JMJ, Ting MM. 2003. Bargaining in bicameral legislatures: When and why does malapportionment matter. *American Polical Science Review* 97:471–481

Arrow KJ. 1951. Social choice and individual values. New York: Wiley

Austen-Smith D, Banks J. 1988. Elections, coalitions, and legislative outcomes. *American Political Science Review* 82:405–422

Austen-Smith D, Banks J. 1990. Stable governments and the allocation of policy portfolios. *American Political Science Review* 84:891–906

Austen-Smith D, Banks JS. 1999. Positive political theory I. University of Michigan Press

Austen-Smith D, Banks JS. 2005. Positive political theory II: strategy and structure. University of Michigan Press

Austen-Smith D, Feddersen T. 2005. Deliberation and voting rules. In *Social Choice and Strategic Decisions*. Springer, 269–316

Austen-Smith D, Feddersen T. 2006. Deliberation, preference uncertainty, and voting rules. *American political science review* 100:209–217

Baliga S, Serrano R. 1995. Multilateral bargaining with imperfect information. *Journal of Economic Theory* 67:578–589

Banks JS. 1985. Sophisticated voting outcomes and agenda control. *Social Choice and Welfare* 1:295–306

Banks JS, Duggan J. 2000. A bargaining model of collective choice. *American Political Science Review* 94:73–88

Banks JS, Duggan J. 2006. A general bargaining model of legislative policy-making. *Quarterly Journal of Political Science* 1:49–85

Banzhaf JF. 1965. Weighted voting doesn't work: A mathematical analysis. *Rutgers Law Review* 19:317–343

Baranski A. 2016a. Pre-distribution: Bargaining over incentives with endogenous production. Maastricht University

Baranski A. 2016b. Voluntary contributions and collective redistribution. *American Economic Journal: Microeconomics* 8:149–73

Baranski A. 2018. Endogenous claims and collective production: An experimental study on the timing of profit-sharing negotiations and production. Maastricht University

Baron DP. 1991. A spatial theory of government formation in parliamentary system. *American Political Science Review* 85:137–164

Baron DP. 1993. Government formation and endogenous parties. *American Political Science Review* 87:34–47

Baron DP. 1996. A dynamic theory of collective goods programs. *American Political Science Review* 90:316–330

Baron DP. 1998. Comparative dynamics of parliamentary governments. *American Political Science Review* 92:593–609

Baron DP. 2017. Simple dynamics of legislative bargaining: coalitions and proposal power. *Economic Theory* :1–26

Baron DP. 2018. Elections and durable governments in parliamentary governments. *Journal of Theoretical Politics* 30:74–118

Baron DP, Bowen TR. 2015. Dynamic coalitions. Graduate School of Business, Stanford University

Baron DP, Bowen TR, Nunnari S. 2017. Durable coalitions and communication: Public versus private negotiations. *Journal of Public Economics* 156:1–13

Baron DP, Diermeier D. 2001. Elections, governments, and parliaments in proportional representation systems. *The Quarterly Journal of Economics* 116:933–967

Baron DP, Ferejohn JA. 1989. Bargaining in legislatures. *American Political Science Review* 83:1181–1206

Baron DP, Kalai E. 1993. The simplest equilibrium of a majority-rule division game. *Journal of Economic Theory* 61:290–301

Bassi A. 2013. A model of endogenous government formation. *American Journal of Political Science* 57:777–793

Battaglini M, Coate S. 2007. Inefficiency in legislative policymaking: a dynamic analysis. *American Economic Review* 97:118–149

Battaglini M, Coate S. 2008. A dynamic theory of public spending, taxation, and debt. *American Economic Review* 98:201–36

Battaglini M, Nunnari S, Palfrey TR. 2012. Legislative bargaining and the dynamics of public investment. *American Political science review* 106:407–429

Bennedsen M, Feldmann SE. 2002. Lobbying legislatures. *Journal of political Economy* 110:919–946

Bernheim BD, Rangel A, Rayo L. 2006. The power of the last word in legislative policy making. *Econometrica* 74:1161–1190

