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by

T. Renee Bowen, Ying Chen, Hülya Eraslan, Jan Zápal

RICE
Department of Economics
Baker Hall, MS22
6100 Main Street, Houston, Texas 77005
https://economics.rice.edu

Efficiency of Flexible Budgetary Institutions

T. Renee Bowen* Ying Chen† Hülya Eraslan‡ Jan Zápal§

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Abstract

Which budgetary institutions result in efficient provision of public goods? We analyze a model with two parties deciding the allocation to a public good each period. Parties place different values on the public good, but these values may change over time. We model a budgetary institution as the rules governing feasible allocations to mandatory and discretionary spending programs. We model mandatory spending as an endogenous status quo since it is enacted by law and remains in effect until changed, and discretionary spending as periodic appropriations that are not allocated if no new agreement is reached. We consider budgetary institutions that either allow only discretionary programs, only mandatory programs, an endogenous choice of mandatory and discretionary programs, or state-contingent mandatory programs. We show that discretionary only institutions can lead to dynamic inefficiencies, mandatory only institutions can lead to static and dynamic inefficiencies, whereas allowing mandatory programs with appropriate flexibility results in static and dynamic efficiency.

* Stanford University
† Johns Hopkins University
‡ Rice University
§ IAE-CSIC, Barcelona GSE and CERGE-EI Prague

1 Introduction

Allocation of resources to public goods are typically decided through government budget negotiations. In many democracies these negotiations occur annually, and are constrained by the *budgetary institutions* in place. We focus on budgetary institutions that specify the rules governing feasible allocations to mandatory and discretionary spending programs.[1] Discretionary programs require periodic appropriations; no spending is allocated if no new agreement is reached. By contrast, mandatory programs are enacted by law, and spending continues into the future until changed. Thus under mandatory programs, spending decisions today determine the status quo level of spending for tomorrow. We focus on three budgetary institutions: those that allow only discretionary programs, those that allow only mandatory program and those that allow for endogenously chosen combination of both types of programs.

Naturally, there may be disagreement among groups on the appropriate level of public spending, thus the final spending outcome is the result of negotiations between parties that represent these groups' interests. Negotiations are typically led by the party in power whose identity may change over time, and thus there is turnover in agenda-setting power. Furthermore, the economic environment is also changing over time, potentially resulting in changes in preferences. Hence the party in power today must consider how current spending on the public good affects future spending, when preferences and the agenda-setter possibly change in the future. In this paper we investigate the role of budgetary institutions in the efficient provision of public goods in an environment with these features - disagreement over the value of the public good, a changing economic environment, and turnover in political power.

We begin by analyzing a model in which two parties bargain over the allocation to a public good in each of two periods. The parties place different values on the public good, reflecting possible disagreement, and these values may change over time, reflecting changes in the underlying economic environment. To capture turnover in political power we assume

[1]This terminology is used in the United States budget. Related institutions exist in other budget negotiations, for example the budget of the European Union is categorized into commitment and payment appropriations. The main distinction is that one has dynamic consequences because agreements are made for future budgets, and the other does not.

the proposing party is selected at random each period. Unanimity is required to implement the proposed spending on the public good, thus the proposing party requires the responding to party to accept the proposal. We compare the equilibrium outcome of this bargaining game under different budgetary institutions to Pareto efficient outcomes.

We distinguish between a *statically Pareto efficient allocation* and a *dynamically Pareto efficient allocation*. A statically Pareto efficient allocation in a given period is an allocation such that there is no alternative allocation that would make both parties better off and at least one of them strictly better off in that period. A dynamically Pareto efficient allocation is a sequence of allocations, one for each period, that needs to satisfy a similar requirement except that the utility possibility frontier is constructed using the discounted sum of utilities. Dynamic efficiency puts intertemporal restrictions on allocations in addition to requiring static efficiency for each period, making it is a stronger requirement than static Pareto efficiency. We show that if parties disagree about the value of the public good in all periods, then any equilibrium in which spending varies with the identity of the proposer cannot be dynamically Pareto efficient. That is, dynamic Pareto efficiency requires that parties insure against *political risk*.

Comparing equilibrium public good allocations with the efficient ones, we show that discretionary only institutions lead to static efficiencies *but* dynamic inefficiencies, mandatory only institutions can lead to static *and* dynamic inefficiencies, whereas allowing an *endogenous* choice between mandatory and discretionary programs results in public good allocations that are statically *and* dynamically Pareto efficient if the value of the public good is decreasing over time. Furthermore, we show that if temporary cuts to mandatory programs are allowed, an endogenous choice of mandatory and discretionary programs results in public good allocations that are dynamically and statically Pareto efficient, for any deterministic change in the value of the public good.

The intuition behind dynamic inefficiency of discretionary only budget institutions is that they lead to equilibrium public good allocations that fluctuate with the change of political power. Because the status quo size of a discretionary spending program is exogenous, the political party in power, which possesses proposal power in our model, can implement the level

of spending that reflects its preference. Because there could be turnover in political power in the future, resulting public good allocations cannot satisfy the intertemporal requirements needed for dynamic Pareto efficiency. That is, when the budgetary institution allows for only discretionary spending, the parties are exposed to political risk.

To show dynamic inefficiency that arises under the mandatory only budget institutions, we first show that in any equilibrium, the second period's public good allocation either varies with the identity of the proposing party or is equal to the first period's level. In the former case, dynamic inefficiency is immediate because the allocation depends on the identity of the proposing party. In the latter case, we obtain dynamic inefficiency if the parties' values of the public good vary across time. This is because when the parties' preferences vary across time, dynamic efficiency requires that allocations also vary across time.

Static efficiency may also obtain with mandatory only budget institutions. The reason is that the first period's public good spending becomes the status quo in the second period and thus determines the parties' bargaining positions in the second period. Concerns about their future bargaining positions can lead the parties to reach an outcome that disregards their first-period preferences, resulting in static inefficiency.

In spite of this dynamic and static inefficiency with mandatory only budgetary institutions, we obtain dynamic efficiency in two special cases. The first is if the values of the public good of the two parties are constant across time. This precludes variation in any dynamically efficient allocation. The second is if parties agree on the value of the public good in the first period. In this case dynamic efficiency does not restrict the second period allocation to be constant across proposers.

Dynamic efficiency when both discretionary and mandatory programs are allowed arises due to two ingredients. First, the party in power in the first period finds it optimal to set the size of the mandatory program so that it is statically efficient in the second period. If the status quo allocation is statically efficient in the second period, then it is maintained, and hence the second period's public good allocation is independent of the identity of the party in power, thereby eliminating the political risk. Second, the party in power in the first period can use discretionary spending to tailor the level of the first period's public good spending to

her preferences. We obtain dynamic efficiency if the discretionary spending can be negative, which we interpret as a temporary cut of mandatory programs, as this allows the party in power in the first period to accommodate any time profile of public good spending (either increasing or decreasing) that dynamic efficiency might require. If we restrict discretionary spending to be positive, we obtain dynamic efficiency only if the value of the public good decreases for the two parties since dynamic efficiency requires a decreasing time profile of public good spending in that case. In both cases, the flexibility afforded by the combination of mandatory and discretionary programs delivers dynamic efficiency.

We show that when preferences are stochastic, allowing state-contingent mandatory spending programs provides sufficient flexibility to achieve dynamic efficiency. To do this we consider an extension of the model in which preferences are random and the time horizon is arbitrary. We consider a budgetary institution in which proposers choose a spending rule that gives spending levels conditional on the realization of the state. Examples of state-contingent programs include those that are inflation adjusted or depend on the unemployment rate. The first period proposer chooses a rule that is dynamically Pareto efficient and once selected, this spending rule does not change because no future proposer can make a different proposal that is better for itself and acceptable to the other party.

Our work is related to several strands of literature. A large body of political economy literature studies policies that arise from bargaining of political actors under different decision-making rules governing their interaction (see for example Acemoglu et al., 2010; Acemoglu and Robinson, 2012; Austen-Smith and Banks, 1988; Besley and Persson, 2011; Dixit et al., 2000; Persson and Tabellini, 2000, 2003; Persson et al., 1997; Lizzeri and Persico, 2001). Still in the same category but more closely related to our are papers highlighting inefficiency of policies that arise in political equilibrium (for example Alesina and Tabellini, 1990; Azzimonti, 2011; Besley and Coate, 1998; Battaglini and Coate, 2007; Krusell and Ríos-Rull, 1996; Bai and Lagunoff, 2011; Persson and Svensson, 1984; Van Weelden, 2013). Inefficiency in these papers arises either because efficient policies yield benefits in the future when the current political representation is not in the position to enjoy them, or because efficient policies alter the choices of future policy makers, or because efficient policies lower the

probability of the current political representation remaining in power (see Besley and Coate, 1998; Bai and Lagunoff, 2011, for discussion of possible sources of inefficiencies). Our paper shares with the rest of the literature the first two sources of inefficiency, but unlike the rest of the literature, our main focus is on linking these sources of inefficiency to budgetary institutions that specify the rules governing feasible allocations to mandatory and discretionary spending programs.

Modeling mandatory spending programs as an endogenous status quo links our work to a growing dynamic bargaining literature (including Anesi and Seidmann, 2012; Baron, 1996; Baron and Bowen, 2013; Bowen et al., 2014; Battaglini and Palfrey, 2012; Chen and Eraslan, 2014; Diermeier and Fong, 2011; Duggan and Kalandrakis, 2012; Dziuda and Loeper, 2013; Forand, 2010; Kalandrakis, 2004, 2010; Levy and Razin, 2013; Nunnari and Zapal, 2012). With the exception of Bowen, Chen and Eraslan (2014) this literature has focused on studying models with policies that only have the endogenous status quo property. In the language of our model, this literature has focused on mandatory spending programs only. Bowen, Chen and Eraslan (2014) model discretionary and mandatory public good spending programs, but do not allow for endogenous choice between these two types of programs (transfers between parties in their model are modeled as discretionary). In addition, unlike in their model, we allow the values parties put on the public good to vary over time which plays an important role in our results. Bowen, Chen and Eraslan (2014) show that mandatory programs may improve the efficiency of public good provision, whereas we show that with changing preferences mandatory programs with appropriate flexibility achieves dynamic Pareto efficiency.

In the next section we describe the model of budgetary institutions. In Section 3 we discuss Pareto efficient allocations and define Pareto efficient equilibria. We discuss an institution with only discretionary spending in Section 4. In Section 5 we give properties of equilibria when the institution allows mandatory spending (with or without discretionary), and give efficiency properties of mandatory only institutions. Section 6 discusses institutions that allow for an endogenous choice of mandatory and discretionary spending, and Section 7 considers state-contingent mandatory spending. Section 8 concludes.

2 Model

Consider a stylized economy and political system with two parties labeled A and B. There are two time periods indexed by $t \in \{1, 2\}$.[2] In each period t, the two parties decide an allocation to a public good $x_t \in \mathbb{R}_+$. The stage utility for party i from the public good in period t is $u_{it}(x_t)$. We assume $u_{it}(\cdot)$ is twice continuously differentiable, strictly concave, and attains a maximum at θ_{it} for all $i \in \{A, B\}$ and $t \in \{1, 2\}$. This implies $u_{it}(\cdot)$ is single-peaked with θ_{it} denoting party i's static ideal level of the public good in period t.[3]

We assume parties' ideal levels of the public good are positive and party A's ideal is lower than party B's. That is, $0 < \theta_{At} \leq \theta_{Bt}$ for all t. Parties' ideal levels of the public good may vary across periods. In particular, these may be *increasing* with $\theta_{i1} < \theta_{i2}$ for all $i \in \{A, B\}$, *decreasing* with $\theta_{i1} > \theta_{i2}$ for all $i \in \{A, B\}$, *divergent* with $\theta_{A2} < \theta_{A1} < \theta_{B1} < \theta_{B2}$, or *convergent* with $\theta_{A1} < \theta_{A2} < \theta_{B2} < \theta_{B1}$.

