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How to Measure Spillover Effects of Public Capital Stock: A Spatial Autoregressive Stochastic Frontier Model

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Abstract

This paper aims to investigate spillover effects of public capital stock in a production function model that accounts for spatial dependencies. In many settings, ignoring spatial dependency yields inefficient, biased and inconsistent estimates in cross country panels. Although there are a number of studies aiming to estimate the output elasticity of public capital stock, many of those fail to reach a consensus on refining the elasticity estimates. We argue that accounting for spillover effects of the public capital stock on the production efficiency and incorporating spatial dependences are crucial. For this purpose we employ a spatial autoregressive stochastic frontier model based on a number of specifications of the spatial dependency structure. Using the data of 21 OECD countries from 1960 to 2001, we estimate a spatial autoregressive stochastic frontier model and derive the mean indirect marginal effects of public capital stock, which are interpreted as spillover effects. We found that spillover effects can be an important factor explaining variations in technical inefficiency across countries as well as in explaining the discrepancies among various levels of output elasticity of public capital stock in traditional production function approaches.

Keywords: Public capital, Spillover effect, Stochastic frontier model, Spatial panel model, Time-varying spatial weights

JEL Classification: C23; D24; O47

1 Introduction

Public capital includes many types of goods which are used to produce final goods and services for consumers. The infrastructures such as highways, streets, roads, and public educational buildings take the largest components of public capital and also electric, gas and water supply facilities, administration, police, military service, hospital facilities, and many other forms of goods and services are included in public capital. In the United States, the real government gross fixed capital

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formation is between 3% and 4% of real GDP and about 20% of the real gross fixed capital formation in the private sector. Based on the estimates of Kamps (2006), the public capital stock is sizable, taking about 55% of real GDP and more than 20% of private capital stock on average of the 22 OECD countries during the sample periods: 1960-2001. This implies that ignoring public capital can be problematic when analyzing productivity and efficiency. Delorme et al. (1999) also argue that public infrastructure reduces the technical inefficiency of private-sector production.

The economic impact of public investment has been received great attention in numerous studies for the last few decades. Many studies have tried to estimate the output elasticity of public capital and there has been a debate on the effects of public capital on output. The output elasticities of public capital estimated by various models and samples range from 0.1 to 0.4. Specifically, the earlier studies such as Aschauer (1989) or Munnell (1990) report relatively large elasticities while they are considered implausibly high by subsequent studies such as Tatom (1991) or Holtz-Eakin (1994). Recently, Bom and Ligthart (2013) estimate an average output elasticity of public capital to be around 0.15 using meta-analysis with 68 studies for the 1983 - 2008 period. Even though the magnitudes of the effects of public capital stock are far from consensus, there is little doubt on the positive sign and statistical significance (Pereira and Andraz, 2013). There has been several explanations for the disagreement on the magnitude of the effects of public capital. Major explanations are related to econometric issues. Tatom (1991) points out the significant influence of the relative price of energy on productivity, the omitted time trend, and possibility of spurious regressions as main shortcomings of the previous studies. Moreover, in general, ignorance of cross sectional dependency results in inefficient and biased estimates and invalid inference when it actually exists. Another explanation is the possible existence of spillover effects and the different levels of samples of studies. Pereira and Andraz (2013) pointed that the large effects of public investment observed at the aggregate level cannot be replicated at the regional level. Also, they mentioned that the spillover effects captured by aggregate level studies can give a clue for this paradox. They said that the significant spillover effects are observed in some empirical studies and this can explain some of the divergences found between regional and aggregate studies. The issue of the possible existence of spillovers of public capital has received relatively little attention. Holtz-Eakin and Schwartz (1995) examine the degree of geographical spillovers in productivity with state highway investment but no evidence of important regional spillovers is found. Pereira and Roca-Sagales (2003) investigate the regional effects of public investment and the spillovers in Spain using vector autoregressive(VAR) models. Recently spatial econometric methods are extensively used in regional science studies to assess spatial spillovers.

Spatial econometrics consists of econometric techniques dealing with interactions of economic units in space. For a cross sectional model, the spatial autoregressive model by Cliff and Ord (1973) has received the most attention. The technique is extended to panel data models. Anselin (1988) provides a panel regression model with error components and spatial autoregressive disturbances. Kapoor et al. (2007) propose a method of moments estimation. Lee and Yu (2010c) investigate the quasi-maximum likelihood estimation of spatial panel models under the fixed effects specification.

We form exogenous spatial weights matrices that capture the interactions between individuals. The interactions can be of geographical or economic characteristics. The spatial weights matrices tend to be time invariant because mostly spatial weights are based on geographical concepts such as border sharing characteristics or centroid distances, which are not change over time. However, we can also consider the economic/socioeconomic distances or demographic characteristics as sources of cross sectional dependence. As Lee and Yu (2012) point out, when spatial weights matrix is constructed from those characteristics in a panel or dynamic setting, the characteristics might change over time.

Spatial dependency and the heterogeneity of the spatial dependence structure influence on the productivity or efficiency of the economic units. The standard stochastic frontier models do not take spatial interactions into account. Pavlyuk (2012) estimates the efficiency in presence of spatial dependency and presents a general specification of the spatial stochastic frontier model. Glass et al. (2013) develop a spatial autoregressive frontier model with time-varying efficiency. They shed lights on the possible interpretations of spillover of returns to scale.

In this paper, we mainly concern how to estimate the effects of public capital stock on output separating out the direct and indirect effects. The direct effects include the feedback effects which pass through neighboring regions and back to the region itself. The indirect effects are interpreted as spillover effects. We consider the possible existence of spatial dependence by incorporating the spatially correlated terms with a variety of spatial dependence structures. Also, we measure the technical efficiencies of countries. We expect to improve the estimation of the technical efficiency by adding public capital as a factor input and controlling the possible cross sectional dependence. To this end we investigate the quasi-maximum likelihood(QML) estimation of Spatial Autoregressive Stochastic Frontier Model. Finally, we apply the model to a dataset from 21 OECD countries under the settings of both time-invariant and time-varying spatial weights matrices. We found significant and sizable output elasticity of public capital, and significant spillover effects, and also we estimated the relative technical efficiency scores of each models.

The paper continues with the following structure. Section 2 introduces the standard spatial models and presents the associate frontier model we are interested in. Also, we discuss on the direct, indirect, and total effects of the inputs on output and connect the interpretation to the spillover effects. In section 3, we modifies the quasi-maximum likelihood estimation provided by Lee and Yu (2012) for the efficiency analysis. In section 4, we apply the model to a dataset from 21 OECD countries with a variety of weight matrices specifications. Section 5 concludes the paper.

2 Spatial Autoregressive Stochastic Frontier Model

We begin with a non-spatial production function. The production function is of the form:

$$y_{it} = \beta_0 + X_{it}\beta + \varepsilon_{it} \quad i = 1, \cdots, N; \quad t = 1, \cdots, T, \tag{1}$$

where i indexes cross-section of economic units and t indexes time periods. y_{it} is output of the ith unit at time t, whereas X_{it} is a $(1 \times K)$ input vector of the ith unit at time t. β is the $(K \times 1)$ parameter vector to be estimated, and ε_{it} is an $i.i.d$ disturbance term with zero mean and variance σ_ε^2.

In general, three types of spatial interaction effects can be given on the non-spatial production function. The first are endogenous interaction effects, which are explaining dependence between the dependent variable, y, of each unit. The second are exogenous interaction effects, which are explaining dependence between the dependent variable of a specific unit, y, and the independent variable of another unit, X. The last type are the interaction effects among the error terms. A full model with all types of spatial effects can be written as:

$$\begin{aligned} y_{it} &= \rho \sum_{j=1}^{N} w_{ij} y_{jt} + \beta_0 + X_{it}\beta + \theta \sum_{j=1}^{N} w_{ij} X_{jt} + \varepsilon_{it}, \\ \varepsilon_{it} &= \lambda \sum_{j=1}^{N} w_{ij} \varepsilon_{jt} + u_{it}. \end{aligned} \tag{2}$$

The model in matrix notation is given:

$$\begin{aligned} Y &= \rho W Y + \beta_0 \iota_n + X\beta + W X \theta + \varepsilon, \\ \varepsilon &= \lambda W \varepsilon + u, \end{aligned} \tag{3}$$

where W is an exogenous $(N \times N)$ spatial weights matrix with non-negative elements, WY, WX, and $W\varepsilon$ represent the endogenous interaction effects, exogeneous interaction effects, and interaction effects among the disturbance term, respectively[1].