Besley T, Coate S. 1998. Sources of inefficiency in a representative democracy: a dynamic analysis. *American Economic Review* :139–156

Bils P, Spaniel W. 2017. Policy bargaining and militarized conflict. *Journal of Theoretical Politics* 29:647–678

Binmore KG. 1985. Bargaining and coalitions. In *Game Theoretic Models of Bargaining*, chap. 13. Cambridge University Press, 269–304

Black D. 1948. On the rationale of group decision-making. *Journal of political economy* 56:23–34

Bond P, Eraslan H. 2010. Strategic voting over strategic proposals. *The Review of Economic Studies* 77:459–490

Bondareva ON. 1963. Some applications of linear programming methods to the theory of cooperative games. *Problemy kibernetiki* 10:119–139

Bowen TR, Chen Y, Eraslan H. 2014. Mandatory versus discretionary spending: The status quo effect. *American Economic Review* 104:2941–2974

Bowen TR, Chen Y, Eraslan H, Zápal J. 2017. Efficiency of flexible budgetary institutions. *Journal of Economic Theory* 167:148–176

Breitmoser Y. 2009. Demand commitments in majority bargaining or how formateurs get their way. *International Journal of Game Theory* 38:183

Breitmoser Y. 2011. Parliamentary bargaining with priority recognition for committee members. *Social Choice and Welfare* 37:149–169

Britz V, Herings PJJ, Predtetchinski A. 2010. Non-cooperative support for the asymmetric nash bargaining solution. *Journal of Economic Theory* 145:1951–1967

Britz V, Herings PJJ, Predtetchinski A. 2014. On the convergence to the nash bargaining solution for action-dependent bargaining protocols. *Games and Economic Behavior* 86:178–183

Britz V, Herings PJJ, Predtetchinski A. 2015. Delay, multiplicity, and non-existence of equilibrium in unanimity bargaining games. *Journal of Mathematical Economics* 61:192–202

Cai H. 2000. Delay in multilateral bargaining under complete information. *Journal of Economic Theory* 93:260–260

Cai H. 2003. Inefficient markov perfect equilibria in multilateral bargaining. *Economic Theory* 22:583–606

Calvert RL, Dietz N. 2005. Legislative coalitions in a bargaining model with externalities. In *Choice and Strategic Decisions: Essays in Honor of Jeffrey S. Banks*, eds. D Austen-Smith, J.Duggan. Springer-Verlag, 228–247

Cardona D, Polanski A. 2013. Voting rules and efficiency in one-dimensional bargaining games with endogenous protocol. *Social Choice and Welfare* 41:217–240

Cardona D, Ponsatí C. 2007. Bargaining one-dimensional social choices. *Journal of Economic Theory* 137:627–651

Cardona D, Ponsatí C. 2011. Uniqueness of stationary equilibria in bargaining one-dimensional policies under (super) majority rules. *Games and Economic Behavior* 73:65–75

Cardona D, Rubí-Barceló A. 2014. On the efficiency of equilibria in a legislative bargaining model with particularistic and collective goods. *Public Choice* 161:345–366

Çelik L, Karabay B. 2016. Veto players and equilibrium uniqueness in the baron–ferejohn model. *Theory and Decision* 81:33–52

Çelik L, Karabay B, McLaren J. 2013. Trade policy-making in a model of legislative bargaining. *Journal of International Economics* 91:179–190

Chatterjee K, Dutta B, Ray D, Sengupta K. 1993. A noncooperative theory of coalitional bargaining. *The Review of Economic Studies* 60:463–477

Chatterjee K, Sabourian H. 2000. Multiperson bargaining and strategic complexity. *Econometrica* 68:1491–1509

Chen Y, Eraslan H. 2013. Informational loss in bundled bargaining. *Journal of Theoretical Politics* 25:338–362

Chen Y, Eraslan H. 2014. Rhetoric in legislative bargaining with asymmetric information. *Theoretical Economics* 9:483–513

Chen Y, Eraslan H. 2017. Dynamic agenda setting. *American Economic Journal: Microeconomics* 9:1–32