The parties have a common discount factor $\delta \in (0, 1]$. Party i seeks to maximize its discounted dynamic payoff from the sequence of public good allocations, $u_{i1}(x_1) + \delta u_{i2}(x_2)$.

Political system

We consider a political system with unanimity rule.[4] Each period a party is randomly selected to make a proposal for the allocation to the public good. The probability that party i proposes in a period is $p_i \in (0, 1)$.

At the beginning of each period, the identity of the proposing party is realized. The proposing party makes a proposal for the allocation to the public good. Spending on the public good may be allocated via different programs - a discretionary program, which expires after the first period, or a mandatory program, for which spending will continue in the next period unless the parties agree to change it. Denote the proposed amount allocated to a

[2] In Section 3 we characterize Pareto efficient allocations for a model with any number of periods, and in Section 7 we consider a more general model with any number of periods and random preferences.

[3] Because of the opportunity cost of providing public goods, it is reasonable to model parties' utility functions as single-peaked as in, for example, Baron (1996).

[4] Most political systems are not formally characterized by unanimity rule, however, many have institutions that limit a single party's power, for example, the "checks and balances" included in the U.S. Constitution. Under these institutions, if the majority party's power is not sufficiently high, then it needs approval of the other party to set new policies.

discretionary program in period t as k_t, and to a mandatory program as g_t. If the responding party agrees to the proposal, the implemented allocation to the public good for the period is the sum of the discretionary and mandatory allocations proposed, so $x_t = k_t + g_t$; otherwise, $x_t = g_{t-1}$.

Denote a proposal by $z_t = (k_t, g_t)$. We require $g_t \geq 0$ to ensure a positive status quo each period. Let $Z \subseteq \mathbb{R} \times \mathbb{R}_+$ be the set of feasible proposals. The set Z is determined by the rules governing mandatory and discretionary programs, hence we call Z the *budgetary institution*. We explore efficiency implications of mandatory and discretionary programs for different budgetary institutions. Specifically, we consider the following institutions: only discretionary programs, in which case $Z = \mathbb{R}_+ \times \{0\}$ and $g_{t-1} = 0$ for all t; only mandatory programs, in which case $Z = \{0\} \times \mathbb{R}_+$; both mandatory and positive discretionary, in which case $Z = \mathbb{R}_+ \times \mathbb{R}_+$; and both mandatory and discretionary where discretionary spending may be positive or negative, in which case $Z = \{(k_t, g_t) \in \mathbb{R} \times \mathbb{R}_+ : k_t + g_t \geq 0\}$. It is natural to think of spending as positive, however, it is also possible to have temporary cuts to spending on mandatory programs, for example government furloughs that temporarily reduce salaries to public employees. This temporary reduction in mandatory spending can be thought of as negative discretionary spending as it reduces total spending in the current period on a particular good, but does not affect the status quo for the next period.

We consider subgame perfect equilibria of the game between parties A and B. A pure strategy for party i in period t is a pair of functions $\sigma_{it} = (\pi_{it}, \alpha_{it})$, where $\pi_{it} : \mathbb{R}_+ \to Z$ is a proposal strategy for party i in period t and $\alpha_{it} : \mathbb{R}_+ \times Z \to \{0, 1\}$ is an acceptance strategy for party i in period t. Party i's proposal strategy $\pi_{it} = (\kappa_{it}, \gamma_{it})$ associates with each status quo g_{t-1} an amount of public good spending in discretionary programs, denoted by $\kappa_{it}(g_{t-1})$ and an amount in mandatory programs, denoted by $\gamma_{it}(g_{t-1})$. Party i's acceptance strategy $\alpha_{it}(g_{t-1}, z_t)$ takes the value 1 if party i accepts the proposal z_t offered by party $j \neq i$ when the status quo is g_{t-1}, and 0 otherwise.[5]

We restrict attention to equilibria in which (i) $\alpha_{it}(g_{t-1}, z_t) = 1$ when party i is indifferent

[5]We are interested in efficiency properties of budgetary institutions. Because the utility functions are strictly concave, Pareto efficient allocations do not involve randomization. Hence, if any pure strategy equilibrium is inefficient, allowing mixed strategies does not improve efficiency.

between g_{t-1} and z_t; and (ii) $\alpha_{it}(g_{t-1}, \pi_{jt}(g_{t-1})) = 1$ for all t, $g_{t-1} \in \mathbb{R}_+$, $i, j \in \{A, B\}$ with $j \neq i$. That is, the responder accepts any proposal that it is indifferent between accepting and rejecting, and the equilibrium proposals are always accepted. We henceforth refer to a subgame perfect equilibrium that satisfies (i) and (ii) simply as an equilibrium.

Denote an equilibrium as σ^*. Given conditions (i) and (ii), if party $i \in \{A, B\}$ is the proposer and party $j \neq i$ is the responder in period 2, then for any g_1 admissible under Z the equilibrium proposal strategy $(\kappa^*_{i2}(g_1), \gamma^*_{i2}(g_1))$ of party i in period 2 solves

$$\begin{aligned} \max_{(k_2,g_2)\in Z} \quad & u_{i2}(k_2 + g_2) \\ \text{s.t.} \quad & u_{j2}(k_2 + g_2) \geq u_{j2}(g_1). \end{aligned} \tag{P_2}$$

Let $V_i(g; \sigma_2)$ be the expected second-period payoff for party i given first-period mandatory spending g and second-period strategies $\sigma_2 = (\sigma_{A2}, \sigma_{B2})$. That is

$$V_i(g; \sigma_2) = p_A u_{i2}(\kappa_{A2}(g) + \gamma_{A2}(g)) + p_B u_{i2}(\kappa_{B2}(g) + \gamma_{B2}(g)).$$

If party $i \in \{A, B\}$ is the proposer and party $j \neq i$ is the responder in period 1, then for any g_0 admissible under Z the equilibrium proposal strategy $(\kappa^*_{i1}(g_0), \gamma^*_{i1}(g_0))$ of party i in period 1 solves

$$\begin{aligned} \max_{(k_1,g_1)\in Z} \quad & u_{i1}(k_1 + g_1) + \delta V_i(g_1; \sigma^*_2) \\ \text{s.t.} \quad & u_{j1}(k_1 + g_1) + \delta V_j(g_1; \sigma^*_2) \geq u_{j1}(g_0) + \delta V_j(g_0; \sigma^*_2). \end{aligned} \tag{P_1}$$

3 Pareto efficiency

In this section we characterize Pareto efficient allocations and define Pareto efficient equilibria, both in the static and the dynamic sense.

3.1 Pareto efficient allocations

As a benchmark we characterize the Pareto efficient allocations. We distinguish between the social planner's static problem (SSP) which determines static Pareto efficient allocations, and the social planner's dynamic problem (DSP) which determines dynamic Pareto efficient allocations.[6]

[6]Previous authors, for example, Dziuda and Loeper (2013) or Riboni and Ruge-Murcia (2008) have demonstrated that an endogenous status quo can lead to static inefficiency due to dynamic incentives. The

We define a statically Pareto efficient allocation in period t as the solution to the following maximization problem

$$\begin{aligned} \max_{x_t \in \mathbb{R}_+} \quad & u_{it}(x_t) \\ \text{s.t.} \quad & u_{jt}(x_t) \geq \overline{u} \end{aligned} \tag{SSP}$$

for some $\overline{u} \in \mathbb{R}$, $i, j \in \{A, B\}$ and $i \neq j$.[7] By Proposition 1, statically Pareto efficient allocations are all those between the ideal points of the parties.

Proposition 1. *An allocation x_t is statically Pareto efficient in period t if and only if $x_t \in [\theta_{At}, \theta_{Bt}]$.*

Denote a sequence of allocations by $\mathbf{x} = (x_1, x_2)$ and party i's discounted dynamic payoff from $\mathbf{x}$ by $U_i(\mathbf{x}) = \sum_{t=1}^{2} \delta^{t-1} u_{it}(x_t)$. We define a dynamically Pareto efficient allocation as the solution to the following maximization problem

$$\begin{aligned} \max_{\mathbf{x} \in \mathbb{R}^2_+} \quad & U_i(\mathbf{x}) \\ \text{s.t.} \quad & U_j(\mathbf{x}) \geq \overline{U} \end{aligned} \tag{DSP}$$

for some $\overline{U} \in \mathbb{R}$, $i, j \in \{A, B\}$ and $i \neq j$. Denote the sequence of party i's static ideals by $\boldsymbol{\theta}_i = (\theta_{i1}, \theta_{i2})$ for all $i \in \{A, B\}$, and denote the solution to (DSP) as $\mathbf{x}^* = (x_1^*, x_2^*)$.[8] Proposition 2 characterizes the dynamically Pareto efficient allocations.

Proposition 2. *A dynamically Pareto efficient allocation $\mathbf{x}^*$ satisfies the following properties:*

1. *For all t, x_t^* is statically Pareto efficient. That is, $x_t^* \in [\theta_{At}, \theta_{Bt}]$ for all t.*
2. *Either $\mathbf{x}^* = \boldsymbol{\theta}_A$, or $\mathbf{x}^* = \boldsymbol{\theta}_B$, or $u'_{At}(x_t^*) + \lambda^* u'_{Bt}(x_t^*) = 0$ for some $\lambda^* > 0$, for all t.*

Proposition 2 part 2 implies that if $\mathbf{x}^* \neq \boldsymbol{\theta}_i$ for all $i \in \{A, B\}$, and $\theta_{At} \neq \theta_{Bt}$ in period t then we must have

$$-\frac{u'_{At}(x_t^*)}{u'_{Bt}(x_t^*)} = \lambda^* \tag{1}$$

latter paper also shows that the endogenous status-quo does not necessarily lead to dynamic efficiencies in a different model, and in the context of central bank decision-making.

[7] The social planner's static problem (SSP) is a standard concave programming problem so the solution is unique for a given $\overline{u}$ if it exists.

[8] Note the solution to (DSP) depends on $\overline{U}$, but for notational simplicity we suppress this dependency and denote the solution to (DSP) as $\mathbf{x}^*$. The solution to (DSP) is unique for a given $\overline{U}$ if it exists. In the proof of Proposition 2 in the Appendix we present (DSP) for any number of periods, and prove Proposition 2 for this more general problem.

for some $\lambda^* > 0$. This is because if $u'_{Bt}(x_t^*) = 0$, then part 2 of Proposition 2 implies that we must also have $u'_{At}(x_t^*) = 0$ which is not possible when $\theta_{At} \neq \theta_{Bt}$.

By (1) if parties A and B do not have the same ideal level of the public good in any two periods t and t', then in a dynamically Pareto efficient allocation, either the allocation is equal to party A's or party B's ideal in both periods, or the ratio of their marginal utilities is equal across these two periods, i.e., $\frac{u'_{At}(x_t^*)}{u'_{Bt}(x_t^*)} = \frac{u'_{At'}(x_{t'}^*)}{u'_{Bt'}(x_{t'}^*)}$. In both cases there is a *dynamic link* across periods.