For model selection, Anselin et al. (2008) provide Lagrange Multiplier tests for a spatially lagged dependent variable and for a spatial error term under panel data setting. The test starts from the non-spatial pooled model

$$y = X\beta + \varepsilon. \tag{4}$$

The tests center on the null hypotheses $H_0 : \rho = 0$ and/or $H_0 : \lambda = 0$. The Lagrange Multiplier statistics are of the forms:

$$LM_L = \frac{[e'(I_T \otimes W)Y/(e'e/NT)]^2}{[(W\hat{Y})'M(W\hat{Y})/\hat{\sigma}^2] + T \cdot tr(WW + W'W)}, \tag{5}$$

$$LM_E = \frac{[e'(I_T \otimes W)e/(e'e/NT)]^2}{T \cdot tr(WW + W'W)}, \tag{6}$$

[1]Elhorst (2014) named this equation as the general nesting spatial (GNS) model.

where $e = Y - X\hat{\beta}$ denotes the residuals of a regression model without any spatial terms, $W\hat{Y}$ is the spatially lagged predicted values in the regression, and $M = I_{NT} - X(X'X)^{-1}X'$. The statistics are asymptotically distributed as $\chi^2(1)$ Also, Elhorst (2010) suggested the robust counterparts of the LM tests as

$$robustLM_L = \frac{[e'(I_T \otimes W)Y/\hat{\sigma}_v^2]^2 - [e'(I_T \otimes W)e/\hat{\sigma}_v^2]^2}{J - T \cdot T_W}, \tag{7}$$

$$robustLM_E = \frac{[e'(I_T \otimes W)e/\hat{\sigma}_v^2]^2 - [(TT_W/J) \times e'(I_T \otimes W)Y/\hat{\sigma}_v^2]^2}{T \cdot T_W(1 - T \cdot T_W/J)}, \tag{8}$$

where $J = [(W\hat{Y})'M(W\hat{Y})/\hat{\sigma}^2] + T \cdot tr(WW + W'W)]$ and $T_W = tr(WW + W'W)$. Among the interaction effects, we are interested in the endogenous interaction effects. Hence we impose the restrictions of $\theta = 0$ and $\lambda = 0$. The the model (2) reduces to

$$y_{it} = \rho \sum_{j=1}^{N} w_{ij} y_{jt} + \beta_0 + X_{it}\beta + \varepsilon_{it}, \tag{9}$$

The model is called Spatial Autoregressive Model; hereafter SAR. This is the Cliff-Ord type production function, suggested by Cliff and Ord (1981)

The associated stochastic frontier specification of (9) can be obtained by assuming the disturbance term ε_{it} is a composite error with a non-negative random variable u_i, which represents the technical inefficiency, and a systematic random noise v_{it}. We assume u_i to be time-invariant following the assumption of Schmidt and Sickles (1984). ε_{it} is no longer assumed to have zero mean. Instead, we assume v_{it} is $i.i.d.$ with zero mean and variance σ_v^2. Then the spatial autoregressive stochastic frontier (SARSF) model is given

$$y_{it} = \rho \sum_{j=1}^{N} w_{ij} y_{jt} + \beta_0 + X_{it}\beta - u_i + v_{it}. \tag{10}$$

Define $\alpha_i \equiv \beta_0 - u_i$ then the model (10) becomes

$$y_{it} = \rho \sum_{j=1}^{N} w_{ij} y_{jt} + \alpha_i + X_{it}\beta + v_{it}. \tag{11}$$

A relative inefficiency(or efficiency) measure accounts for the output of each unit to the output that could be produced by a fully-efficient unit. Because the most efficient unit has the largest α_i, the relative inefficiency measure can be derived by defining u_i^* as the distance between $max(\hat{\alpha}_i)$ and α_i[2].

[2]Since output is in logarithms, relative technical efficiency is defined as $\hat{r}_i \equiv exp(-u_i^*)$.

$$u_i^* \equiv max(\hat{\alpha}_i) - \hat{\alpha}_i. \tag{12}$$

We can write the stacked form of (12) as follows:

$$Y_t = \rho W Y_t + X_t\beta + \boldsymbol{\alpha} + V_t, \quad t = 1, \cdots, T \tag{13}$$

where $Y_t = (y_{1t}, y_{2t}, \cdots, y_{Nt})'$ and $V_t = (v_{1t}, v_{2t}, \cdots, v_{Nt})'$ are $(N \times 1)$ column vectors, and v_{it}'s are i.i.d across i and t with zero mean and variance σ^2. The X_t is an $(N \times K)$ matrix matrix of non-stochastic regressors, and $\boldsymbol{\alpha}$ is an $(N \times 1)$ column vector of individual effects. The spatial weights matrix W_t is a non-stochastic and row-normalized, and time varying $(N \times N)$ matrix.

The spatial weights matrix, W, is taken to be exogenous and captures cross-section dependence among observations. The spatial weights matrix is mostly specified to be time invariant because it is usually based on the characteristics that are hardly changing over time such as geographical distance or border sharing feature. However, it can be formed from other concepts that are time varying such as economic/socioeconomic distances or demographic characteristics. We incorporate time varying characteristics of the spatial weights matrix, so we add time-subscript t in (13) as follows:

$$Y_t = \rho W_t Y_t + X_t\beta + \boldsymbol{\alpha} + V_t \tag{14}$$

The reduced form of (14) is

$$Y_t = (I_N - \rho W_t)^{-1}(X_t\beta + \boldsymbol{\alpha} + V_t). \tag{15}$$

One advantage of incorporating the spatial dependence is that it captures the direct and indirect effects separately. If the OLS model is adopted, only the direct effects of explanatory variables are captured as the coefficient estimates of the variables. When a spatial lag term is introduced in the model, the direct effects of an explanatory variable, X_k, are the diagonal elements of $(I_N - \rho W_t)^{-1}\beta_k$, while the indirect effects are the off-diagonal elements of the matrix. The premultiplied matrix can be decomposed as

$$(I_N - \rho W_t)^{-1} = I_N + \rho W_t + \rho^2 W_t^2 + \rho^3 W_t^3 + \cdots. \tag{16}$$

Hence the direct effect will be greater than or equal to β_k. The first two matrix terms of the right hand side of (16) represent a direct effect of a change in X_k only and an indirect effect of a change in X_k only, respectively. The rest terms represent higher order direct and indirect effects, which include the feedback effects of other units. To obtain a single direct effect and indirect effect for an explanatory variable in the model, LeSage and Pace (2009) suggest reporting the average of the direct effects and the average of the indirect effects.

To test whether spatial spillovers exist or not, the estimated indirect effects should be used, not the estimate of ρ. This is because the indirect effects are derived from the multiplication of $(I_N - \rho W_t)^{-1}$ and β_k, and the variation of the indirect effects depends on the variation of all coefficient estimates. In other words, even though each coefficients are estimated to be significant, this does not mean that the mean indirect effect is significant, and vice versa. LeSage and Pace (2009) suggest simulating the distribution of the direct and indirect effects. The simulation procedure is basically in two steps: 1) computing the mean value over D draws of direct/indirect effects for the approximation of the overall effects and 2) obtaining t-statistics by dividing the sample mean by the corresponding standard deviation.

3 QML estimation of Spatial Autoregressive Model with Time-varying spatial weights matrices

Spatial models can be estimated by maximum likelihood, quasi-maximum likelihood, instrumental variables, generalized method of moments, or by Bayesian Markov Chain Monte Carlo methods. One advantage of QML is that it does not rely on the assumption of normality of the disturbances. For the standard panel data model with fixed effects, one can estimate jointly the common parameters of interest and fixed effects by the maximum likelihood estimation. However, it is well-known that the MLE of the variance is inconsistent when T is finite. Similar consequences are found for the spatial panel data model with fixed effects. To avoid the incidental parameter problem, we can use a data transformation, which is a demeaning procedure of each variables. Lee and Yu (2010c) provide asymptotic properties of quasi-maximum likelihood estimators for spatial dynamic panel data with both time and individual fixed effects. Elhorst (2014) summarizes how to estimate spatial autoregressive model with fixed effects when spatial weight matrix is time invariant. In this section, we derive quasi-maximum likelihood estimator for spatial autoregressive model with fixed effects when spatial weight matrix is time variant. Maximum likelihood estimator accounts for the endogeneity of the spatial lag term.