Cho Sj, Duggan J. 2003. Uniqueness of stationary equilibria in a one-dimensional model of bargaining. *Journal of Economic Theory* 113:118–130

Cho Sj, Duggan J. 2009. Bargaining foundations of the median voter theorem. *Journal of Economic Theory* 144:851–868

Cho Sj, Duggan J. 2015. A folk theorem for the one-dimensional spatial bargaining model. *International Journal of Game Theory* 44:933–948

Choate T, Weymark JA, Wiseman AE. 2018a. Legislative bargaining and partisan delegation. Graduate School of Business, Stanford University

Choate T, Weymark JA, Wiseman AE. 2018b. Partisan strength and legislative bargaining. Graduate School of Business, Stanford University

Christiansen N. 2013. Strategic delegation in a legislative bargaining model with pork and public goods. *Journal of Public Economics* 97:217–229

Christiansen N, Georganas S, Kagel JH. 2014. Coalition formation in a legislative voting game. *American Economic Journal: Microeconomics* 6:182–204

Deegan J, Packel EW. 1978. A new index of power in simple *n*-person games. *International Journal of Game Theory* 7:113–123

Denzau AT, Mackay RJ. 1981. Structure-induced equilibria and perfect-foresight expectations. *American Journal of Political Science* :762–779

Diermeier D, Egorov G, Sonin K. 2017. Political economy of redistribution. *Econometrica* 85:851–870

Diermeier D, Eraslan H, Merlo A. 2002. Coalition governments and comparative constitutional design. *European Economic Review* 46:893–907

Diermeier D, Eraslan H, Merlo A. 2003. A structural model of government formation. *Econometrica* 71:27–70

Diermeier D, Eraslan H, Merlo A, et al. 2007. Bicameralism and government formation. *Quarterly Journal of Political Science* 2:227–252

Diermeier D, Feddersen TJ. 1998. Cohesion in legislatures and the vote of confidence procedure. *American Political Science Review* 92:611–621

Diermeier D, Merlo A. 2000. Government turnover in parliamentary democracies. *Journal of Economic Theory* 94:46–79

Diermeier D, Merlo A. 2004. An empirical investigation of coalitional argaining procedures. *Journal of Public Economics* 88:783–797

Diermeier D, Morton R. 2005. Experiments in majoritarian bargaining. In *Social Choice and Strategic Decisions*. Springer, 201–226

Diermeier D, Myerson RB. 1999. Bicameralism and its consequences for the internal organization of legislatures. *American Economic Review* 89:1182–1196

Diermeier D, Prato C, Vlaicu R. 2015. Procedural choice in majoritarian organizations. *American Journal of Political Science* 59:866–879

Diermeier D, Prato C, Vlaicu R. 2016. A bargaining model of endogenous procedures. *Social Choice and Welfare* 47:985–1012

Drouvelis M, Montero M, Sefton M. 2010. Gaining power through enlargement: Strategic foundations and experimental evidence. *Games and Economic Behavior* 69:274–292

Dziuda W, Loeper A. 2016. Dynamic collective choice with endogenous status quo. *Journal of Political Economy* 124:1148–1186

Eguia JX, Shepsle KA. 2015. Legislative bargaining with endogenous rules. *The Journal of Politics* 77:1076–1088

Epple D, Riordan MH. 1987. Cooperation and punishment under repeated majority voting. *Public Choice* 55:41–73

Eraslan H. 2002. Uniqueness of stationary equilibrium payoffs in the Baron-Ferejohn model. *Journal of Economic Theory* 103:11–30

Eraslan H. 2008. Corporate bankruptcy reorganizations: Estimates from a bargaining model. *International Economic Review* 49:659–681

Eraslan H. 2016. Uniqueness of stationary equilibrium payoffs in the baron-ferejohn model with risk-averse players. *International Journal of Economic Theory* 12:29–40

Eraslan H, Evdokimov KS, Zápal J. 2019. Legislative bargaining with endogenous status quo. *Work in progress*. Rice University

Eraslan H, McLennan A. 2013. Uniqueness of stationary equilibrium payoffs in coalitional bargaining. *Journal of Economic Theory* 148:2195–2222