3.2 Pareto efficient equilibrium

We define a dynamically Pareto efficient equilibrium given an initial status quo g_0 as an equilibrium that results in a dynamically Pareto efficient allocation for any realization of the sequence of proposers. More precisely, denote a strategy profile as $\sigma = ((\sigma_{A1}, \sigma_{A2}), (\sigma_{B1}, \sigma_{B2}))$ with $\sigma_{it} = ((\kappa_{it}, \gamma_{it}), \alpha_{it})$. Recall that the total spending on the public good in period t is the sum of the mandatory spending and the discretionary spending. An *equilibrium allocation* for σ given initial status quo g_0 is a possible realization of total public good spending for each period $\mathbf{x}^\sigma(g_0) = \{x_t^\sigma(g_0)\}_{t=1}^2$, where $x_1^\sigma(g_0) = \kappa_{i1}(g_0) + \gamma_{i1}(g_0)$, and $x_2^\sigma(g_0) = \kappa_{j2}(\gamma_{i1}(g_0)) + \gamma_{j2}(\gamma_{i1}(g_0))$ for some $i, j \in \{A, B\}$. The random determination of proposers each period induces a probability distribution over allocations given an equilibrium σ, thus any element in the support of this distribution is an equilibrium allocation for σ.[9] We require every allocation in the support of this distribution to be dynamically Pareto efficient for the equilibrium to be dynamically Pareto efficient.

Definition 1. A profile of strategies σ is a *dynamically Pareto efficient equilibrium given initial status quo* $g_0 \in \mathbb{R}_+$ if and only if

1. σ constitutes an equilibrium.

2. Every equilibrium allocation $\mathbf{x}^\sigma(g_0)$ for σ given initial status quo g_0 is dynamically Pareto efficient.

[9] For example, if A is the proposer in period 1 and B is the proposer in period 2, then the equilibrium allocation is $x_1^\sigma(g_0) = \kappa_{A1}(g_0) + \gamma_{A1}(g_0)$, and $x_2^\sigma(g_0) = \kappa_{B2}(\gamma_{A1}(g_0)) + \gamma_{B2}(\gamma_{A1}(g_0))$.

A *statically Pareto efficient equilibrium given initial status quo* g_0 is analogously defined as an equilibrium in which the realized allocation to the public good is statically Pareto efficient in all periods t given initial status quo g_0. Thus a necessary condition for a strategy profile σ to be a dynamically Pareto efficient equilibrium is that σ is a statically Pareto efficient equilibrium. In the next sections we analyze which budgetary institutions result in a statically or dynamically Pareto efficient equilibrium.

The analysis of a dynamically Pareto efficient equilibrium is aided by the following lemmas.

Lemma 1. *Suppose* $\theta_{At} \neq \theta_{Bt}$ *for any* t*. If allocations* $\mathbf{x}$ *and* $\tilde{\mathbf{x}}$ *are both dynamically Pareto efficient and* $x_{t'} = \tilde{x}_{t'}$ *for some* t'*, then* $\mathbf{x} = \tilde{\mathbf{x}}$*.*

An implication of Lemma 1 is that if parties' ideal levels of the public good are different in both periods, then given an allocation in the first period, the dynamically Pareto efficient allocation in the second period is uniquely determined. This means that if the equilibrium level of spending in period 2 varies with the identity of the period-2 proposer, then the equilibrium cannot be dynamically Pareto efficient.

In general dynamic Pareto efficiency is stronger than static Pareto efficiency, but from part 1 of Proposition 2 it is clear that if $\boldsymbol{\theta}_A = \boldsymbol{\theta}_B$ then an allocation $\mathbf{x}$ is dynamically Pareto efficient if and only if x_t is statically Pareto efficient in all periods. The next lemma shows that this is also true if the preferences differ in only one period.

Lemma 2. *Suppose there exists at most one* t' *such that* $\theta_{At'} \neq \theta_{Bt'}$*. Then an allocation* $\mathbf{x}$ *is dynamically Pareto efficient if and only if* x_t *is statically Pareto efficient in period* t *for all* t*.*

This is true because for all $t \neq t'$ it must be that $x_t = \theta_{it}$, and only in period t' must (1) be true, thus there is no dynamic link across periods.

4 Discretionary spending

Suppose spending is allocated through discretionary programs only, implying that the status quo in each period is exogenous and equal to 0. In this case there is no dynamic link

between the previous period's policy and the current period's status quo, and $Z = \mathbb{R}_+ \times \{0\}$. Without the dynamic link between periods, the bargaining between the two parties is a static problem, similar to the monopoly agenda-setting model in Romer and Rosenthal (1978, 1979).[10] We next characterize the equilibrium and its efficiency properties.

For this section we denote a proposal in period t by k_t since g_t is zero. Consider any period t. Since u_{Bt} is single-peaked at θ_{Bt} and $0 < \theta_{At} \leq \theta_{Bt}$, we have $u_{Bt}(0) \leq u_{Bt}(\theta_{At})$. Hence, if party A is the proposer in period t, it proposes its ideal policy $k_t = \theta_{At}$, which is accepted by party B. If party B is the proposer in period t, however, whether it can implement its ideal policy depends on the locations of the parties' ideal points relative to the status quo, which is equal to 0. Specifically, let ϕ^o_{At} be the highest policy that makes party A as well off as the status quo in period t. That is,

$$\phi^o_{At} = \max\{x \in \mathbb{R}_+ | u_{At}(x) \geq u_{At}(0)\}.$$

Note that $\phi^o_{At} \geq \theta_{At}$. Since status quo spending is 0, party A accepts any proposal k_t such that $0 \leq k_t \leq \phi^o_{At}$. To find party B's optimal proposal, there are two cases to consider. (i) Suppose $\theta_{Bt} \leq \phi^o_{At}$. Then, since u_{At} is single-peaked at $\theta_{At} \leq \theta_{Bt}$, we have $u_{At}(\theta_{Bt}) \geq u_{At}(\phi^o_{At}) = u_{At}(0)$ and therefore party A accepts $k_t = \theta_{Bt}$ in period t. In this case, party B's optimal proposal in period t is equal to its ideal point θ_{Bt}. (ii) Suppose $\theta_{Bt} > \phi^o_{At}$. Then, given the single-peakedness of u_{Bt}, the optimal policy for B that is acceptable to A is equal to ϕ^o_{At}. In this case, party B proposes $k_t = \phi^o_{At} < \theta_{Bt}$. To summarize, party B's optimal proposal is equal to $\min\{\theta_{Bt}, \phi^o_{At}\}$.

Since the policy implemented in period t is equal to θ_{At} when party A is the proposer and equal to $\min\{\theta_{Bt}, \phi^o_{At}\} \geq \theta_{At}$ when party B is the proposer, the policy implemented in period t is in $[\theta_{At}, \theta_{Bt}]$ and therefore statically Pareto efficient by Proposition 1. To discuss the equilibrium's dynamic efficiency properties, we consider the following three cases. (i) Suppose $\theta_{At} \neq \theta_{Bt}$ for all t. In this case, $\min\{\theta_{Bt}, \phi^o_{At}\} > \theta_{At}$, which implies that the policy implemented in period t varies with the identity of the proposer. By Lemma 1, this implies dynamic Pareto inefficiency. (ii) Suppose there is at most one t' such that $\theta_{At'} \neq \theta_{Bt'}$. By

[10] Note that when only discretionary spending is allowed, $g_0 = g_1 = 0$, and hence the proposer's first-period problem (P_1) becomes a static problem identical to (P_2).

Lemma 2, an allocation $\mathbf{x}$ is dynamically Pareto efficient if and only if x_t is statically Pareto efficient in all periods. The following proposition summarizes these results.[11]

Proposition 3. *Under a budgetary institution that allows only discretionary spending programs, given the initial status quo of zero:*

1. *The equilibrium is statically Pareto efficient.*

2. *The equilibrium is dynamically Pareto inefficient if and only if $\theta_{At} \neq \theta_{Bt}$ for all t.*

Specifically, the equilibrium level of spending in period t is $x_t = \theta_{At}$ if party A is the proposer and is $x_t = \min\{\theta_{Bt}, \phi^o_{At}\} \in [\theta_{At}, \theta_{Bt}]$ if party B is the proposer.

5 Mandatory spending

When spending can be allocated by way of mandatory programs, the party selected to propose in the first period has to take into account the effect of the amount of public good allocated to mandatory program on the second-period spending because it becomes the status quo spending in the second period. This creates a dynamic link between periods.

5.1 Preliminaries

We first show that this dynamic game admits an equilibrium and give properties of the equilibrium proposal strategies in period 2.[12] The proposition applies to any budgetary institution that allows mandatory spending programs, in combination with discretionary spending or in isolation.

To state the proposition we define the functions $\phi_{At} : \mathbb{R}_+ \to \mathbb{R}_+$ and $\phi_{Bt} : \mathbb{R}_+ \to \mathbb{R}_+$. The value $\phi_{At}(g_{t-1})$ is the highest spending level that makes party A as well off as under the

[11] The result is stated for the two-period model, but a straightforward generalization of the argument shows that in a model with an arbitrary number of periods, Proposition 3 holds if the condition in part 2 is replaced by $\theta_{At} \neq \theta_{Bt}$ for at least two periods.

[12] Equilibrium existence is not immediate because the constraint set in the second period is determined by the responder's acceptance condition [see (P_2)]. The constraint is not monotone in the proposer's first-period choice variable g_1. Rather, g_1 determines both the lowest and the highest policy the second-period responder is willing to accept. This requires a non-trivial proof of lower-hemicontinuity of the second-period acceptance correspondence to show continuity of the first period payoff. This proof is given in the Appendix.

status quo g_{t-1}, and $\phi_{Bt}(g_{t-1})$ is the lowest spending level that makes party B as well off as under the status quo. That is,

$$\begin{aligned}\phi_{At}(g_{t-1}) &= \max\{x \in \mathbb{R}_+ | u_{At}(x) \geq u_{At}(g_{t-1})\} \\ \phi_{Bt}(g_{t-1}) &= \min\{x \in \mathbb{R}_+ | u_{Bt}(x) \geq u_{Bt}(g_{t-1})\}.\end{aligned}$$

These are illustrated below in Figure 1. If $g_{t-1} < \theta_{At}$, then $\phi_{At}(g_{t-1}) > g_{t-1}$ and if $g_{t-1} \geq \theta_{At}$, then $\phi_{At}(g_{t-1}) = g_{t-1}$. If $g_{t-1} \leq \theta_{Bt}$, then $\phi_{Bt}(g_{t-1}) = g_{t-1}$ and if $g_{t-1} > \theta_{Bt}$, then $\phi_{Bt}(g_{t-1}) < g_{t-1}$.