Let $\theta = (\beta, \rho, \sigma_v^2)'$. The log-likelihood function of the model 14 is

$$LogL_{N,T}(\theta, \boldsymbol{\alpha}) = -\frac{NT}{2} ln(2\pi\sigma_v^2) + \sum_{t=1}^{T} ln|I_N - \rho W_t| - \frac{1}{2\sigma_v^2} \sum_{t=1}^{T} V_t'(\theta) V_t(\theta), \tag{17}$$

where $V_t(\theta) = (I_N - \rho W_t)Y_t - X_t\beta - \boldsymbol{\alpha}$. We can get the analytic solution to α_i by solving the first-order conditions of (17) with respect to α_i as follows:

$$\alpha_i = \frac{1}{T} \sum_{t=1}^{T} \left(y_{it} - \rho \sum_{j=1}^{N} w_{ij}^t y_{jt} - X_{it}\beta \right) \qquad i = 1, \cdots, N. \tag{18}$$

By substituting (18) into (17), we concentrate out $\boldsymbol{\alpha}$ in (17) and get the concentrated log-likelihood function as follows:

$$LogL_{N,T}(\theta) = -\frac{NT}{2} ln(2\pi\sigma_v^2) + \sum_{t=1}^{T} ln|I_N - \rho W_t| - \frac{1}{2\sigma_v^2} \sum_{t=1}^{T} \widetilde{V}_t'(\theta)\widetilde{V}_t(\theta), \tag{19}$$

where $\widetilde{V}_t(\theta) = \widetilde{Y}_t - \rho\widetilde{W_tY_t} - \widetilde{X}_t\beta$, $\widetilde{Y}_t = Y_t - \frac{1}{T}\sum_{t=1}^{T} Y_t$, $\widetilde{W_tY_t} = W_tY_t - \frac{1}{T}\sum_{t=1}^{T} W_tY_t$, and $\widetilde{X}_t = X_t - \frac{1}{T}\sum_{t=1}^{T} X_t$. As one can see, the concentrated log-likelihood function (19) is same as the log-likelihood function from (14) after demeaning variables. We can write the transformed model after demeaning as follows:

$$\widetilde{Y}_t = \rho\widetilde{W_tY_t} + \widetilde{X}_t\beta + \widetilde{V}_t. \tag{20}$$

Assuming we know the true value of $\rho = \rho^*$, the ordinary least squares estimates of β and σ_v^2 of (20) can be found analytically as follows [3]:

$$\hat{\beta} = (\widetilde{X}'\widetilde{X})^{-1}\widetilde{X}'[\widetilde{Y} - \rho^*\widetilde{WY}], \tag{21}$$

$$\hat{\sigma}_v^2 = \frac{1}{NT}(\widetilde{Y} - \rho\widetilde{WY} - \widetilde{X}\hat{\beta})'(\widetilde{Y} - \rho\widetilde{WY} - \widetilde{X}\hat{\beta}). \tag{22}$$

Substituting (21) and (22) into the likelihood (19) gives a concentrated likelihood function depending on a single unknown parameter ρ:

$$LogL_{N,T}(\rho) = C - \frac{NT}{2}(\widetilde{Y} - \rho\widetilde{WY} - \widetilde{X}\hat{\beta})'(\widetilde{Y} - \rho\widetilde{WY} - \widetilde{X}\hat{\beta}) + \sum_{t=1}^{T} ln|I_N - \rho W_t|, \tag{23}$$

where C is a constant term which is not relying on ρ. Finally, by maximizing (23), we can obtain a solution to ρ. Even though a closed-form solution of ρ does not exist, the numerical solution is unique because the concentrated log-likelihood (23) is a concave function in ρ. Once we have a ML estimator of ρ, we can compute the estimates of β and σ_v^2 by replacing it in (21) and (22).

For the asymptotic properties of the QML estimators, we need the following regularity assumptions for the spatial weight matrix, W_t.

Assumption A1 *W_t's are row-normalized non-stochastic spatial weights matrices with zero diagonals for all t.*

Assumption A2 *$I_N - \rho W_t$ is invertible for all t and for all $\rho \in \Lambda$, where the parameter space Λ is compact and ρ_0 is in the interior of Λ.*

Assumption A3 *W_t's are uniformly bounded in both row and column sums in absolute value, uniformly in t. Also, $(I_N - \rho W_t)^{-1}$'s are uniformly bounded, uniformly in $\rho \in \Lambda$ and t.*

[3] The bias-corrected $\hat{\sigma}_{v,BC}^2$ proposed by Lee and Yu (2010a) to avoid possible bias caused by the demeaning procedure can be obtained as $\hat{\sigma}_{v,BC}^2 = \frac{T}{T-1}\hat{\sigma}_v^2$.

The asymptotic variance matrix of the parameters[4] is derived as

$$Asy.Var(\beta,\rho,\sigma_v^2) = \begin{bmatrix} \frac{1}{\sigma^2}\tilde{X}'\tilde{X} & - & - \\ \frac{1}{\sigma^2}\tilde{X}'W^{*\prime}\tilde{X}\beta & \sum_{t=1}^{T} tr(W_t^* W_t^* + W_t^{*\prime}W_t^*) + \frac{1}{\sigma^2}\beta'\tilde{X}'W^{*\prime}W^*\tilde{X}\beta & - \\ 0 & \frac{1}{\sigma^2}\sum_{t=1}^{T} tr(W_t^*) & \frac{NT}{2\sigma^4} \end{bmatrix}^{-1}, \tag{24}$$

where $W_t^* = W_t(I_N - \rho W_t)^{-1}$.

4 An Empirical Application

In this section, we apply the SARSF model to a dataset from 21 Countries[5] in the Organisation for Economic Co-operation and Development(OECD) for the period of 1960 - 2001. For the analysis, we construct the spatial weights matrices using both geographical distance and economic distance as described in Sec. 4.1. Also, for each cross-sectional interaction, we try contiguity approach and distance decaying approach with and without thresholds for the weights matrix construction. Moreover, we allow the weights matrices for changing over time if the underlying distance concept is time dependent. If a spatial weights matrix is based on economic or socio-economic distance concepts or demographic characteristics, time-varying weights matrix may need be considered for more accurate specification.

4.1 Specifications of the Spatial Weights Matrices

Prior to the estimation, we need to specify the spatial dependence structure between observations. The approaches to construct the spatial weight matrices often used in practice are roughly categorized into two groups: weights based on a contiguity or distance. We utilize both approaches following the tradition. Typically geographical relations are used for specifying the dependence, but we can use economic relations. For the geographical relations, we used the border sharing feature and the centroid distance between capital cities of each countries. For the economic relations, the bilateral trade volume measured by sum of bilateral exports and bilateral imports is used. The main distance concepts we use in this section are as follows:

- **Contiguity :** D_{G1}

  For $\forall i \neq j$,

$$d_{ij} = \begin{cases} 1 & \text{if countries } i \text{ and } j \text{ share a border;} \\ 0 & \text{otherwise.} \end{cases}$$

[4] This is a modification of the asymptotic variance matrix provided by Elhorst and Freret (2009).

[5] The 21 countries are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Iceland, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, and United States. Germany is excluded because a truncation in data.

- **Geographical Distance :** D_{G2}
  For $\forall i \neq j$,
  $$d_{ij} = bd_{ij}^{-\gamma},$$
  where bd_{ij} denotes the bilateral distance between capital cities of countries i and j, and α is any positive exponent[6].

- **Average Bilateral Trade :** D_E^A
  $$d_{ij} = \frac{1}{T}\sum_{t=1}^{T}\left(ex_{ij}^t + im_{ij}^t\right),$$
  where ex_{ij}^t and im_{ij}^t represent the bilateral exports and imports volume from country i to j at time t, respectively.

- **Yearly Bilateral Trade :** D_E^t
  $$d_{ij}^t = ex_{ij}^t + im_{ij}^t$$

D_{G1} is the most typical weights concept. In many cases the boundaries shared between spatial units play an important role in terms of spatial influence. Moreover, the contiguity approach lessens burden of calculation making the weights matrix sparser than one from other specifications. On the other hands, the distance weights such as D_{G2}, D_E^A, and D_E^t, have the advantages in that they provide more affluent information. However, all the elements of the weights matrices tend to be non-zero, so if the number of cross-sectional units gets larger, the numerical calculation can be burdensome. Hence, we generate distance weights with thresholds for the purpose of robustness check[7]. We order the centroid distances and pick the k closest (or biggest trade volume) units. In this paper, we used $k = 5$, and $k = 10$. To satisfy the regularity conditions for the asymptotic analysis, the distance concepts should be row-normalized before estimation. Denote the normalized weights matrices by W instead of D, which represents the non-normalized weights matrices.

4.2 Data Description

For the estimation of the typical production function, we need real GDP as output, and labor participation and private capital stock as factor inputs. To get those variables, we first extracted PPP converted GDP per capita($rgdpl$), PPP converted GDP per worker($rgdpl2wok$), population(pop) and investment share of $rgdpl(ki)$ from Penn World Table Version 7.1 (Heston et al., 2012). With these variables, we can easily calculate real GDP, y, and labor participation, l, as $y = rgdpl \times pop$ and

[6] We assume $\gamma = 1$ in this paper.

[7] Spatial models have a major weakness in the exogeneity of the spatial weights matrix W. W needs to be specified prior to the estimation but the specification of W is not tested in most cases. For this reason, it is examined whether the empirical results are robust to the specifications of W by estimating the same model with different spatial weights matrices in empirical research. In this paper, we check the sensitivity of the results to the choice of W using the 10 specifications of W.