Eraslan H, Merlo A. 2002. Majority rule in a stochastic model of bargaining. *Journal of Economic Theory* 103:31–48

Eraslan H, Merlo A. 2017. Some unpleasant bargaining arithmetic? *Journal of Economic Theory*

171:293–315

Feddersen T, Pesendorfer W. 1996. The swing voter's curse. *The American economic review* :408–424

Feddersen T, Pesendorfer W. 1997. Voting behavior and information aggregation in elections with private information. *Econometrica: Journal of the Econometric Society* :1029–1058

Feddersen T, Pesendorfer W. 1998. Convicting the innocent: The inferiority of unanimous jury verdicts under strategic voting. *American Political science review* 92:23–35

Feddersen T, Pesendorfer W. 1999. Abstention in elections with asymmetric information and diverse preferences. *American Political Science Review* 93:381–398

Fréchette GR, Kagel JH, Morelli M. 2005a. Behavioral identification in coalitional bargaining: An experimental analysis of demand bargaining and alternating offers. *Econometrica* 73:1893–1937

Fréchette GR, Kagel JH, Morelli M. 2005b. Gamson's law versus non-cooperative bargaining theory. *Games and Economic Behavior* 51:365–390

Gamson WA. 1961. A theory of coalition formation. *American Sociological Review* :373–382

Gilligan TW, Krehbiel K. 1987. Collective decisionmaking and standing committees: An informational rationale for restrictive amendment procedures. *Journal of Law, Economics, & Organization* 3:287–335

Gilligan TW, Krehbiel K. 1990. Organization of informative committees by a rational legislature. *American Journal of Political Science* :531–564

Gomes A. 2005. Multilateral contracting with externalities. *Econometrica* 73:1329–1350

Gomes A. 2015. Multilateral negotiations and formation of coalitions. *Journal of Mathematical Economics* 59:77–91

Gomes A. 2017. Coalitional bargaining games: A new concept of value and coalition formation. Washington University in St. Louis

Gomes A, Jehiel P. 2005. Dynamic processes of social and economic interactions: On the persistence of inefficiencies. *Journal of Political Economy* 113:626–667

Gül F. 1989. Bargaining foundations of shapley value. *Econometrica* 57:81–95

Harrington JEJ. 1990a. The power of the proposal maker in a model of endogenous agenda formation. *Public Choice* 64:1–20

Harrington JEJ. 1990b. The role of risk preferences in bargaining when acceptance of a proposal requires less than unanimous approval. *Journal of Risk and Uncertainty* 3:135–154

Helpman E, Persson T. 2001. Lobbying and legislative bargaining. *Advances in economic analysis & policy* 1

Herings PJJ, Houba H. 2016. The condorcet paradox revisited. *Social Choice and Welfare* 47:141–186

Herings PJJ, Meshalkin A, Predtetchinski A. 2017. A one-period memory folk theorem for multilateral bargaining games. *Games and Economic Behavior* 103:185–198

Herings PJJ, Meshalkin A, Predtetchinski A. 2018. Subgame perfect equilibria in majoritarian bargaining. *Journal of Mathematical Economics* 76:101–112

Herings PJJ, Predtetchinski A. 2010. One-dimensional bargaining with markov recognition probabilities. *Journal of Economic Theory* 145:189–215

Hyndman K, Ray D. 2007. Coalition formation with binding agreements. *The Review of Economic Studies* 74:1125–1147

Iaryczower M, Oliveros S. 2016. Power brokers: Middlemen in legislative bargaining. *Journal of Economic Theory* 162:209–236

Inman RP. 1987. Markets, governments, and the "new" political economy. In *Handbook of public economics*, vol. 2. Elsevier, 647–777

Jackson MO, Moselle B. 2002. Coalition and party formation in a legislative voting game. *Journal of Economic Theory* 103:49–87

Johnston RJ. 1978. On the measurement of power: Some reactions to Laver. *Environmental Planning A* 10:907–914

Kalandrakis T. 2004a. Bicameral winning coalitions and equilibrium federal legislatures. *Legislative Studies Quarterly* 29:49–79