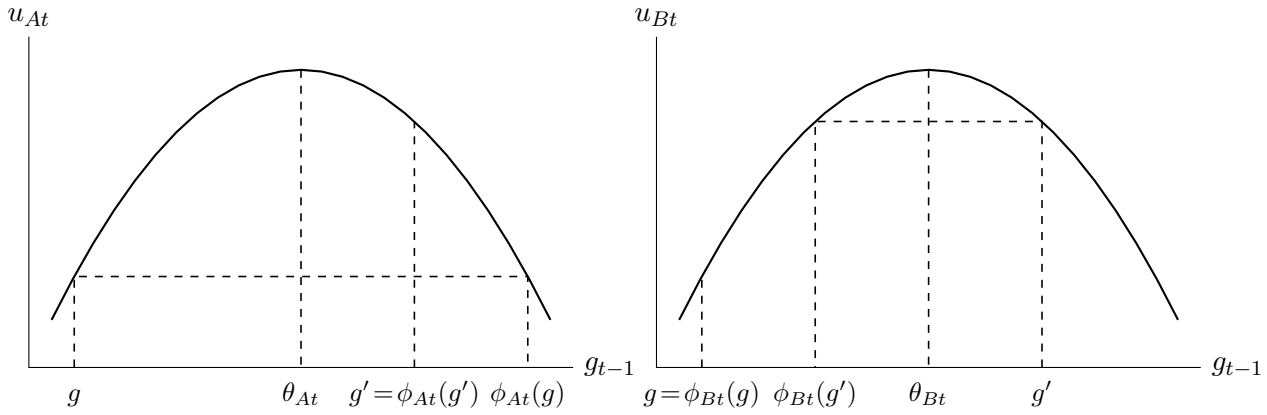


Figure 1: ϕ_{At} and ϕ_{Bt}

Proposition 4. *Under any budgetary institution that allows mandatory spending programs, an equilibrium exists. For any $g_1 \in \mathbb{R}_+$, any equilibrium $\sigma^* = ((\sigma^*_{A1}, \sigma^*_{A2}), (\sigma^*_{B1}, \sigma^*_{B2}))$ with $\sigma^*_{it} = ((\kappa^*_{it}, \gamma^*_{it}), \alpha^*_{it})$ satisfies*

$$\begin{aligned}\kappa^*_{A2}(g_1) + \gamma^*_{A2}(g_1) &= \max\{\theta_{A2}, \phi_{B2}(g_1)\} \\ \kappa^*_{B2}(g_1) + \gamma^*_{B2}(g_1) &= \min\{\theta_{B2}, \phi_{A2}(g_1)\}.\end{aligned}$$

Furthermore:

1. *$\kappa^*_{i2}(g_1) + \gamma^*_{i2}(g_1) \in [\theta_{A2}, \theta_{B2}]$ for all $i \in \{A, B\}$ and all $g_1 \in \mathbb{R}_+$.*

2. *If $\theta_{A2} \neq \theta_{B2}$, then*

$$\begin{aligned}&\kappa^*_{A2}(g_1) + \gamma^*_{A2}(g_1) = \kappa^*_{B2}(g_1) + \gamma^*_{B2}(g_1) = g_1 \qquad \text{if } \ g_1 \in [\theta_{A2}, \theta_{B2}], \\ &\kappa^*_{A2}(g_1) + \gamma^*_{A2}(g_1) < \kappa^*_{B2}(g_1) + \gamma^*_{B2}(g_1) \qquad \text{if } \ g_1 \notin [\theta_{A2}, \theta_{B2}].\end{aligned}$$

Proposition 4 gives the equilibrium level of total spending in the second period. The proposition implies that the equilibrium total spending is unique for any status quo. Proposition 4 part 1 implies that the equilibrium level of spending in period 2 is statically Pareto efficient. Part 2 gives properties of the equilibrium spending if the parties' ideals are different. If the status quo is statically Pareto efficient, then it is maintained. If the status quo is not statically Pareto efficient, then the equilibrium proposal is different from the status quo and depends on the identity of the proposer - specifically, it is lower when A is the proposer than when B is the proposer.

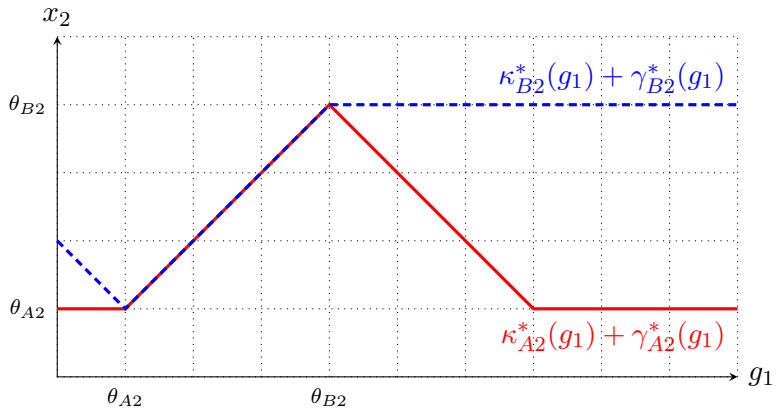


Figure 2: Period 2 equilibrium strategies with mandatory spending for $u_{it}(x_t) = -(x_t - \theta_{it})^2$

Figure 2 is an example of equilibrium spending in period 2 for quadratic loss utility. While the exact form depends on the specific utility function, any second-period strategy has similar properties. Consider party A as the proposer in period 2. If the status quo is $g_1 < \theta_{A2}$, then, since $u_{B2}(g_1) < u_{B2}(\theta_{A2})$, party A proposes its ideal policy $x_2 = \theta_{A2}$, which is accepted. If $g_1 \in [\theta_{A2}, \theta_{B2}]$, then, since any $x_2 < g_1$ would be rejected by party B and party A prefers g_1 to any $x_2 > g_1$, party A proposes $x_2 = g_1$. If $g_1 > \theta_{B2}$, then party B accepts any proposal in $[\phi_{B2}(g_1), g_1]$, the interval of policies closer to θ_{B2} than g_1 is. Since $\theta_{A2} < \theta_{B2} < g_1$, either $\theta_{A2} \in [\phi_{B2}(g_1), g_1]$ or $\theta_{A2} < \phi_{B2}(g_1)$. If $\theta_{A2} \in [\phi_{B2}(g_1), g_1]$, then party A proposes $x_2 = \theta_{A2}$. If $\theta_{A2} < \phi_{B2}(g_1)$, then party A proposes the policy closest to θ_{A2} that is acceptable to B, which is $\phi_{B2}(g_1)$. For quadratic loss utility function, if $g_1 \geq \theta_{B2}$ and $\phi_{B2}(g_1) \geq \theta_{A2}$, we have $\phi_{B2}(g_1) = 2\theta_{B2} - g_1$, which is decreasing in g_1. For general strictly concave u_{B2}, if $g_1 \geq \theta_{B2}$

and $\phi_{B2}(g_1) \geq \theta_{A2}$, then $\phi_{B2}(g_1)$ is decreasing in g_1.

5.2 Inefficiency with mandatory spending only

Suppose now that spending is allocated through mandatory programs only, such that $Z = \{0\} \times \mathbb{R}_+$. Since k_t is zero for any t, the equilibrium discretionary proposal $\kappa^*_{it}(g_{t-1})$ is zero for all $i \in \{A, B\}$, all t and all $g_{t-1} \in \mathbb{R}_+$. For the rest of the section we thus denote a proposal in period t by g_t.

We show that equilibrium allocations can in general be dynamically Pareto inefficient and even statically Pareto inefficient with mandatory spending programs only. Proposition 5 gives conditions under which we obtain dynamic inefficiency, but we first define two regularity conditions satisfied by commonly used utility functions.

Definition 2. We say u_{it} is *regular* if $u_{it}(x_t) = u_i(x_t, \theta_{it})$ for all t. We say u_{it} is *regular with increasing marginal returns* if u_{it} is regular and $\frac{\partial u_i}{\partial x_t}$ is strictly increasing in θ_{it} for all t.

Proposition 5. *Under a budgetary institution that allows only mandatory spending programs, any equilibrium σ^* is dynamically Pareto inefficient for any initial status quo $g_0 \in \mathbb{R}_+$, if any of the following conditions hold:*

1. *Parties' ideals are increasing or decreasing and not overlapping, that is $\theta_{A1} < \theta_{B1} < \theta_{A2} < \theta_{B2}$ or $\theta_{A2} < \theta_{B2} < \theta_{A1} < \theta_{B1}$.*

2. *Parties' ideals are either increasing or decreasing, $\theta_{At} \neq \theta_{Bt}$ for all t and u_{it} is regular with increasing marginal returns for all $i \in \{A, B\}$.*

Furthermore, if parties' ideals are divergent or convergent and $u_{it}(x_t) = -(|x_t - \theta_{it}|)^r$ with $r > 1$, there exists a set $\mathcal{I}$, where $\mathbb{R}_+ \setminus \mathcal{I}$ is a finite set, such that any equilibrium σ^* is dynamically Pareto inefficient for any initial status quo $g_0 \in \mathcal{I}$.

Proposition 5 covers all possible ways in which ideal levels of public good spending of the two parties can vary: increasing, decreasing, convergent or divergent. Proposition 5 parts 1 and 2 give conditions under which dynamic inefficiency is obtained for increasing and decreasing ideals of the parties. The final part of the proposition states that when preferences

are divergent or convergent, for a special class of utility functions, dynamic efficiency is obtained only for a set of non-generic status quos.[13]

The reason behind the dynamic Pareto inefficiency of any equilibrium in the proposition is the combination of the variance in parties' ideals with budgetary institution restricted to mandatory spending programs. Without discretionary spending, the level of public good spending in period 1 becomes the status quo in period 2. Because of the second-period conflict between the two parties, either none of the parties is willing to change the status quo and the level of public good spending is constant, or the level of public good spending in period 2 depends on the identity of the proposing party. In each case the equilibrium allocation violates dynamic Pareto efficiency, which requires variation in allocations across time, but precludes variation in allocation in a given period.

The next result shows that equilibrium allocations under mandatory spending programs can violate not only dynamic, but also static Pareto efficiency.[14]

Proposition 6. *Suppose* $u_{it}(x_t) = -(x_t - \theta_{it})^2$ *for all* $i \in \{A, B\}$ *and all* t. *Under a budgetary institution that allows only mandatory spending programs, if either* $\frac{\theta_{A1}-\theta_{A2}}{\theta_{B2}-\theta_{A2}} \in (0,1)$ *or* $\frac{\theta_{B2}-\theta_{B1}}{\theta_{B2}-\theta_{A2}} \in (0,1)$, *then there exists set* $\mathcal{I}$ *of non-zero measure such that any equilibrium* σ^* *is statically Pareto inefficient for any initial status quo* $g_0 \in \mathcal{I}$.

The key condition of Proposition 6 is stated in terms of $\frac{\theta_{A1}-\theta_{A2}}{\theta_{B2}-\theta_{A2}}$ and $\frac{\theta_{B2}-\theta_{B1}}{\theta_{B2}-\theta_{A2}}$. Each of these fractions has natural interpretation as the ratio of preference variation of party $i \in \{A, B\}$ to future polarization between the two parties in period 2. If this ratio is small, static Pareto inefficiency arises in equilibrium. Because the proposition requires the ratio to be positive, static Pareto inefficiency potentially arises when the ideal levels are increasing, decreasing or divergent. And since the proposition does not rule out $\theta_{A1} = \theta_{B1}$, static Pareto inefficiency can arise even in the absence of first-period conflict between the two parties.

Figure 3 illustrates Proposition 6. The parameters used satisfy the conditions in Proposition 6. Specifically, the ideal levels of the two parties diverge and $\frac{\theta_{A1}-\theta_{A2}}{\theta_{B2}-\theta_{A2}} = \frac{\theta_{B2}-\theta_{B1}}{\theta_{B2}-\theta_{A2}} = \frac{1}{3}$.

[13]Dynamic inefficiency also obtains in a finite-horizon model with more than two periods under the conditions in Proposition 5, when the changes in preferences apply to the last two periods.

[14]The proposition assumes quadratic stage utilities. This is for convenience, as it facilitates derivation of analytical expressions, rather than out of necessity.

The figure plots equilibrium public good spending in period 1 proposed by each party for initial status quo $g_0 \in [0, 2]$. What the figure shows is that unless $g_0 \in [\theta_{A1}, \theta_{B1}]$, we have $\gamma^*_{i1}(g_0) \notin [\theta_{A1}, \theta_{B1}]$ for at least one of the parties, i.e., the equilibrium is statically Pareto inefficient.