$l = (rgdpl \times pop)/rgdpl2wok$, respectively. For the estimation of private capital stock, kp, we use the perpetual inventory method after obtaining real aggregate investment, which is $rgdpl \times ki \times pop/100$. In using the perpetual inventory method, we assume that the real capital stock in 1960 is depreciated real aggregate investment in 1959 and the fixed depreciation rate of 6% for all countries for all periods. Since we are interested in the effects of public capital, we also need the public capital variable. Major challenge of empirical study on public capital productivity is shortage of public capital data. Kamps (2006) provides the public capital stock estimates of 22 OECD countries from 1960 to 2001. Unfortunately, the study is outdated so the estimates are not compatible with the up-to-date Penn World Table dataset and also the estimates are in national currencies, which is not suitable for international comparison[8]. Hence we obtained the public capital stock using the multiplication of *public capital to real GDP ratio* in Kamps (2006) and the real GDP from PWT 7.1. The summary statistics of the key variables are shown in Table 1.

To construct spatial weight matrices, we need the contiguity relations, geographical distances, and economic distances. The contiguity relations are examined using queen criterion. The bilateral geographical distances(both in miles and km) between capital cities are accessible from the website of Professor Kristian Skrede Gleditsch in University of Essex[9]. For the economic distances, we obtained the bilateral trade data from NBER-UN Trade Data 1962-2000 (Feenstra et al., 2005). The dataset is based on the reports by the importing country assuming that they are more accurate than the reports by the exporting countries. For a limitation, the dataset contains several data points that indicate a country imports from itself. This happens because some aggregate items of the statistics are created simply by adding up the subcategories. To keep the regularity assumption A1, we ignored the 'self-trade' data points and we set them zeros.

4.3 Empirical Findings

Table 2 reports the LM test statistics and the robust LM test statistics for a spatially lagged dependent variable and a spatial error term and the associated $p - values$ to determine which spatial terms are appropriate. This is so called specific-to-general approach that tests the non-spatial model against the spatial lag and/or the spatial error model (Elhorst, 2014). The hypothesis of no spatially lagged dependent variable is rejected at 1% significance in both classical LM tests and robust LM tests with all time invariant spatial weights matrices. However, the test results on spatial error correlation are arguable; the hypothesis of no spatially autocorrelated error term cannot be rejected with all spatial weights matrices when using the classical LM tests while the Elhorst's robust LM test rejects the nulls in all spatial weights except for the contiguity case[10].

Tables 3 - 5 show the estimation results when adopting a non-spatial panel fixed effects model and SARSF with selected spatial weights matrices. There are 11 estimated models depending on the spatial matrix specifications: the five main models of (1) Non-spatial fixed effects, (2) Geographical

[8]It is hard to find appropriate price indexes and PPP exchange rates for all countries for the periods.

[9]`http://privatewww.essex.ac.uk/~ksg/`

[10]Because these tests are not nested each other, we are not able to choose one between SAR and SEM against each other.

contiguity, (3) Geographical distance without cut-off, (4) Average bilateral trade, and (5) Yearly bilateral trade, and the two k-nearest sub-models with $k = 5, 10$ for (3), (4), and (5), respectively. The first column of Table 3 gives the results of non-spatial fixed effects model, which is a benchmark model. The coefficients of thed three factor inputs are significantly different from zero and have the expected signs. However, we found the spatial lag model is more appropriate, so we regard the results as biased. A caveat is that we are not able to compare the coefficient estimates in the non-spatial model and their counterparts in the other models because the coefficient estimates in the models (2) - (4) and the associated sub-models (3-1), (3-2), (4-1), and (4-2) are no longer interpreted as output elasticities of the factor inputs because the existence of $(I_N - \rho W_t)^{-1}$ in the equation (15). For appropriate comparisons, we need to obtain the mean direct effects, mean indirect effects and total effects as we discussed in Sec. 2.

In Tables 6 and 7, we present the direct and indirect effects estimates based on the coefficient estimates in Tables 3 - 5. The direct effects are computed by averaging the diagonal elements of $(I_N - \rho W_t)^{-1}\beta_k$, and the indirect effects are computed by averaging the row sums of the off-diagonal elements of $(I_N - \rho W_t)^{-1}\beta_k$ with 1,000 parameter combinations draws. Tables 6 shows the results when we use the time invariant spatial weights matrices. The total effects are sum of direct and indirect effects. Overall, the total effects estimates have similar values with the results of non-spatial fixed effects displayed in Table 3. In Model (3), the direct effects of factor inputs are 0.331, 0.175, and 0.239. The indirect effects appear to be 0.128, 0.067, and 0.093, which are 27.9%, 27.7%, and 28.0% of their own total effects. By comparing the Tables 3 - 5 and the Tables 6 - 7, we can back out the feedback effects, which pass through neighbors and back to the country itself[11]. Since the coefficients estimates in Model (3) are 0.327, 0.175, and 0.236, the feedback effects are 0.004, 0.000, and 0.003 for labor, private capital, and public capital, respectively. The other columns of Table 6 show more or less similar results with the results from Model (3) except for Model (2). The estimations are robust to spatial weights matrix specification. One may find reasons for this results, but we conclude that this is resulted from a wrong specification of the contiguity matrix.

Table 7 gives the direct and indirect effects estimates when we use the time varying spatial weights matrix. We constructed the spatial weights matrix using the yearly bilateral trade volume hence we can compute the output elasticities along the time. We are almost not able to observe the variations of the output elasticities across the period. This is because the main trade partners of the most countries do not vary along the time, even though each countries prefer different countries for their partners. The total effects of the factor inputs range from 0.293 to 0.300 for labor, from 0.251 to 0.253 for private capital, from 0.380 to 0.384 for public capital. The indirect effects are around 33% of the total effects for all factor inputs across the period. We find that the total effects of labor are smaller than the results of the time invariant weights specifications, while the total effects of private capital and public capital are greater than the corresponding total effects. Especially, the difference in the total effects of labor are mainly due to the difference in the direct effects, while the difference in the total effects of private and public capital are due to the difference in the indirect

[11] A feedback effect is computed as the average of the diagonal elements of $(\rho W_t + \rho^2 W_t^2 + \rho^3 W_t^3 + \cdots)\beta_k$.

effects.

Our interests in this paper are the spillover effects of public capital stock. Especially, examining the spillover effects of a particular country to other countries or the spillover effects from other countries to a specific country is crucial. However, the average indirect effects do not give the information related to our interests. For that purpose, we separated out the marginal indirect effects for each country. First of all, as we discussed in Sec. 2, the marginal effects of spatial model can be found as:

$$\frac{\partial Y_t}{\partial X_{k,t}} = (I_N - \rho W_t)^{-1}\beta_k. \tag{25}$$

Here, $(I_N - \rho W_t)^{-1}$ is a $(N \times N)$ matrix, which makes β_k be no longer interpreted as marginal effects. Because of the pre-multiplication of $(I_N - \rho W_t)^{-1}$, the marginal effects $\frac{\partial Y_t}{\partial X_{k,t}}$ is also $(N \times N)$ matrix. The diagonal elements represent the direct effects of regressors. In other words, the effects of a unit increment of an input in a particular country on the production of the country. The off-diagonal elements can be interpreted as indirect effects. For instance, the ijth element$(i \neq j)$ represent the effect of a unit increment of an input in country i on the production of country j. Hence, the row-sum, $\sum_{i \neq j} \frac{\partial Y_i}{\partial X_{k,j}}$, can be interpreted as the spillover effects from other countries to a specific country when all other countries increase a unit in a factor input. On the other hands, the column-sum, $\sum_{j \neq i} \frac{\partial Y_i}{\partial X_{k,j}}$, can be interpreted as the total spillover effects of a particular country to the outputs of other countries. We obtain the measures using the simulation approach suggested by LeSage and Pace (2009) as we discussed in Sec. 2.

Table 8 shows the spillover effects from other countries to a specific country. Interestingly, the spillover effects of increasing a unit of factor inputs in all other countries are almost similar across the countries. Table 9 shows the total spillover effects of a particular country to other countries. In this case, the spillover effects of increasing a unit of factor inputs in a particular country shows some variations across the countries. Specifically, the countries with large economies, such as United States, United Kingdom, and France, have larger effects on other countries than smaller economies.

Let us turn our attention to the results of the efficiency analyses. With the estimation results, we obtained the efficiency scores and rankings of countries. Table 10 displays the relative efficiency scores estimates and the rankings. In the non-spatial fixed effects model, Iceland appears to be the most efficient country, while Japan is the least efficient country. However, we regard the estimation results as biased because we found the spatial lag model is more appropriate from the Lagrange Multiplier tests, hence the efficiency estimates are also biased. The other four models show relatively low efficiency scores than the efficiency scores of non-spatial fixed effects model. Moreover, the efficiency scores change dramatically when we incorporate the spatial autoregressive term. Except for Model (2), which uses geographical contiguity as spatial weights, Model (3), (4), (5) are showing quite robust results. In these models, United States is the most efficient, while Iceland is the least efficient. From the change between the Model (1) and other Models, we conclude that smaller economies get relatively less benefits from other countries, while larger economies get more benefits from other countries. The countries that have relatively low average GDP, such as Iceland, Ireland, and New Zealand, get lower ranks in efficiency, while the countries that have relatively high average GDP, such as France, Italy, Japan, UK, and US, get higher ranks in SARSF models.