Kalandrakis T. 2004b. Equilibria in sequential bargaining games as solutions to systems of equations. *Economics Letters* 84:407–411

Kalandrakis T. 2004c. A three-player dynamic majoritarian bargaining game. *Journal of Economic Theory* 116:294–322

Kalandrakis T. 2006. Proposal rights and political power. *American Journal of Political Science* 50:441–448

Kalandrakis T. 2010. Minimum winning coalitions and endogenous status quo. *International Journal of Game Theory* 39:617–643

Kalandrakis T. 2015. Computation of equilibrium values in the baron and ferejohn bargaining model. *Games and Economic Behavior* 94:29–38

Kalandrakis T. 2016. Pareto efficiency in the dynamic one-dimensional bargaining model. *Journal of Theoretical Politics* 28:525–536

Kawamori T. 2004. Legislative bargaining and parties' patience. Master's thesis, University of Tokyo

Kawamori T. 2005. Players' patience and equilibrium payoffs in the Baron-Ferejohn model. *Economics Bulletin* 3:1–5

Kawamori T. 2013. Rejecter-proposer legislative bargaining with heterogeneous time and risk preferences. *Social Choice and Welfare* 40:27–40

Kennan J, Wilson R. 1993. Bargaining with private information. *Journal of Economic Literature* 31:45–104

Kim DG. 2018a. The cycle-stationary subgame perfect equilibrium in legislative bargaining without replacement. University of Mannheim

Kim DG. 2018b. "One bite at the apple": Legislative bargaining without replacement. University of Mannheim

Knight B. 2004. Parochial interests and the centralized provision of local public goods: evidence from congressional voting on transportation projects. *Journal of Public Economics* 88:845–866

Knight B. 2005. Estimating the value of proposal power. *American Economic Review* 95:1639–1652

Knight B. 2008. Legislative representation, bargaining power and the distribution of federal funds: Evidence from the us congress. *The Economic Journal* 118:1785–1803

Koh Y. 2018. Universalism and the value of political power. *International Economic Review*

Konishi H, Ray D. 2003. Coalition formation as a dynamic process. *Journal of Economic theory* 110:1–41

Krishna V, Morgan J. 2001. Asymmetric information and legislative rules: Some amendments. *American Political Science Review* 95:435–452

Krishna V, Serrano R. 1996. Multilateral bargaining. *The Review of Economic Studies* 63:61–80

Kurz S. 2012. On the inverse power index problem. *Optimization* 61:989–1011

Kurz S. 2016. The inverse problem for power distributions in committees. *Social Choice and Welfare* 47:65–88

Kurz S, Napel S, Nohn A. 2014. The nucleolus of large majority games. *Economics Letters* 123:139–143

Le Breton M, Montero M, Zaporozhets V. 2012. Voting power in the EU council of ministers and fair decision making in distributive politics. *Mathematical Social Sciences* 63:159–173

LeBlanc W, Snyder JM, Tripathi M. 2000. Majority rule and the underprovision of public investment. *Journal of Public Economics* 75:21–47

Levy G, Razin R. 2013. Dynamic legislative decision making when interest groups control the agenda. *Journal of Economic Theory* 148:1862–1890

Li H, Suen W. 2009. Decision-making in committees. *Canadian Journal of Economics/Revue canadienne d'économique* 42:359–392

Lupia A, Strøm K. 1995. Coalition termination and the strategic timing of parliamentary elections. *American Political Science Review* 89:648–665

Martin GJ. 2018. Dividing the dollar with formulas. *The Journal of Politics* 80:479–493

Matthews SA. 1989. Veto threats: Rhetoric in a bargaining game. *The Quarterly Journal of Economics* 104:347–369

McCarty NM. 2000a. Presidential pork: Executive veto power and distributive politics. *American Political Science Review* 94:117–129

McCarty NM. 2000b. Proposal rights, veto rights, and political bargaining. *American Journal of Political Science* :506–522

McKelvey RD. 1976. Intransitivities in multidimensional voting models and some implications for agenda control. *Journal of Economic theory* 12:472–482