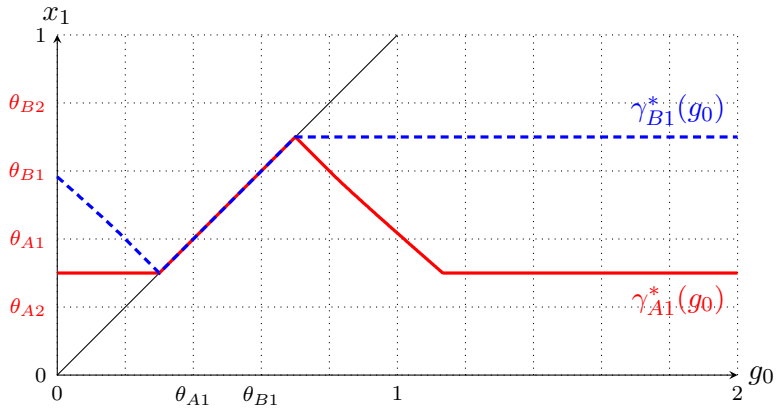


Figure 3: Period 1 equilibrium strategies when all spending is mandatory
$\boldsymbol{\theta}_A = (0.4, 0.2)$, $\boldsymbol{\theta}_B = (0.6, 0.8)$, $p_A = p_B = \frac{1}{2}$, $\delta = 1$, $u_{it}(x_t) = -(x_t - \theta_{it})^2$

The tendency of the parties to propose $g_1 \notin [\theta_{A1}, \theta_{B1}]$ is due to the dual role of first-period public good spending. It represents spending in a standard sense but it also determines the status quo in period 2. Party i selected to propose has an incentive to propose g_1 trading off these two roles. The first one pushes towards proposing θ_{i1} while the second one pushes towards θ_{i2}. When party i is unconstrained by the acceptance of party $j \neq i$, it proposes g_1 that is a weighted average of θ_{i1} and θ_{i2}. For $i = A$, when $\theta_{i2} < \theta_{i1}$ this weighted average is below θ_{A1}, giving rise to static Pareto inefficiency.

5.3 Efficiency with mandatory spending only

We show that despite the restriction to mandatory spending only, equilibrium allocations can be dynamically Pareto efficient in the absence of a conflict in period 2 or in the absence of variation in ideal levels of public good spending.

Proposition 7. *Under a budgetary institution that allows only mandatory spending programs, any equilibrium σ^* is dynamically Pareto efficient for any initial status quo $g_0 \in \mathbb{R}_+$, if any of the following conditions hold:*

1. $\theta_{A2} = \theta_{B2}$.

2. u_{it} *is regular and* $\theta_{i1} = \theta_{i2}$ *for all* $i \in \{A, B\}$.

To understand the dynamic Pareto efficiency result in Proposition 7 part 1, recall that by Lemma 2, if $\theta_{A2} = \theta_{B2}$, an allocation $\mathbf{x}$ is dynamically Pareto efficient if and only if x_t is statically Pareto efficient in all periods, so that there is no dynamic link between allocations. When $\theta_{A2} = \theta_{B2}$, Proposition 4 part 1 also implies that, for any second-period status quo g_1, both parties propose $g_2 = \theta_{A2}$. The equilibrium allocation in period 2 is thus $x_2^{\sigma^*}(g_0) = \theta_{A2}$ for any $g_0 \in \mathbb{R}_+$ and the expected second-period payoff of party i, $V_i(g_1)$, is constant in g_1. Hence the proposer's first-period problem (P_1) becomes a static problem identical to (P_2) and the equilibrium allocation in period 1 satisfies $x_1^{\sigma^*}(g_0) \in [\theta_{A1}, \theta_{B1}]$ for any $g_0 \in \mathbb{R}_+$. By Proposition 1, $x_t^{\sigma^*}(g_0)$ is statically Pareto efficient in period t for all t, and hence $\mathbf{x}^{\sigma^*}(g_0)$ is dynamically Pareto efficient for any $g_0 \in \mathbb{R}_+$.

The logic underlying the dynamic Pareto efficiency result in Proposition 7 part 1 differs from the one underlying Proposition 7 part 2. When $\theta_{i1} = \theta_{i2}$ for all $i \in \{A, B\}$, the parties may have different ideal levels of public good spending in both periods, and the definition of dynamic Pareto efficiency does create a dynamic link between allocations x_1 and x_2. When u_{it} is regular for all $i \in \{A, B\}$, the dynamic link requires the allocations to be constant, and any $x_1 = x_2 \in [\theta_{A1}, \theta_{B1}]$ is dynamically Pareto efficient. Proposition 7 then follows via showing that the equilibrium allocation $\mathbf{x}^{\sigma^*}(g_0)$ satisfies $x_1^{\sigma^*}(g_0) = x_2^{\sigma^*}(g_0)$ for any $g_0 \in \mathbb{R}_+$. That is, the level of public good spending is fully determined by the party selected to propose in the first period, along with the initial status quo g_0, and is constant across the two periods.

6 Mandatory and discretionary spending

In this section we consider that parties can endogenously choose how much public good to allocate to mandatory and discretionary spending. We begin by showing that when discretionary spending can only be positive, that is $Z = \mathbb{R}_+ \times \mathbb{R}_+$, we obtain dynamic Pareto efficiency under some conditions.

Proposition 8. *Under a budgetary institution that allows positive discretionary and mandatory spending, if u_{it} is regular with increasing marginal returns, and each party's ideal value of the public good is decreasing, then every equilibrium is dynamically Pareto efficient for any status quo $g_0 \in \mathbb{R}_+$.*

The proof of Proposition 8 is instructive so we include it. First, consider the following alternative way of writing the social planner's dynamic problem:

$$\begin{aligned} \max_{(x_1, x_{A2}, x_{B2}) \in \mathbb{R}^3_+} \quad & u_{i1}(x_1) + \delta[p_A u_{i2}(x_{A2}) + p_B u_{i2}(x_{B2})] \\ \text{s.t.} \quad & u_{j1}(x_1) + \delta[p_A u_{j2}(x_{A2}) + p_B u_{j2}(x_{B2})] \geq \overline{U}, \end{aligned} \tag{DSP'}$$

for some $\overline{U} \in \mathbb{R}$, $i, j \in \{A, B\}$ and $i \neq j$. The difference between the original social planner's problem (DSP) and the modified social planner's problem (DSP') is that in the modified problem, the social planner is allowed to choose a *distribution* of allocations in period 2. Since we assume that utility functions are concave, it is not optimal for the social planner to randomize and therefore the solution to the original problem (DSP) is also the solution to the modified problem (DSP'). To state this result formally, recall that given $\overline{U} \in \mathbb{R}$ we denote the solution to the original social planner's dynamic problem (DSP) as $\mathbf{x}^*(\overline{U}) = (x_1^*(\overline{U}), x_2^*(\overline{U}))$.

Lemma 3. *The solution to the modified social planner's problem* (DSP') *is $x_1 = x_1^*(\overline{U})$ and $x_{A2} = x_{B2} = x_2^*(\overline{U})$.*

Now fix the initial status quo g_0. Denote $f_j(g_0)$ as responder j's status quo payoff. The next result characterizes the equilibrium proposal in period 1.

Lemma 4. *Under a budgetary institution that allows positive discretionary and mandatory spending, if u_{it} is regular with increasing marginal returns, and each party's ideal value of the public good is decreasing, then for any equilibrium σ^*, given initial status quo g_0, the equilibrium proposal strategy for party i in period 1 satisfies $\gamma_{i1}^*(g_0) = x_2^*(\overline{U})$ and $\kappa_{i1}^*(g_0) = x_1^*(\overline{U}) - x_2^*(\overline{U})$, for $\overline{U} = f_j(g_0)$ and $j \neq i$.*

Proof. If party i is the proposer in period 1, then party i's equilibrium proposal strategy

$(\kappa^*_{i1}(g_0), \gamma^*_{i1}(g_0))$ is a solution to

$$\begin{aligned} \max_{(k_1,g_1)\in\mathbb{R}^2_+} \quad & u_{i1}(k_1+g_1) + \delta V_i(g_1;\sigma^*) \\ \text{s.t.} \quad & u_{j1}(k_1+g_1) + \delta V_j(g_1;\sigma^*) \geq u_{j1}(g_0) + \delta V_j(g_0;\sigma^*), \end{aligned} \tag{P_1}$$

where

$$V_i(g;\sigma^*) = p_A u_{i2}(\kappa^*_{A2}(g) + \gamma^*_{A2}(g)) + p_B u_{i2}(\kappa^*_{B2}(g) + \gamma^*_{B2}(g)).$$

For the proof we write x^*_1 and x^*_2 instead of $x^*_1(\overline{U})$ and $x^*_2(\overline{U})$. We first show that $(x^*_1 - x^*_2, x^*_2)$ is in the feasible set for (P_1). By Lemma A2, if the parties' ideal levels of the public good are decreasing, the Pareto efficient allocation is also decreasing. This implies $x^*_1 > x^*_2$, and thus $(x^*_1 - x^*_2, x^*_2) \in \mathbb{R}^2_+$ is feasible.

We next show that if $\gamma^*_{i1}(g_0) = x^*_2$ and $\kappa^*_{i1}(g_0) = x^*_1 - x^*_2$, then the induced equilibrium allocation is x^*_1 in period 1 and x^*_2 in period 2. It is straightforward to see $x^{\sigma^*}_1(g_0) = \gamma^*_{i1}(g_0) + \kappa^*_{i1}(g_0) = x^*_1$. Now to see that $x^{\sigma^*}_2(g_0) = x^*_2$, first note that by Proposition 2 part 1, $x^*_2 \in [\theta_{A2}, \theta_{B2}]$. Then by Proposition 4 part 2, we have $\gamma^*_{A2}(x^*_2) = \gamma^*_{B2}(x^*_2) = x^*_2$.

Finally, we show by contradiction that $(x^*_1 - x^*_2, x^*_2)$ is the maximizer of (P_1). Suppose not. Then proposing $(\kappa^*_{i1}(g_0), \gamma^*_{i1}(g_0))$ is better than proposing $(x^*_1 - x^*_2, x^*_2)$. That is, proposing $(\kappa^*_{i1}(g_0), \gamma^*_{i1}(g_0))$ gives proposer i a higher dynamic payoff while giving the responder j a dynamic payoff at least as high as $f_j(g_0)$. Recall the modified social planner's problem (DSP') allowed randomization over allocations in period 2, so if $(\kappa^*_{i1}(g_0), \gamma^*_{i1}(g_0)) \neq (x^*_1 - x^*_2, x^*_2)$, this implies that the allocation with $x_1 = \gamma^*_{i1}(g_0) + \kappa^*_{i1}(g_0)$, $x_{A2} = \gamma^*_{A2}(\gamma^*_{i1}(g_0))$, $x_{B2} = \gamma^*_{B2}(\gamma^*_{i1}(g_0))$ does better than x^*_1 and x^*_2. Since the solution to the social planner's problem is unique, this contradicts Lemma 3. ■

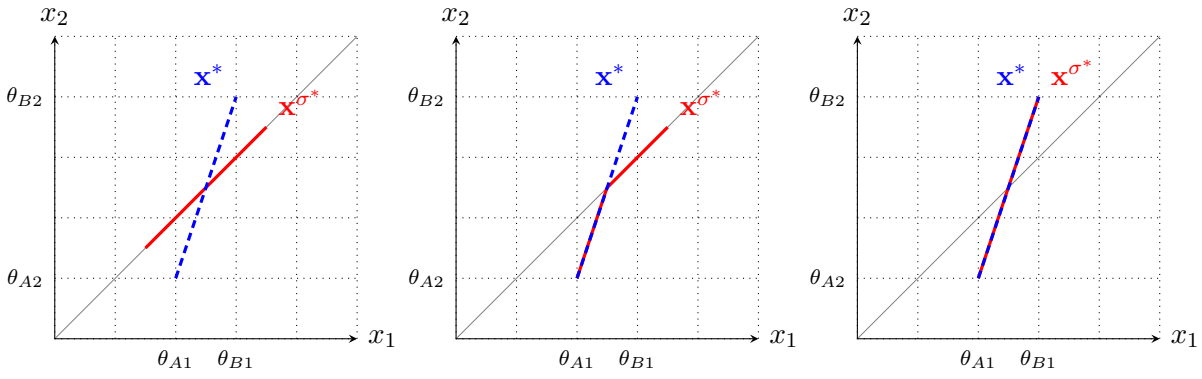
By Lemma 4, for status quo g_0 and period 1 proposer $i \in \{A, B\}$, the equilibrium outcome is $\mathbf{x}^{\sigma^*}(g_0) = \{x^*_t(\overline{U})\}^2_{t=1}$ for $\overline{U} = f_j(g^0)$ and $j \neq i$. Hence σ^* is a dynamically Pareto efficient equilibrium for any sequence of proposers, and for any $g_0 \in \mathbb{R}_+$. This completes the proof of Proposition (8).