From the results, one can observe some variations in efficiency ranking between non-spatial model and spatial models. As our spatial weights rely on the economic distance, specifically on the relative importance as trade partner, we roughly guess that the changes in efficiency ranking are caused by the (relative) size of trade or the size of the economy. Hence, we examined the relationship between the changes in efficiency scores and either GDP share (% of World GDP) or Trade-to-GDP share[12]. Assuming all spatial models deliver similar effects, we confine the analysis on the comparison of Model (1) and (4). Fig. 1 shows the relation between the absolute change in efficiency ranking and GDP share. We observe the positive correlation between the two indices. The correlation coefficient is 0.149. The relation is reasonable because we have checked the indirect effects tend to be larger if the size of economy is larger(See Table 9). The non-spatial Fixed effects model cannot capture the indirect effects appropriately that the efficiency rankings might be inaccurate. Fig. 2 display the same plots with GDP share $< 1\%$. We still can see the positive correlation[13]. For the next, Fig. 3 illustrates the relation between the changes in efficiency scores and the trade openness. The trade openness is measured by Trade-to-GDP share. We observe a negative correlation. The correlation coeffcient is -0.10. This is possible because the correlation coefficient between GDP share and Trade-to-GDP is -0.45. In other word, the relative trades to GDPs of the countries that have relatively small economies tend to be large. Hence, for those countries the indirect effects are relatively small, which results in the negative relation between the changes in efficiency scores and trade openness.

For the distributional comparisons of the models, we plot the kernel densities of the efficiency scores in Fig. 5. We found the distribution from the non-spatial fixed effects model located slightly to the right side of the density plots of other four models. Finally, in Fig. 6 we compared the efficiency scores distributions when we include the public capital stock as a factor input with the distributions when we exclude the public capital stock. We observe the efficiency scores distributions are shifted to the right when we add public capital as a factor input except for the case of Model (2), which implies that the inclusion of public capital stock variable helps to explain some of variations in production.

5 Conclusions

In this paper, we investigate the spillover effects of public capital stock in a production frontier model that accounts for cross sectional dependency among countries. We estimate the output elas-

[12]The data sources of GDP share and Trade-to-GDP are `https://www.quandl.com/collections/economics/gdp-as-share-of-world-gdp-at-ppp-by-country` and World Bank(`http://data.worldbank.org/indicator/NE.TRD.GNFS.ZS`), respectively.

[13]For Iceland, we guess that the spatial effects might be reflected excessively because the size of economy is too small relative to other observations in the sample. One needs to be careful for interpretation on this sample.

ticity of public capital stock as well as labor and private capital stock in a Cliff-Ord type production frontier function. Especially, we separate out the direct and indirect effects of factor inputs and interpret the indirect effects as the spillovers. Moreover, the spatial autoregressive stochastic frontier model allow us to gain the relative efficiencies of countries as well. The spatial weights matrix is the essential characteristic of the approach and we exploit the geographical and economic interactions to construct our spatial weights matrices. Specifically, we allow for the spatial weights matrix to vary across time, which is reasonable for economic/socioeconomic spatial weights. In addition, we compare the estimation results based on a number of specifications of the spatial dependency structure and observe robustness of our model.

We estimate the output elasticities of factor inputs at international level by the empirical application to the data for 21 OECD countries from 1960 to 2001. We found relatively large total output elasticities of public capital stock and observe that the indirect effects comprise around 30% of the total elasticities, which supports the argument of Pereira and Andraz (2013). We obtained larger total output elasticity of public capital stock when we use the time varying weights matrix. Finally, we measure the relative technical efficiencies of countries. Concerning the spatial dependency, SARSF model is expected to give the less biased relative efficiency estimates than the non-spatial counterpart, but more delicate comparison between efficiency estimates from different models should be analyzed by the future works. Compare to the analysis that utilizes only the traditional factor inputs, labor and private capital stock, inclusion of public capital stock results in shift of the efficiency score distribution to the right, which implies that some of production process is explained by public capital stock variable additionally.

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Table 1: Summary Statistics

	Mean	St.Dev.	Min	Max
Real GDP(millions)	875.8	1,598.5	3.92	11,265.5
Labor participation(thousands)	16,448	26,188	79	144,746
Private Capital(millions)	1,941.3	3,434.6	7.46	24,753.4
Public Capital(millions)	528.0	1,004.8	2.12	5,705.4

Table 2: Lagrange Multiplier Tests for a spatially lagged dependent variable and spatial error correlation

	Spatial lag ($H_0 : \rho = 0$)				**Spatial error ($H_0 : \lambda = 0$)**			
Spatial Weights	LM		Robust LM		LM		Robust LM	
Geo: Contiguity	36.17	(0.000)	36.75	(0.000)	0.07	(0.784)	0.65	(0.419)
Geo: Distance (no cut off)	2106.36	(0.000)	3184.42	(0.000)	0.36	(0.550)	1078.42	(0.000)
Geo: Distance (closest 5)	755.43	(0.000)	947.97	(0.000)	0.18	(0.667)	192.73	(0.000)
Geo: Distance (closest 10)	1730.85	(0.000)	2322.63	(0.000)	0.31	(0.575)	592.09	(0.000)
Average Trade (no cut off)	902.68	(0.000)	1128.23	(0.000)	0.14	(0.705)	225.69	(0.000)
Average Trade (biggest 5)	813.92	(0.000)	1116.02	(0.000)	0.10	(0.756)	302.19	(0.000)
Average Trade (biggest 10)	907.07	(0.000)	1174.70	(0.000)	0.12	(0.726)	267.76	(0.000)

Note: The numbers in parentheses are p-values.

Table 3: Estimation results of a non-spatial panel fixed effects model and SARSF with time invariant spatial weights matrices

	(1)		(2)		(3)		(4)	
Determinants	**Non-spatial FE**		**Geo: Contiguity**		**Geo: Distance**		**Average Trade**	
$Log(L)$	0.489	(15.922)	0.646	(17.795)	0.327	(9.954)	0.300	(8.994)
$Log(Kp)$	0.240	(22.279)	0.296	(21.587)	0.175	(10.692)	0.177	(11.605)
$Log(Kg)$	0.302	(15.191)	0.332	(15.049)	0.236	(12.092)	0.241	(12.490)
$W \cdot Log(Y)$	-	-	-0.236	(-10.741)	0.284	(7.464)	0.276	(8.004)
$\hat{\sigma}_v^2$	0.008		0.010		0.008		0.008	
$Log - likelihood$			NaN		909.202		912.892	

Note: The numbers in parentheses are t-stats.

Table 4: Estimation results of SARSF with k-nearest spatial weights matrices

Determinants	**Geo: Distance**				**Average Trade**			
	(3-1) Closest 5		(3-2) Closest 10		(4-1) Biggest 5		(4-2) Biggest 10	
$Log(L)$	0.341	(10.191)	0.330	(9.980)	0.288	(8.466)	0.301	(8.972)
$Log(Kp)$	0.193	(12.110)	0.183	(11.014)	0.186	(12.674)	0.179	(11.827)
$Log(Kg)$	0.261	(13.383)	0.243	(12.446)	0.244	(12.686)	0.243	(12.630)
$W \cdot Log(Y)$	0.213	(6.001)	0.259	(6.688)	0.261	(7.979)	0.270	(7.955)
$\hat{\sigma}_v^2$	0.008		0.008		0.008		0.008	
$Log - likelihood$	894.227		903.506		908.300		911.369	

Note: The numbers in parentheses are t-stats.

Table 5: Estimation results of SARSF with time varying spatial weights matrices

Determinants	(5) Yearly Trade		(5-1) Biggest 5		(5-2) Biggest 10	
$Log(L)$	0.197	(5.811)	0.233	(6.823)	0.190	(5.622)
$Log(Kp)$	0.167	(10.903)	0.194	(12.903)	0.163	(10.876)
$Log(Kg)$	0.253	(12.550)	0.262	(13.085)	0.254	(12.617)
$W \cdot Log(Y)$	0.337	(19.015)	0.252	(7.664)	0.336	(10.106)
$\hat{\sigma}_v^2$	0.008		0.008		0.008	
$Log - likelihood$	2,154.899		2,158.198		2,157.316	

Note: The numbers in parentheses are t-stats.