McKelvey RD. 1979. General conditions for global intransitivities in formal voting models. *Econometrica:* :1085–1112

McKelvey RD. 1986. Covering, dominance, and institution-free properties of social choice. *American Journal of Political Science* :283–314

McKelvey RD, Niemi RG. 1978. A multistage game representation of sophisticated voting for binary procedures. *Journal of Economic Theory* 18:1–22

McKelvey RD, Riezman R. 1992. Seniority in legislatures. *American Political Science Review* 86:951–965

McKelvey RD, Riezman R. 1993. Initial versus continuing proposal power in legislative seniority systems. *Political Economy: Institutions, Competition, and Representation* :279–292

Merlo A. 1997. Bargaining over governments in a stochastic environment. *Journal of Political Economy* 105:101–131

Merlo A, Wilson C. 1995. A stochastic model of sequential bargaining with complete information. *Econometrica* 63:371–399

Merlo A, Wilson C. 1998. Efficient delays in a stochastic model of bargaining. *Economic Theory* 11:39–55

Miller NR. 1980. A new solution set for tournaments and majority voting: Further graph-theoretical approaches to the theory of voting. *American Journal of Political Science* :68–96

Montero M. 2006. Noncooperative foundations of the nucleolus in majority games. *Games and Economic Behavior* 54:380–397

Montero M. 2017. Proportional payoffs in legislative bargaining with weighted voting: A characterization. *Quarterly Journal of Political Science* 12:325–346

Montero M, Vidal-Puga JJ. 2007. Demand commitment in legislative bargaining. *American Political Science Review* 101:847–850

Montero M, Vidal-Puga JJ. 2011. Demand bargaining and proportional payoffs in majority games. *Games and Economic Behavior* 71:395–408

Morelli M. 1999. Demand competition and policy compromise in legislative bargaining. *American Political Science Review* 93:809–820

Muthoo A, Shepsle KA. 2014. Seniority and incumbency in legislatures. *Economics & Politics* 26:13–37

Nakamura K. 1979. The vetoers in a simple game with ordinal preferences. *International Journal of Game Theory* 8:55–61

Nash J. 1953. Two-person cooperative games. *Econometrica: Journal of the Econometric Society* :128–140

Noe TH, Wang J. 2004. Fooling all of the people some of the time: A theory of endogenous sequencing in confidential negotiations. *The Review of Economic Studies* 71:855–881

Norman P. 2002. Legislative bargaining and coalition formation. *Journal of Economic Theory* 102:322–353

Nunnari S, Zápal J. 2016. Gambler's fallacy and imperfect best response in legislative bargaining. *Games and Economic Behavior* 99:275–294

Okada A. 1996. A noncooperative coalitional bargaining game with random proposers. *Games and Economic Behavior* 16:97–108

Okada A. 2000. The efficiency principle in non-cooperative coalitional bargaining. *The Japanese Economic Review* 51:34–50

Okada A. 2011. Coalitional bargaining games with random proposers: Theory and application. *Games and Economic Behavior* 73:227–235

Osborne MJ, Rubinstein A. 1990. Bargaining and markets. Academic Press

Osborne MJ, Rubinstein A. 1994. A course in game theory. MIT press

Parameswaran G. 2018. Bargaining and bicameralism. *Legislative Studies Quarterly* 43:101–139

Parameswaran G, Cameron CM, Kornhauser LA. 2017. Bargaining on appellate courts. Haverford College

Persson T. 1998. Economic policy and special interest politics. *The Economic Journal* 108:310–327

Persson T, Roland G, Tabellini G. 1998. Towards micropolitical foundations of public finance. *European Economic Review* 42:685–694

Persson T, Roland G, Tabellini G. 2000. Comparative politics and public finance. *Journal of political Economy* 108:1121–1161

Plott CR. 1967. A notion of equilibrium and its possibility under majority rule. *The American Economic Review* 57:787–806

Predtetchinski A. 2011. One-dimensional bargaining. *Games and Economic Behavior* 72:526–543

Proost S, Zaporozhets V. 2013. The political economy of fixed regional public expenditure shares with an illustration for belgian railway investments. *Regional Science and Urban Economics* 43:808–815