By the arguments above it is clear that if the parties' ideal levels of the public good is increasing, and only positive discretionary spending is allowed, in general we do not obtain Pareto efficiency. This is because with increasing ideal levels of the public good $\kappa^*_{i1} =$

$x_1^* - x_2^* < 0$ and is not feasible. This implies however that allowing negative discretionary spending restores dynamic Pareto efficiency, and indeed we have this result.

Proposition 9. *Under a budgetary institution that allows positive or negative discretionary spending and positive mandatory spending, that is* $Z = \{(k_t, g_t) \in \mathbb{R} \times \mathbb{R}_+ : k_t + g_t \geq 0\}$*, then every equilibrium* σ^* *is dynamically Pareto efficient for any status quo* $g_0 \in \mathbb{R}_+$*.*

Proposition 9 is straightforward from the proof of Proposition 8 so we omit the proof. To compare the result to other budgetary institutions, Figure 4 below provides an example of equilibrium allocations for budgetary institutions that allow for mandatory spending programs, either in isolation or in combination with discretionary spending. The parameters used are the same as in Figure 3.



(a) Mandatory only

(b) Mandatory and positive only discretionary

(c) Mandatory and positive or negative discretionary

Figure 4: Equilibrium allocations under different budgetary institutions
$\boldsymbol{\theta}_A = (0.4, 0.2)$, $\boldsymbol{\theta}_B = (0.6, 0.8)$, $p_A = p_B = \frac{1}{2}$, $\delta = 1$, $u_{it}(x_t) = -(x_t - \theta_{it})^2$

In Figure 4 the dashed blue line gives all the dynamically Pareto efficient allocations for parties A and B, and the red line gives the set of equilibrium allocations. An initial status quo and a sequence of proposers induces an equilibrium allocation in the set. As shown in panels 4a and 4b, when mandatory spending is allowed but negative discretionary spending is not, equilibrium allocations do not coincide with any dynamically Pareto efficient allocation for some status quos and some realization of proposers. By contrast, when positive and negative discretionary spending is allowed, the equilibrium allocation coincides with a dynamically

Pareto efficient allocation for any status quo and any realization of proposers.

The reason mandatory and discretionary spending achieves dynamic Pareto efficiency is because the proposer in the first period can perfectly tailor the spending in the first period to first period preferences, and simultaneously tailor the next period's status quo to avoid the political risk. Positive or negative discretionary spending allows sufficient flexibility to achieve this. This suggests that the efficiency result may no longer hold in a model with $T > 2$ periods, unless further flexibility is allowed. In the next section we consider a model with more than two periods and stochastic preferences and show that state-contingent mandatory spending provides such flexibility.

7 State-contingent mandatory spending

In this section we consider a richer environment in which parties bargain in $T \geq 2$ periods and preferences are stochastic in each period reflecting uncertain changes in the economy. The economic state (henceforth we refer to the economic state as simply the state) in each period t is $s_t \in S$ where S is a finite set of $n = |S|$ possible states. We assume the distribution of states has full support in every period, but we do not require the distribution to be the same across periods. The utility of party i in period t when the spending is x and the state is s is $u_i(x, s)$. As before, we assume $u_i(x, s)$ is twice continuously differentiable and strictly concave in x. Further, $u_i(x, s)$ attains a maximum at θ_{is} and we assume $0 < \theta_{As} < \theta_{Bs}$ for all $s \in S$. The state is drawn at the beginning of each period before a proposal is made.

In this setting, we consider a budgetary institution that allows state-contingent mandatory spending. A mandatory spending program indexed to inflation or the unemployment rate, for example, might be thought of as a state-contingent mandatory spending program. A proposal in period t is a *spending rule* $g_t : S \to \mathbb{R}_+$ where $g_t(s)$ is the level of public good spending proposed to be allocated to the mandatory program at time t in state s. If the responding party agrees to the proposal, the allocation implemented in period t is $g_t(s_t)$, otherwise the allocation in period t is given by the status quo spending rule $g_{t-1}(s_t)$. In this environment, a strategy for party i in period t is $\sigma_{it} = (\gamma_{it}, \alpha_{it})$. Let $\mathcal{M}$ be the space of all functions from S to $\mathbb{R}_+$, then $\gamma_{it} : \mathcal{M} \times S \to \mathcal{M}$ is a proposal strategy for party i in period t

and $\alpha_{it} : \mathcal{M} \times S \times \mathcal{M} \to \{0, 1\}$ is an acceptance strategy for party i in period t. A strategy for party i is $\sigma_i = (\sigma_{i1}, \ldots, \sigma_{iT})$ and a profile of strategies is $\sigma = (\sigma_1, \sigma_2)$.

7.1 Pareto efficiency with stochastic preferences

With stochastic preferences the social planner chooses an allocation rule $x_t : S \to \mathbb{R}_+$ for all $t \in \{1, \ldots, T\}$ to maximize the expected payoff of one of the parties subject to providing the other party with a minimum expected dynamic payoff. Formally, a dynamically Pareto efficient allocation rule solves the following maximization propblem:

$$\begin{aligned} \max_{\{x_t : S \to \mathbb{R}_+\}_{t=1}^T} & \sum_{t=1}^{T} \delta^{t-1} \mathbb{E}_{s_t}[u_i(x_t(s_t), s_t)] \\ \text{s.t.} & \sum_{t=1}^{T} \delta^{t-1} \mathbb{E}_{s_t}[u_j(x_t(s_t), s_t)] \geq \overline{U}, \end{aligned} \tag{DSP-S}$$

for some $\overline{U} \in \mathbb{R}$, $i, j \in \{A, B\}$ and $i \neq j$. We denote the solution to (DSP-S) by the sequence of functions $\mathbf{x}^* = \{x_t^*\}_{t=1}^T$. The next proposition characterizes dynamically Pareto efficient allocation rules, anologous to Proposition 2.

Proposition 10. *Any dynamically Pareto efficient allocation rule satisfies:*

1. *For any t and t', $x_t^* = x_{t'}^*$.*

2. *For all $s \in S$ and all t, either*

$$-\frac{u_i'(x_t^*(s), s)}{u_j'(x_t^*(s), s)} = \lambda^*$$

for some $\lambda^* > 0$, or $x_t^*(s) = \theta_{As}$ or $x_t^*(s) = \theta_{Bs}$.

Proposition 10 first states that the dynamically Pareto efficient allocation rule is independent of time, i.e., the same allocation rule is used each period. The second part of the proposition states that the dynamically Pareto efficient allocation rule is either one party's ideal each period, or satisfies the condition that the ratio of the parties' marginal utilities is constant across *states*.

We next formalize a dynamically Pareto efficient equilibrium and state the efficiency result for state-contingent mandatory spending. Define recursively $x_t^\sigma(g_0)$ for $t \in \{1, \ldots, T\}$

by $x_1^\sigma(g_0) = \gamma_{i1}(g_0, s_1)$ for some $i \in \{1, 2\}$ and some $s_1 \in S$ and $x_t^\sigma(g_0) = \gamma_{it}(x_{t-1}^\sigma(g_0), s_t)$ for some $i \in \{1, 2\}$ and some $s_t \in S$ and $t \in \{2, \ldots, T\}$. An *equilibrium allocation rule* for σ given initial status quo g_0 is a possible realization of a spending rule for each period, $\mathbf{x}^\sigma(g_0) = \{x_t^\sigma(g_0)\}_{t=1}^T$. The random determination of proposers and states in each period induces a probability distribution over allocation rules given an equilibrium σ, thus any element in the support of this distribution is an equilibrium allocation rule for σ.

Definition 3. A profile of strategies σ is a *dynamically Pareto efficient equilibrium given initial status quo* $g_0 \in \mathcal{M}$ if and only if

1. σ constitutes an equilibrium.

2. Every equilibrium allocation rule $\mathbf{x}^\sigma(g_0)$ for σ given initial status quo g_0 is dynamically Pareto efficient.

Proposition 11. *Under state-contingent mandatory spending, an equilibrium is either dynamically Pareto efficient for any initial status quo* $g_0 \in \mathcal{M}$ *or is outcome-equivalent to a dynamically Pareto efficient equilibrium.*

The result in Proposition 11 is in stark contrast to the inefficiency results for mandatory spending given in Propositions 5 and 6. Note that dynamic efficiency fails in the model with changing (deterministic) preferences and fixed mandatory spending because the proposer in period 1 cannot specify spending in the current period separately from the status quo for the next period. Thus either spending is constant across periods or varies with the identity of the proposer, both of which violates dynamic Pareto efficiency.

The result with stochastic preferences is understood in an analogous way to the model with discretionary and mandatory spending. In the first period the proposer can tailor the status quo for each state such that the future proposer in that state has no incentive to change it. This mitigates the political risk. This also implies the proposer in the first period chooses the spending rule that persists across periods, satisfying the first condition of a dynamically Pareto efficient allocation rule. The first period proposer's maximization problem has the same solution as the social planner's problem with the responder's minimum payoff appropriately specified.

8 Conclusion

In this paper we study the efficiency properties of budgetary institutions. We demonstrate that discretionary only and mandatory only budget institutions may result in dynamic inefficiency, and static inefficiency in the case of mandatory only budget institutions. However, we show that by introducing flexibility through either the endogenous choice of mandatory and discretionary programs (in the case of two periods and deterministic, but fluctuating, preferences), or through a state-contingent mandatory program, parties' bargaining achieves dynamic Pareto efficiency. We show that these budgetary institutions eliminate political risk by allowing the proposer to choose a status quo that is not changed by future proposers because they fully account for future changes in preferences.

We have considered mandatory spending programs that are completely state-contingent, but it is possible that factors influencing parties' preferences, such as the mood of the electorate, cannot be contracted on. In this case it seems there is room for inefficiency even with mandatory spending that depends on the part of the state that can be contracted on. It is possible that further flexibility with discretionary spending may be helpful, but this may also imply more room for political risk. That is, including discretionary spending may allow too much flexibility resulting in proposals that depend on the identity of the proposer. We leave for future work exploring efficiency implications of discretionary and mandatory spending when a part of the state may not be contracted on.

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Appendix

A1 Pareto efficiency

A1.1 Proof of Proposition 1

First, we show that if x_t is statically Pareto efficient, then $x_t \in [\theta_{At}, \theta_{Bt}]$. Consider $x_t \notin [\theta_{At}, \theta_{Bt}]$. Then we can find x'_t in either (x_t, θ_{At}) or (θ_{Bt}, x_t) such that $u_{At}(x'_t) > u_{At}(x_t)$ and $u_{Bt}(x'_t) > u_{Bt}(x_t)$, and therefore x_t is not a solution to (SSP).