Table 6: Direct and indirect effects estimates based on the coefficient estimates in Table 3 and 4

Effects	Geo: Contiguity		Geo: Distance						Average Trade					
	(2)		(3) **No cut off**		(3-1) **Closest 5**		(3-2) **Closest 10**		(4) **No cut off**		(4-1) **Biggest 5**		(4-2) **Biggest 10**	
log(L)														
Total	0.529	(18.990)	0.459	(11.451)	0.447	(11.099)	0.433	(11.356)	0.416	(10.336)	0.413	(11.083)	0.387	(9.887)
Direct	0.663	(17.345)	0.331	(10.507)	0.333	(9.973)	0.343	(10.202)	0.304	(9.475)	0.305	(9.802)	0.290	(8.761)
Indirect	-0.134	(-8.262)	0.128	(5.814)	0.114	(5.502)	0.090	(5.510)	0.112	(6.421)	0.109	(6.710)	0.097	(6.796)
log(Kp)														
Total	0.242	(23.975)	0.242	(16.159)	0.247	(17.077)	0.246	(17.852)	0.244	(16.140)	0.244	(16.408)	0.251	(17.776)
Direct	0.304	(20.453)	0.175	(11.086)	0.184	(11.281)	0.195	(12.738)	0.178	(11.679)	0.180	(11.780)	0.188	(13.551)
Indirect	-0.061	(-8.435)	0.067	(7.436)	0.063	(6.900)	0.051	(6.460)	0.066	(8.370)	0.064	(8.308)	0.063	(8.043)
log(Kg)														
Total	0.272	(14.420)	0.332	(11.822)	0.328	(11.522)	0.332	(12.566)	0.333	(11.410)	0.333	(11.721)	0.330	(12.252)
Direct	0.340	(14.660)	0.239	(12.068)	0.244	(12.430)	0.262	(13.667)	0.243	(12.175)	0.245	(12.573)	0.247	(12.874)
Indirect	-0.069	(-8.679)	0.093	(5.474)	0.084	(4.944)	0.070	(4.877)	0.090	(5.801)	0.088	(5.720)	0.083	(5.995)

Note: The numbers in parentheses are t-stats.

Table 7: Direct and indirect effects estimates based on the coefficient estimates in Table 5

Effects	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980
log(L)																					
Total	0.295	0.296	0.297	0.296	0.294	0.293	0.294	0.296	0.296	0.296	0.297	0.297	0.297	0.297	0.296	0.295	0.295	0.298	0.296	0.300	0.296
	(6.555)	(6.193)	(6.451)	(6.392)	(6.496)	(6.222)	(6.245)	(6.278)	(6.226)	(6.264)	(6.278)	(6.362)	(6.238)	(6.209)	(6.455)	(5.938)	(6.348)	(6.470)	(6.259)	(6.237)	(6.471)
Direct	0.198	0.199	0.200	0.199	0.198	0.197	0.197	0.199	0.199	0.199	0.200	0.200	0.200	0.200	0.200	0.199	0.198	0.201	0.199	0.202	0.199
	(6.034)	(5.742)	(5.967)	(5.969)	(5.980)	(5.778)	(5.815)	(5.791)	(5.753)	(5.773)	(5.850)	(5.899)	(5.711)	(5.752)	(5.914)	(5.527)	(5.832)	(6.057)	(5.768)	(5.840)	(6.013)
Indirect	0.097	0.096	0.097	0.097	0.096	0.095	0.097	0.097	0.097	0.096	0.097	0.097	0.097	0.097	0.096	0.096	0.097	0.097	0.097	0.098	0.096
	(5.815)	(5.584)	(5.717)	(5.722)	(5.847)	(5.518)	(5.672)	(5.684)	(5.528)	(5.620)	(5.522)	(5.758)	(5.706)	(5.690)	(5.768)	(5.478)	(5.737)	(5.683)	(5.791)	(5.551)	(5.785)
log(Kp)																					
Total	0.251	0.252	0.252	0.251	0.252	0.252	0.251	0.251	0.252	0.252	0.251	0.252	0.251	0.253	0.252	0.252	0.252	0.252	0.251	0.251	0.252
	(15.181)	(14.884)	(15.257)	(15.084)	(15.561)	(14.517)	(15.055)	(14.644)	(14.816)	(15.053)	(14.668)	(15.083)	(15.396)	(15.606)	(15.109)	(15.102)	(14.976)	(14.868)	(14.673)	(14.868)	(15.004)
Direct	0.169	0.170	0.170	0.169	0.169	0.170	0.169	0.169	0.170	0.170	0.169	0.169	0.169	0.170	0.170	0.170	0.169	0.170	0.169	0.169	0.170
	(11.131)	(11.018)	(11.429)	(11.248)	(11.712)	(10.64)	(11.179)	(10.746)	(10.67)	(11.373)	(10.737)	(11.323)	(11.435)	(11.418)	(11.144)	(11.058)	(11.074)	(11.001)	(11.218)	(11.139)	(11.245)
Indirect	0.083	0.082	0.082	0.082	0.082	0.082	0.083	0.082	0.083	0.082	0.082	0.082	0.082	0.082	0.082	0.082	0.083	0.082	0.082	0.082	0.082
	(9.329)	(9.446)	(9.201)	(9.842)	(9.309)	(9.378)	(9.724)	(9.579)	(9.408)	(8.932)	(9.203)	(9.549)	(9.054)	(9.807)	(9.107)	(9.753)	(9.414)	(9.790)	(9.339)	(9.682)	(9.596)
log(Kg)																					
Total	0.384	0.382	0.381	0.384	0.383	0.383	0.384	0.384	0.382	0.381	0.382	0.382	0.382	0.381	0.382	0.382	0.383	0.381	0.384	0.383	0.382
	(11.787)	(11.497)	(12.154)	(11.812)	(11.931)	(11.423)	(11.949)	(11.509)	(11.221)	(11.801)	(11.652)	(11.514)	(11.832)	(11.873)	(11.774)	(11.412)	(11.464)	(11.471)	(11.422)	(11.509)	(11.652)
Direct	0.257	0.257	0.256	0.257	0.257	0.257	0.257	0.257	0.256	0.257	0.257	0.257	0.257	0.256	0.257	0.257	0.257	0.256	0.257	0.257	0.257
	(12.692)	(12.390)	(12.786)	(12.940)	(12.492)	(12.471)	(13.073)	(12.897)	(12.539)	(12.255)	(12.582)	(12.338)	(12.532)	(12.848)	(12.594)	(12.731)	(12.632)	(12.542)	(12.222)	(12.451)	(12.577)
Indirect	0.127	0.125	0.125	0.126	0.126	0.125	0.127	0.127	0.126	0.125	0.126	0.125	0.125	0.125	0.125	0.125	0.127	0.125	0.126	0.126	0.125
	(6.499)	(6.452)	(6.654)	(6.622)	(6.756)	(6.323)	(6.646)	(6.285)	(6.111)	(6.580)	(6.394)	(6.513)	(6.526)	(6.664)	(6.398)	(6.306)	(6.376)	(6.461)	(6.528)	(6.527)	(6.598)

Effects	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
log(L)																					
Total	0.294	0.295	0.297	0.296	0.295	0.293	0.295	0.298	0.299	0.295	0.295	0.295	0.295	0.296	0.295	0.296	0.294	0.295	0.295	0.297	0.296
	(6.386)	(6.317)	(6.376)	(6.360)	(6.224)	(6.472)	(6.164)	(6.478)	(6.115)	(6.158)	(6.479)	(6.470)	(6.142)	(6.165)	(6.355)	(6.408)	(6.410)	(6.337)	(6.481)	(6.322)	(6.410)
Direct	0.198	0.198	0.2	0.2	0.199	0.197	0.198	0.2	0.201	0.199	0.199	0.199	0.199	0.199	0.2	0.199	0.198	0.199	0.199	0.2	0.199
	(5.803)	(5.714)	(5.864)	(5.963)	(5.805)	(5.956)	(5.766)	(6.040)	(5.754)	(5.735)	(5.932)	(5.943)	(5.718)	(5.873)	(5.928)	(5.880)	(5.908)	(5.783)	(6.091)	(5.857)	(5.871)
Indirect	0.096	0.096	0.097	0.097	0.096	0.096	0.096	0.097	0.098	0.096	0.096	0.097	0.096	0.097	0.096	0.097	0.096	0.097	0.096	0.097	0.097
	(5.915)	(5.815)	(5.811)	(5.589)	(5.606)	(5.826)	(5.556)	(5.656)	(5.514)	(5.560)	(5.914)	(5.878)	(5.500)	(5.367)	(5.620)	(5.851)	(5.881)	(5.868)	(5.683)	(5.740)	(5.792)
log(Kp)																					
Total	0.252	0.251	0.252	0.251	0.251	0.253	0.252	0.251	0.252	0.252	0.252	0.251	0.252	0.251	0.252	0.251	0.252	0.252	0.253	0.252	0.251
	(14.717)	(14.362)	(15.569)	(14.502)	(14.733)	(15.154)	(15.524)	(14.243)	(15.111)	(15.245)	(15.228)	(14.276)	(15.007)	(14.830)	(14.327)	(15.511)	(15.510)	(15.263)	(16.037)	(15.238)	(15.229)
Direct	0.170	0.169	0.170	0.169	0.169	0.170	0.169	0.169	0.169	0.170	0.169	0.169	0.170	0.169	0.170	0.169	0.169	0.170	0.171	0.170	0.169
	(10.946)	(10.474)	(11.469)	(10.826)	(10.842)	(11.371)	(11.662)	(10.805)	(11.401)	(11.235)	(11.318)	(10.833)	(11.087)	(11.082)	(10.689)	(11.557)	(11.485)	(11.388)	(11.881)	(11.519)	(11.096)
Indirect	0.082	0.082	0.082	0.082	0.082	0.083	0.082	0.082	0.083	0.082	0.082	0.082	0.082	0.082	0.082	0.082	0.082	0.083	0.082	0.083	0.083
	(9.232)	(9.190)	(9.571)	(9.604)	(9.768)	(9.231)	(9.517)	(9.220)	(9.773)	(9.480)	(9.563)	(9.431)	(9.401)	(9.719)	(9.411)	(9.388)	(9.857)	(9.359)	(9.519)	(9.540)	(9.436)
log(Kg)																					
Total	0.382	0.383	0.382	0.383	0.384	0.382	0.383	0.383	0.382	0.382	0.383	0.384	0.382	0.383	0.382	0.383	0.383	0.382	0.380	0.381	0.384
	(11.232)	(10.906)	(11.626)	(11.345)	(11.707)	(12.151)	(12.119)	(11.017)	(11.961)	(11.686)	(11.795)	(11.346)	(11.785)	(11.549)	(10.981)	(12.115)	(12.006)	(11.627)	(12.510)	(11.937)	(11.675)
Direct	0.257	0.257	0.256	0.257	0.258	0.256	0.257	0.257	0.256	0.256	0.257	0.258	0.257	0.257	0.257	0.257	0.257	0.256	0.256	0.256	0.257
	(12.168)	(12.208)	(12.776)	(12.456)	(13.110)	(13.013)	(12.785)	(11.847)	(12.818)	(12.734)	(12.871)	(12.411)	(12.783)	(12.336)	(11.985)	(12.803)	(13.050)	(12.813)	(13.097)	(12.703)	(12.770)
Indirect	0.125	0.126	0.125	0.126	0.126	0.126	0.126	0.126	0.126	0.125	0.126	0.126	0.125	0.126	0.124	0.126	0.126	0.126	0.124	0.126	0.127
	(6.375)	(6.003)	(6.420)	(6.399)	(6.426)	(6.617)	(6.786)	(6.367)	(6.782)	(6.503)	(6.525)	(6.374)	(6.429)	(6.611)	(6.257)	(6.714)	(6.665)	(6.449)	(6.849)	(6.730)	(6.364)