Ray D. 2007. A game-theoretic perspective on coalition formation. Oxford University Press

Ray D, Vohra R. 1999. A theory of endogenous coalition structures. *Games and Economic Behavior* 26:286–336

Ray D, Vohra R. 2015. Coalition formation. In *Handbook of game theory with economic applications*, vol. 4. Elsevier, 239–326

Romer T, Rosenthal H. 1978. Political resource allocation, controlled agendas, and the status quo. *Public choice* 33:27–43

Roth AE. 1985. A note on risk aversion in a perfect equilibrium model of bargaining. *Econometrica* :207–211

Rubinstein A. 1982. Perfect equilibrium in a bargaining model. *Econometrica: Journal of the Econometric Society* :97–109

Scarf HE. 1967. The core of an n person game. *Econometrica: Journal of the Econometric Society* :50–69

Schmeidler D. 1969. The nucleolus of a characteristic function game. *SIAM Journal on applied mathematics* 17:1163–1170

Schofield N. 2007. The spatial model of politics. Routledge

Seidmann DJ, Winter E. 1998. A theory of gradual coalition formation. *The Review of Economic Studies* 65:793–815

Shapley LS. 1967. On balanced sets and cores. *Naval Research Logistics Quarterly* 14:453–460

Shapley LS, Shubik M. 1954. A method for evaluating the distribution of power in a committee system. *American Political Science Review* 81:935–945

Shepsle KA. 1979. Institutional arrangements and equilibrium in multidimensional voting models. *American Journal of Political Science* :27–59

Shepsle KA, Weingast BR. 1984. Uncovered sets and sophisticated voting outcomes with implications for agenda institutions. *American Journal of Political Science* :49–74

Snyder JM, Ting MM, Ansolabehere S. 2005. Legislative bargaining under weighted voting. *American Economic Review* 95:981–1004

Tarar A. 2005. Constituencies and preferences in international bargaining. *Journal of Conflict Resolution* 49:383–407

Thomas A, Zaporozhets V. 2017. Bargaining over environmental budgets: A political economy model with application to french water policy. *Environmental and Resource Economics* 68:227–

248

Tsai TS, Yang C. 2010a. Minimum winning versus oversized coalitions in public finance: The role of uncertainty. *Social Choice and Welfare* 34:345–361

Tsai TS, Yang C. 2010b. On majoritarian bargaining with incomplete information. *International Economic Review* 51:959–979

Uyanık M. 2017. Computation of stationary equilibrium payoffs in coalitional bargaining. University of Queensland

Vanberg G. 1998. Abstract judicial review, legislative bargaining, and policy compromise. *Journal of Theoretical Politics* 10:299–326

Volden C, Wiseman AE. 2007. Bargaining in legislatures over particularistic and collective goods. *American Political Science Review* 101:79–92

Volden C, Wiseman AE. 2008. Erratum to "Bargaining in legislatures over particularistic and collective goods" American Political Science Review 101 (1): 79–92. *American Political Science Review* 102:385–386

Weber RJ. 1994. Games in coalitional form. *Handbook of Game Theory with Economic Applications* 2:1285–1303

Winter E. 1996. Voting and vetoing. *American Political Science Review* 90:812–823

Woon J, Anderson S. 2012. Political bargaining and the timing of congressional appropriations. *Legislative Studies Quarterly* 37:409–436

Yıldırım H. 2007. Proposal power and majority rule in multilateral bargaining with costly recognition. *Journal of Economic Theory* 136:167–196

Yıldırım H. 2010. Distribution of surplus in sequential bargaining with endogenous recognition. *Public choice* 142:41

Yıldırım M. 2018. Legislative bargaining with a stochastic surplus and costly recognition. *Economics Letters* 163:102–105

Zápal J. 2016. Markovian equilibria in dynamic spatial legislative bargaining: Existence with three players. *Games and Economic Behavior* 98:235–242

Zápal J. 2018. Simple markovian equilibria in dynamic spatial legislative bargaining. CERGE-EI & IAE-CSIC and Barcelona GSE.