Second, we show that if $\tilde{x}_t \in [\theta_{At}, \theta_{Bt}]$, then $\tilde{x}_t$ is statically Pareto efficient. Let $\overline{u} = u_{jt}(\tilde{x}_t)$. Denote the solution to (SSP) as $\hat{x}_t(\overline{u})$. Since $u'_{At}(x_t) < 0$ and $u'_{Bt}(x_t) > 0$ for all $x_t \in (\theta_{At}, \theta_{Bt})$, the solution to (SSP) is $\hat{x}_t(\overline{u}) = \tilde{x}_t$ for any $\tilde{x}_t \in [\theta_{At}, \theta_{Bt}]$. ■

A1.2 Proof of Proposition 2

We prove the result for a more general model with $T \geq 2$. Denote a sequence of allocations by $\mathbf{x} = \{x_t\}_{t=1}^T$ and party i's discounted dynamic payoff from $\mathbf{x} = \{x_t\}_{t=1}^T$ by $U_i(\mathbf{x}) = \sum_{t=1}^T \delta^{t-1} u_{it}(x_t)$. We define a dynamically Pareto efficient allocation in the T-period problem as the solution to the following maximization problem

$$\begin{aligned} \max_{\mathbf{x} \in \mathbb{R}_+^T} \quad & U_i(\mathbf{x}) \\ \text{s.t.} \quad & U_j(\mathbf{x}) \geq \overline{U} \end{aligned} \tag{DSP}$$

for some $\overline{U} \in \mathbb{R}$, $i, j \in \{A, B\}$ and $i \neq j$. Denote the sequence of party i's static ideals by $\boldsymbol{\theta}_i = \{\theta_{it}\}_{t=1}^T$ for all $i \in \{A, B\}$, and denote the solution to (DSP) as $\mathbf{x}^* = \{x_t^*\}_{t=1}^T$.

To prove part 1, by way of contradiction, suppose $x_{t'}^* \notin [\theta_{At'}, \theta_{Bt'}]$. By Proposition 1 there exists $\hat{x}_{t'}$ such that $u_{it'}(\hat{x}_{t'}) \geq u_{it'}(x_{t'}^*)$ for all $i \in \{A, B\}$, and $u_{it'}(\hat{x}_{t'}) > u_{it'}(x_{t'}^*)$ for at least one $i \in \{A, B\}$. Now consider $\hat{\mathbf{x}} = \{\hat{x}_t\}_{t=1}^T$, with $\hat{x}_t = x_t^*$ for all $t \neq t'$. Then $U_i(\hat{\mathbf{x}}) \geq U_i(\mathbf{x}^*)$ for all $i \in \{A, B\}$, and $U_i(\hat{\mathbf{x}}) > U_i(\mathbf{x}^*)$ for at least one $i \in \{A, B\}$. Thus $\mathbf{x}^*$ is not dynamically Pareto efficient.

Next we prove part 2 by considering possible values of $\overline{U}$. Fix $i, j \in \{A, B\}$ with $i \neq j$. For any $\overline{U} > U_j(\boldsymbol{\theta}_j)$ the solution does not exist, so assume $\overline{U} \leq U_j(\boldsymbol{\theta}_j)$.

For $\overline{U} = U_j(\boldsymbol{\theta}_j)$, the solution to (DSP) is $\mathbf{x}^* = \boldsymbol{\theta}_j$ and for any $\overline{U} \leq U_j(\boldsymbol{\theta}_i)$, the solution to (DSP) is $\mathbf{x}^* = \boldsymbol{\theta}_i$. What remains is to consider the case when $\overline{U} \in (U_j(\boldsymbol{\theta}_i), U_j(\boldsymbol{\theta}_j))$. Suppose $\boldsymbol{\theta}_A = \boldsymbol{\theta}_B$, then $U_j(\boldsymbol{\theta}_i) = U_j(\boldsymbol{\theta}_j)$, which implies $\mathbf{x}^* = \boldsymbol{\theta}_i$. So consider $\boldsymbol{\theta}_A \neq \boldsymbol{\theta}_B$.

For the rest of the proof, suppose $\overline{U} \in (U_j(\boldsymbol{\theta}_i), U_j(\boldsymbol{\theta}_j))$ and $\boldsymbol{\theta}_A \neq \boldsymbol{\theta}_B$. By part 1, we can drop the $\mathbf{x} \in \mathbb{R}^T_+$ constraint from (DSP) since this will be satisfied.

The Lagrangian of the modified problem is

$$L(\mathbf{x}, \lambda) = U_i(\mathbf{x}) - \lambda \left[-U_j(\mathbf{x}) + \overline{U} \right]. \tag{A2}$$

By Takayama (1974, Theorem 1.D.4), if the Jacobian of the constraint has rank 1 then the solution to (DSP) satisfies

$$\frac{\partial L(\mathbf{x}^*, \lambda^*)}{\partial x_t} = \delta^{t-1} u'_{it}(x^*_t) + \lambda^* \delta^{t-1} u'_{jt}(x^*_t) = 0 \tag{A3}$$

with $\lambda^* \geq 0$. The Jacobian is

$$\left[\left(\delta^{t-1} \frac{\partial u_{jt}(x^*_t)}{\partial x_t} \right)_{t=1}^{T} \right], \tag{A4}$$

and it has rank 1 if there exists t' such that $x^*_{t'} \neq \theta_{jt'}$, which we show next. Suppose $\mathbf{x}^* = \boldsymbol{\theta}_j$. This implies $U_j(\mathbf{x}^*) > \overline{U}$. Because there exists t' such that $\theta_{it'} \neq \theta_{jt'}$, we can find $\alpha \in (0, 1)$ such that $x_{t'} = \alpha\theta_{it'} + (1 - \alpha)\theta_{jt'}$ satisfies the constraints in (DSP) and strictly increases the value of the objective function relative to $x^*_{t'}$.

If $\lambda^* = 0$, we obtain $\mathbf{x}^* = \boldsymbol{\theta}_i$, violating the $U_j(\mathbf{x}) \geq \overline{U}$ constraint, and hence $\lambda^* > 0$. ■

A1.3 Proof of Lemma 1

Since $\mathbf{x}$ and $\tilde{\mathbf{x}}$ are dynamically Pareto efficient, by Proposition 2 part 1, $x_t \in [\theta_{At}, \theta_{Bt}]$ and $\tilde{x}_t \in [\theta_{At}, \theta_{Bt}]$ for all t. There are three possible cases.

Case (i): $x_{t'} = \tilde{x}_{t'} = \theta_{At'}$. By Proposition 2, part 2 either $\mathbf{x} = \boldsymbol{\theta}_A$, or $\mathbf{x} = \boldsymbol{\theta}_B$, or $u'_{At'}(x_{t'}) + \lambda^* u'_{Bt'}(x_{t'}) = 0$ for some $\lambda^* > 0$. Since $\theta_{At} \neq \theta_{Bt}$ for any t, $x_{t'} = \theta_{At'}$ implies $\mathbf{x} \neq \boldsymbol{\theta}_B$. Next note that $u'_{At}(\theta_{At}) = 0$ for all t and $u'_{Bt}(\theta_{At}) \neq 0$ for any t, hence $u'_{At'}(x_{t'}) + \lambda^* u'_{Bt'}(x_{t'}) \neq 0$ for any $\lambda^* > 0$. Thus, it must be that $\mathbf{x} = \boldsymbol{\theta}_A$. A similar argument shows that $\tilde{\mathbf{x}} = \boldsymbol{\theta}_A$, proving that $\mathbf{x} = \tilde{\mathbf{x}}$.

Case (ii): $x_{t'} = \tilde{x}_{t'} = \theta_{Bt'}$. Analogous to case (i), if $x_{t'} = \tilde{x}_{t'} = \theta_{Bt'}$, then $\mathbf{x} = \tilde{\mathbf{x}} = \boldsymbol{\theta}_B$.

Case (iii): $x_{t'} = \tilde{x}_{t'} \in (\theta_{At'}, \theta_{Bt'})$. Note that $\mathbf{x} \neq \boldsymbol{\theta}_i$ for any i, so by Proposition 2, part 2 it must be that $u'_{At'}(x_{t'}) + \lambda^* u'_{Bt'}(x_{t'}) = 0$ for some $\lambda^* > 0$. This implies $-\frac{u'_{At'}(x_{t'})}{u'_{Bt'}(x_{t'})} = \lambda^*$. Similarly, $u'_{At'}(\tilde{x}_{t'}) + \tilde{\lambda}^* u'_{Bt'}(\tilde{x}_{t'}) = 0$ for some $\tilde{\lambda}^* > 0$, implying $-\frac{u'_{At'}(\tilde{x}_{t'})}{u'_{Bt'}(\tilde{x}_{t'})} = \tilde{\lambda}^*$. Since $x_{t'} = \tilde{x}_{t'}$ it follows that $\lambda^* = \tilde{\lambda}^*$. Then by Proposition 2, part 2 $-\frac{u'_{At}(x_t)}{u'_{Bt}(x_t)} = -\frac{u'_{At}(\tilde{x}_t)}{u'_{Bt}(\tilde{x}_t)} = \lambda^*$ for all t. To prove $x_t = \tilde{x}_t$ for all t, it remains to show that $-\frac{u'_{At}(x)}{u'_{Bt}(x)} = \lambda^*$ has a unique solution for any $\lambda^* > 0$. To see this, first note that $\mathbf{x} \neq \boldsymbol{\theta}_i$ for any i implies $x_t \in (\theta_{At}, \theta_{Bt})$ for all t because otherwise, $x_t = \theta_{it}$ for some i and some t, and by previous arguments this implies $\mathbf{x} = \boldsymbol{\theta}_i$, which is a contradiction. From properties of u_{At} and u_{Bt}, we know that $-\frac{u'_{At}(x)}{u'_{Bt}(x)}$ is continuous on $(\theta_{At}, \theta_{Bt})$, $-\frac{u'_{At}(x)}{u'_{Bt}(x)} > 0$ for all $x \in (\theta_{At}, \theta_{Bt})$, $\lim_{x \to \theta^+_{At}} -\frac{u'_{At}(x)}{u'_{Bt}(x)} = 0$, $\lim_{x \to \theta^-_{Bt}} -\frac{u'_{At}(x)}{u'_{Bt}(x)} = \infty$ and $\frac{\partial}{\partial x}\left[-\frac{u'_{At}(x)}{u'_{Bt}(x)}\right] = -\frac{u''_{At}(x)u'_{Bt}(x) - u'_A(x)u''_B(x)}{(u'_{Bt}(x))^2} > 0$ for all $x \in (\theta_{At}, \theta_{Bt})$. Hence, by the Intermediate Value Theorem, a solution to $-\frac{u'_{At}(x)}{u'_{Bt}(x)} = \lambda^*$ exists and by the strict monotonicity of $-\frac{u'_{At}(x)}{u'_{Bt}(x)}$, it is unique. ■

A1.4 Proof of Lemma 2

The "only if" part follows from Proposition 2 part 1 which states that if $\mathbf{x} = \{x_t\}_{t=1}^T$ is dynamically Pareto efficient, then x_t satisfies static Pareto efficiency for all t.