Note: The numbers in parentheses are t-stats.

Table 8: Indirect effects(row-sum): $\sum_{i \neq j} \frac{\partial Y_i}{\partial X_{k,j}}$

	Labor		Private Capital		Public Capital	
	Indirect effects	t-stat	Indirect effects	t-stat	Indirect effects	t-stat
Australia	0.113	6.725	0.067	8.315	0.091	5.694
Austria	0.113	6.704	0.067	8.277	0.092	5.675
Belgium	0.111	6.781	0.065	8.418	0.090	5.745
Canada	0.106	6.904	0.063	8.645	0.086	5.855
Denmark	0.113	6.714	0.067	8.297	0.092	5.685
Finland	0.113	6.705	0.067	8.280	0.092	5.676
France	0.109	6.833	0.065	8.514	0.089	5.792
Greece	0.114	6.692	0.067	8.256	0.092	5.665
Iceland	0.114	6.684	0.067	8.242	0.093	5.658
Ireland	0.113	6.710	0.067	8.288	0.092	5.681
Italy	0.111	6.785	0.066	8.425	0.090	5.748
Japan	0.109	6.844	0.064	8.535	0.088	5.802
Netherlands	0.111	6.777	0.066	8.410	0.090	5.741
New Zealand	0.113	6.699	0.067	8.269	0.092	5.671
Norway	0.113	6.715	0.067	8.298	0.092	5.686
Portugal	0.113	6.702	0.067	8.274	0.092	5.674
Spain	0.112	6.738	0.066	8.340	0.091	5.706
Sweden	0.112	6.740	0.066	8.344	0.091	5.708
Swaziland	0.113	6.722	0.067	8.311	0.092	5.692
United Kingdom	0.110	6.820	0.065	8.490	0.089	5.780
United States	0.101	7.103	0.060	9.027	0.082	6.039

Table 9: Indirect effects(column-sum): $\sum_{j \neq i} \frac{\partial Y_i}{\partial X_{k,j}}$

	Labor		Private Capital		Public Capital	
	Indirect effects	t-stat	Indirect effects	t-stat	Indirect effects	t-stat
Australia	0.053	7.015	0.031	9.417	0.043	5.953
Austria	0.035	6.802	0.020	8.999	0.028	5.770
Belgium	0.130	6.596	0.076	8.611	0.104	5.596
Canada	0.108	5.735	0.064	7.111	0.087	4.886
Denmark	0.058	7.139	0.034	9.661	0.047	6.061
Finland	0.037	6.967	0.022	9.318	0.030	5.912
France	0.253	6.818	0.149	9.031	0.204	5.784
Greece	0.015	6.659	0.009	8.729	0.012	5.649
Iceland	0.003	6.962	0.002	9.308	0.002	5.907
Ireland	0.032	6.315	0.019	8.099	0.026	5.361
Italy	0.187	6.914	0.110	9.217	0.150	5.866
Japan	0.198	6.665	0.116	8.743	0.159	5.653
Netherlands	0.149	6.782	0.088	8.962	0.120	5.753
New Zealand	0.014	6.913	0.008	9.215	0.011	5.864
Norway	0.055	7.027	0.032	9.438	0.044	5.964
Portugal	0.028	6.885	0.016	9.160	0.022	5.841
Spain	0.090	6.842	0.053	9.077	0.072	5.804
Sweden	0.105	7.189	0.061	9.762	0.084	6.105
Swaziland	0.070	6.735	0.041	8.871	0.056	5.713
United Kingdom	0.303	7.056	0.178	9.495	0.244	5.988
United States	0.427	6.722	0.251	8.850	0.344	5.700

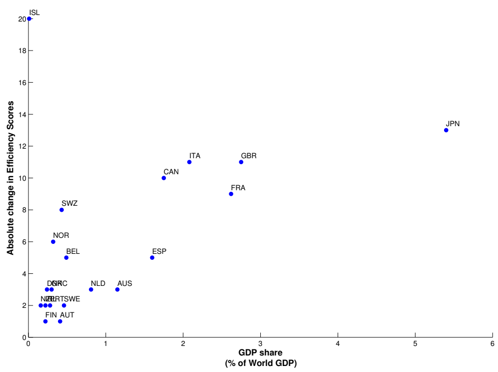

Figure 1: GDP share versus Change in Efficiency Scores

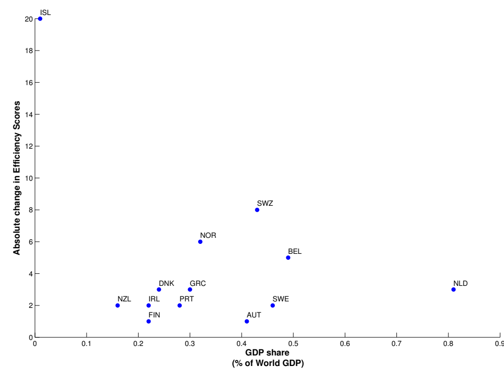

Figure 2: GDP share versus Change in Efficiency Scores (Selected Countries)

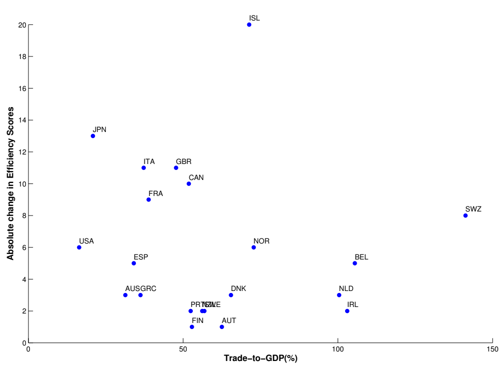

Figure 3: Trade Openness versus Change in Efficiency Scores

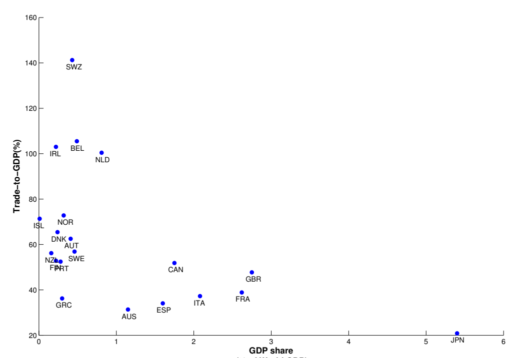

Figure 4: GDP share versus Trade Openness

Table 10: Estimation results of Efficiency Scores

	(1) **Non-spatial FE**		(2) **Geo: Contiguity**		(3) **Geo: Distance**		(4) **Average Trade**		(5) **Yearly Trade**	
Country	Eff. Score	Ranking	Eff. Score	Ranking	Eff. Score	Ranking	Eff. Score	Ranking	Eff. Score	Ranking
Australia	77.3 %	8	22 %	17	48.0 %	7	34.9 %	11	25.2 %	11
Austria	71.7 %	13	38.9 %	8	34.3 %	14	32.4 %	14	23.4 %	13
Belgium	86.8 %	4	51 %	3	39.0 %	12	39.3 %	9	28.4 %	9
Canada	87.3 %	3	66.9 %	2	45.7 %	8	32.8 %	13	23.9 %	12
Denmark	72.7 %	12	38.2 %	9	32.4 %	15	32.1 %	15	22.2 %	15
Finland	69.7 %	16	32 %	12	30.3 %	17	28.9 %	17	20.4 %	16
France	73.0 %	11	25.3 %	16	60.9 %	2	60.1 %	2	52.5 %	3
Greece	67.8 %	19	44.7 %	5	31.3 %	16	29.1 %	16	20.3 %	17
Iceland	100.0 %	1	100 %	1	17.2 %	21	15.5 %	21	7.7 %	21
Ireland	68.2 %	18	31.9 %	13	22.9 %	20	19.3 %	20	12.1 %	20
Italy	70.9 %	15	21.5 %	18	60.2 %	3	56.5 %	4	47.6 %	4
Japan	51.6 %	21	0.1 %	21	58.3 %	5	41.4 %	8	36.5 %	6
Netherlands	76.1 %	9	35.9 %	11	43.1 %	10	42.6 %	6	32 %	7
New Zealand	69.4 %	17	44.5 %	6	23.5 %	19	20.2 %	19	12.4 %	19
Norway	83.2 %	6	36.2 %	10	36.2 %	13	33.7 %	12	23.1 %	14
Portugal	66.8 %	20	39.4 %	7	28.1 %	18	28.1 %	18	19.9 %	18
Spain	76.1 %	10	28.2 %	15	53.7 %	6	48.7 %	5	37.4 %	5
Sweden	83.8 %	5	31.1 %	14	45.3 %	9	42.5 %	7	31.7 %	8
Switzerland	87.6 %	2	49.4 %	4	40.1 %	11	37.3 %	10	26.6 %	10
United Kindom	71.0 %	14	17.7 %	20	60.0 %	4	59.3 %	3	53.9 %	2
United States	78.3 %	7	19.6 %	19	100.0 %	1	100 %	1	100 %	1

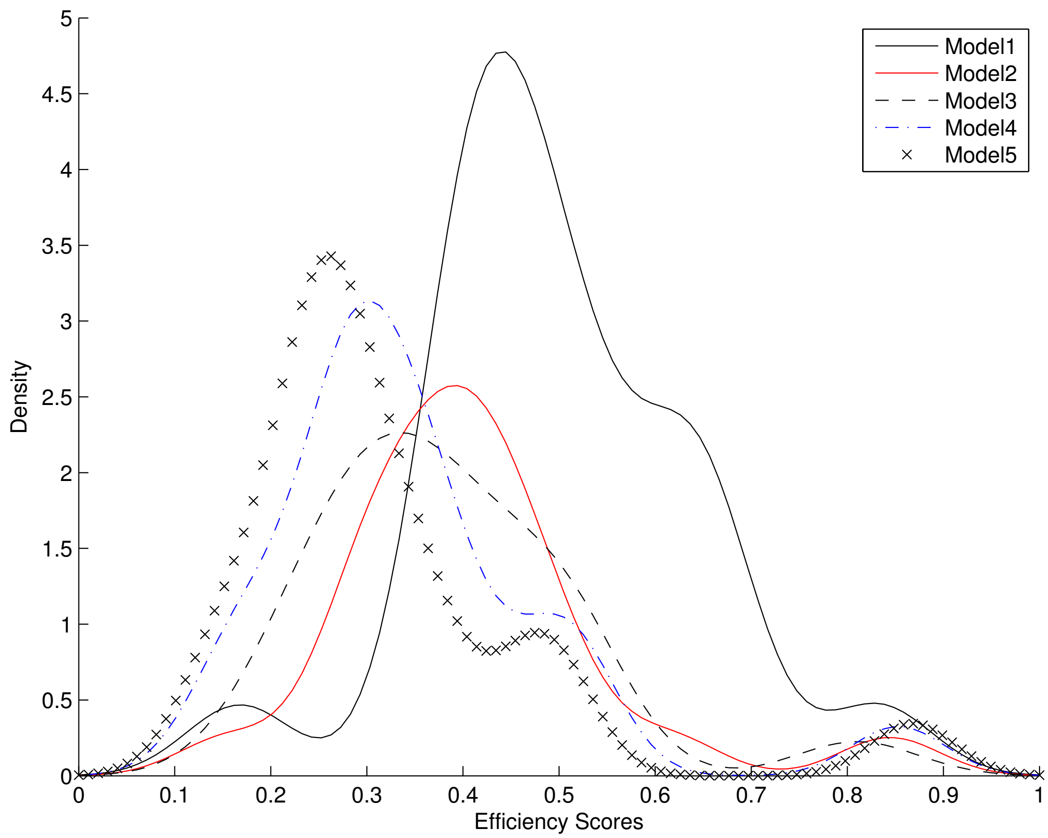

Figure 5: Kernel Densities of Efficiency Scores

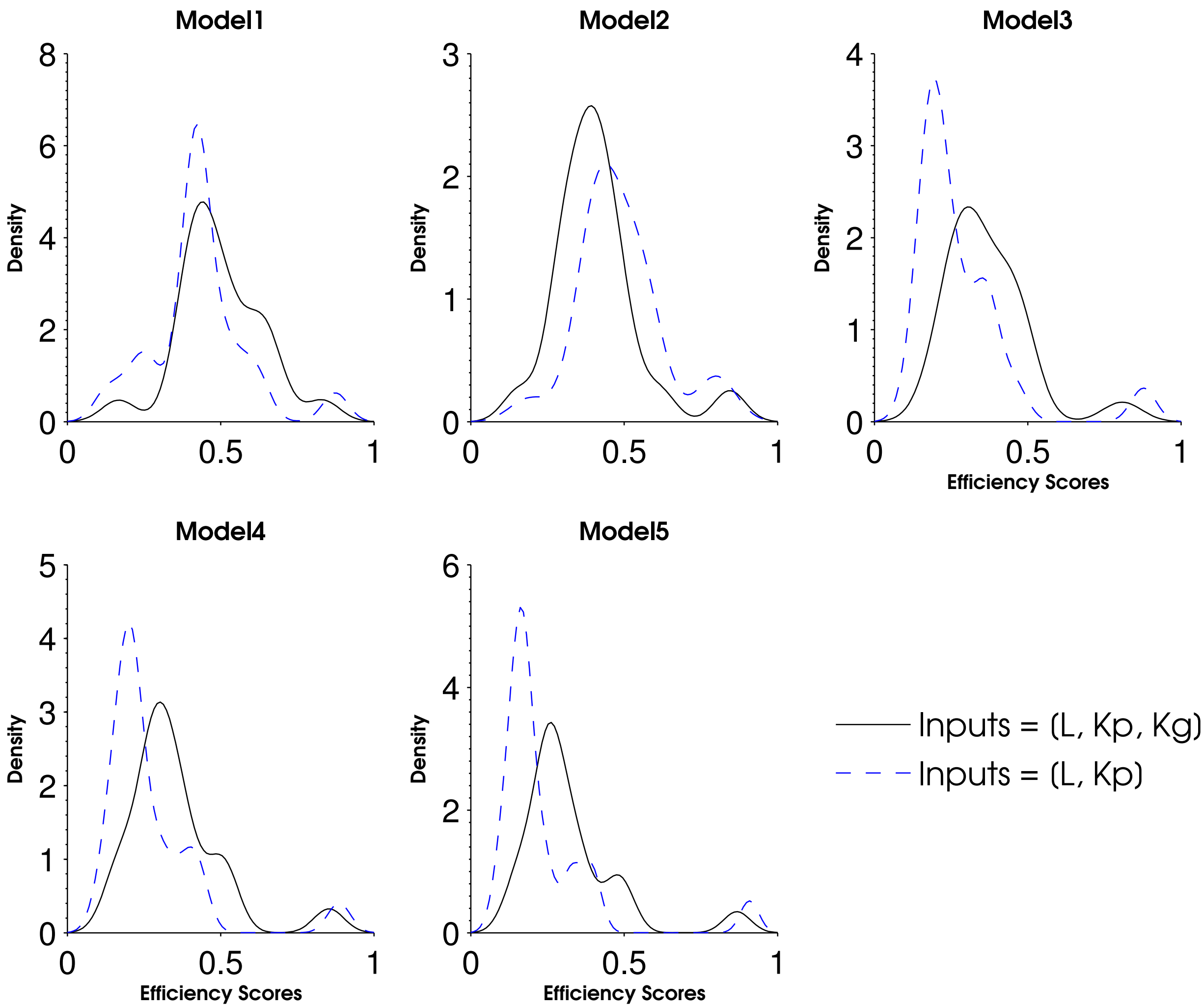

Figure 6: Comparisons of Densities of Efficiency Scores