To show the "if" part, suppose $\mathbf{x} = \{x_t\}_{t=1}^T$ is an allocation such that x_t is statically Pareto efficient in period t for all t. The proof is trivial if $\boldsymbol{\theta}_A = \boldsymbol{\theta}_B$, so we consider the case when there is a unique t' such that $\theta_{At'} \neq \theta_{Bt'}$. We will show that $\mathbf{x}$ solves (DSP). Since $\theta_{At} = \theta_{Bt}$ for $t \neq t'$, by Proposition 1, $x_t = \theta_{At}$ for all $t \neq t'$ and $x_{t'}$ solves

$$\begin{aligned} \max_{x \in \mathbb{R}_+} \quad & u_{it'}(x) \\ \text{s.t.} \quad & u_{jt'}(x) \geq \overline{u} \end{aligned} \tag{SSP*}$$

for some $\bar{u}$. Since $x_{t'}$ solves (SSP*), it also solves

$$\begin{aligned} \max_{x \in \mathbb{R}_+} \quad & u_{it'}(x) + \textstyle\sum_{t \neq t'} u_{it}(\theta_{At}) \\ \text{s.t.} \quad & u_{jt'}(x) + \textstyle\sum_{t \neq 1} u_{jt}(\theta_{At}) \geq \overline{u} + \sum_{t \neq t'} u_{jt}(\theta_{At}). \end{aligned} \tag{SSP**}$$

Since $\{\theta_{At}\}_{t \neq t'}$ maximizes $\sum_{t \neq t'} u_{it}(x_t)$ and $\sum_{t \neq t'} u_{jt}(x_t)$, it follows that $\mathbf{x}$ solves (DSP) with $\overline{U} = \overline{u} + \sum_{t \neq t'} u_{jt}(\theta_{At})$. ■

A2 Mandatory spending

A2.1 Proof of Proposition 4

We first prove equilibrium existence by showing that a solution exists for the proposer's problem in period 2 given any status quo g_1, and given this solution, a solution exists for the proposer's problem in period 1.

Consider the problem of the proposing party $i \in \{A, B\}$ in the second period under status quo $g_1 \in \mathbb{R}_+$ and budgetary institution Z that allows for mandatory spending. The proposing party's maximization problem is:

$$\begin{aligned} \max_{(k_2,g_2)\in Z} \quad & u_{i2}(k_2 + g_2) \\ \text{s.t.} \quad & u_{j2}(k_2 + g_2) \geq u_{j2}(g_1). \end{aligned} \tag{P_2}$$

Consider the related problem

$$\max_{x_2 \in A_{j2}(g_1)} u_{i2}(x_2) \tag{P_2'}$$

where $A_{j2}(g_1) = \{x \in \mathbb{R}_+ | u_{j2}(x) \geq u_{j2}(g_1)\}$ is the responder's acceptance set under status quo g_1. If $\hat{x}_2$ is a solution to (P_2'), then any $(\hat{k}_2, \hat{g}_2) \in Z$ such that $\hat{k}_2 + \hat{g}_2 = \hat{x}_2$ is solution to (P_2).

We next show that for any $g_1 \in \mathbb{R}_+$, a solution exists for (P_2'). To show this, we prove that for any $g_1 \in \mathbb{R}_+$, acceptance set $A_{j2}(g_1)$ is non-empty and compact, and apply the Weierstrass's Theorem. Non-emptiness follows from $g_1 \in A_{j2}(g_1)$ for all $g_1 \in \mathbb{R}_+$. To show compactness, we show that $A_{j2}(g_1)$ is closed and bounded for all $g_1 \in \mathbb{R}_+$. Closedness follows from continuity of u_{j2} and boundedness follow from $\lim_{x\to\infty} u_{j2}(x) = -\infty$. To see that $\lim_{x\to\infty} u_{j2}(x) = -\infty$, note that u_{j2} is continuous, differentiable and strictly concave, which implies that $u_{j2}(x) < u_{j2}(y) + u'_{j2}(y)(x - y)$ for any $x, y \in \mathbb{R}_+$. Selecting $y > \theta_{j2}$ gives $u'_{j2}(y) < 0$ and taking limit as $x \to \infty$ shows the claim.

We next show that a solution exists to the proposer's problem in period 1. The proposer's problem in period 1 is

$$\begin{aligned} \max_{(k_1,g_1)\in Z} \quad & u_{i1}(k_1 + g_1) + \delta V_i(g_1; \sigma_2^*) \\ \text{s.t.} \quad & u_{j1}(k_1 + g_1) + \delta V_j(g_1; \sigma_2^*) \geq u_{j1}(g_0) + \delta V_j(g_0; \sigma_2^*). \end{aligned} \tag{P_1}$$

To apply Weierstrass's Theorem, we show $V_i(g_1; \sigma_2^*)$ is continuous in g_1 and the constraint set is compact. The continuation value V_i is given by

$$V_i(g_1; \sigma_2^*) = p_A u_{i2}(\kappa_{A2}^*(g_1) + \gamma_{A2}^*(g_1)) + p_B u_{i2}(\kappa_{B2}^*(g_1) + \gamma_{B2}^*(g_1)), \tag{A5}$$

where $(\kappa_{i2}^*, \gamma_{i2}^*)$ is a solution to (P_2) for all $i \in \{A, B\}$. We show that $\kappa_{i2}^*(g_1) + \gamma_{i2}^*(g_1)$ is continuous in g_1, by first showing $\kappa_{i2}^*(g_1) + \gamma_{i2}^*(g_1)$ is unique for any $g_1 \in \mathbb{R}_+$, and then using the Maximum Theorem.

Strict concavity of u_{j2} implies that $A_{j2}(g_1)$ is convex. Given that u_{i2} is strictly concave, the solution to (P_2') is unique for any g_1. To apply the Maximum Theorem we need to show that the correspondence A_{j2} has non-empty and compact values and is continuous. We already proved the first two properties. We next establish upper and lower hemicontinuity.

Lemma A1. *Let $X \subseteq \mathbb{R}$ be closed and convex, let $Y \subseteq \mathbb{R}$ and let $f : X \to Y$ be a continuous function. Define $\varphi : X \twoheadrightarrow X$ by*

$$\varphi(x) = \{y \in X | f(y) \geq f(x)\}. \tag{A6}$$

1. *If $\varphi(x)$ is compact $\forall x \in X$, then φ is upper hemicontinuous.*

2. *If f is strictly concave, then φ is lower hemicontinuous.*

Proof. To show part 1, it suffices to prove that if two sequences $x_n \to x$ and $y_n \to y$ satisfy $y_n \in \varphi(x_n)$ for all $n \in \mathbb{N}$, then $y \in \varphi(y)$. Suppose, towards a contradiction, that $y \notin \varphi(y)$. Then $f(y) < f(x)$. Since f is continuous, $x_n \to x$ and $y_n \to y$, we can find n' such that $f(y_n) < f(x_n)$ for all $n' \geq n$, which contradicts $y_n \in \varphi(x_n)$ for all $n \in \mathbb{N}$.

To show part 2, let $x_n \to x$ and $y \in \varphi(x)$. We show that for each sequence $x_n \to x$ there exists a sequence $y_n \to y$ and n' such that $y_n \in \varphi(x_n)$ for all $n \geq n'$. First suppose $f(y) > f(x)$. Set $y_n = y$. Clearly, $y_n \to y$. By continuity of f, there exists n' such that $f(y_n) \geq f(x_n)$ for all $n \geq n'$, that is, $y_n \in \varphi(x_n)$. Next suppose $f(y) = f(x)$. There are two cases to consider. First, if $y = x$, set $y_n = x_n$. Clearly $y_n \to y$ and $y_n \in \varphi(x_n)$ for all n. Second, suppose $y \neq x$. By strict concavity of f, there exist at most one such $y \in X$. Set $y_n = y$ whenever $f(x_n) \leq f(x)$. When $f(x_n) > f(x)$, by strict concavity

of f, $\max_{z\in[\min\{x,y\},\max\{x,y\}]} f(z)$ has a unique solution m. Because $x_n \to x$, passing into subsequence if necessary, $f(x_n) < f(m)$ for all n. For any x_n such that $f(x_n) > f(x)$, strict concavity of f and Intermediate Value Theorem imply that there exists a unique $y_n \neq x_n$ such that $f(y_n) = f(x_n)$. ■

By the Maximum Theorem, the proposer's value function in (P_2') is continuous in g_1 and the correspondence of maximizers is upper hemicontinuous in g_1. Moreover, (P_2') for any $g_1 \in \mathbb{R}_+$ involves maximization of a strictly concave objective function over compact interval, and thus has a unique solution. Because a singleton-valued upper hemicontinuous correspondence is continuous as a function, for any $g_1 \in \mathbb{R}_+$ there exists unique equilibrium level of public good spending $\kappa^*_{i2}(g_1) + \gamma^*_{i2}(g_1)$ that varies continuously in g_1.

Let $V_i(g_1)$ be the continuation value of party $i \in \{A, B\}$ at the beginning of second period with status quo $g_1 \in \mathbb{R}_+$, before the identity of the proposing party has been determined and let $j \neq i$. V_i is given by

$$V_i(g_1) = p_i u_{i2}(\kappa^*_{i2}(g_1) + \gamma^*_{i2}(g_1)) + (1 - p_i)u_{i2}(\kappa^*_{j2}(g_1) + \gamma^*_{j2}(g_1)). \tag{A7}$$

By continuity of $\kappa^*_{i2}(g_1) + \gamma^*_{i2}(g_1)$ for all $i \in \{A, B\}$, V_i is continuous in g_1. We also note that $V_i(g_1) \in [u_{i2}(\theta_{j2}), u_{i2}(\theta_{i2})]$ for any $i \in \{A, B\}$, $j \neq i$ and any $g_1 \in \mathbb{R}_+$. The upper bound because θ_{i2} is the unique maximizer of u_{i2}. The lower bound because, for any $g_1 \in \mathbb{R}_+$, $u_{i2}(\kappa^*_{i2}(g_1) + \gamma^*_{i2}(g_1)) \geq u_{i2}(\theta_{j2})$, if not then i could make alternative proposal j would accept and i would strictly prefer, and $u_{i2}(\kappa^*_{j2}(g_1) + \gamma^*_{j2}(g_1)) \geq u_{i2}(\theta_{j2})$, if not then j could make alternative proposal that i would accept and j would strictly prefer.

We now proceed to the first period. Let $F_i(k_1, g_1)$ for any $i \in \{A, B\}$ denote overall (dynamic) utility of party i under budgetary institutions that allow for both mandatory and discretionary spending programs. Let $f_i(g_1)$ denote the same utility when only mandatory spending programs are allowed. Formally,

$$\begin{aligned} F_i(k_1, g_1) &= u_{i1}(k_1 + g_1) + \delta V_i(g_1) \\ f_i(g_1) &= u_{i1}(g_1) + \delta V_i(g_1). \end{aligned} \tag{A8}$$

Because $V_i(g_1)$ is continuous in g_1, $F_i(k_1, g_1)$ is jointly continuous in (k_1, g_1) and $f_i(g_1)$ is

continuous in g_1.

Depending on the budgetary institution Z, the proposing party $i \in \{A, B\}$ in the first period under status quo $g_0 \in \mathbb{R}_+$ will propose $(k_1, g_1) \in Z$ maximizing her utility subject to the responding party $j \neq i$ acceptance. When only mandatory spending program are allowed, $Z = \{0\} \times \mathbb{R}_+$, and i's equilibrium proposed public good spending is

$$(\kappa_{i1}^*(g_0), \gamma_{i1}^*(g_0)) \in \underset{(k_1,g_1)\in Z | f_j(g_1) \geq f_j(g_0)}{\arg\max} f_i(g_1). \tag{A9}$$

When both types of spending are allowed, either $Z = \mathbb{R}_+ \times \mathbb{R}_+$ or $Z = \{(k_t, g_t) \in \mathbb{R} \times \mathbb{R}_+ | k_t + g_t \geq 0\}$, depending on whether discretionary spending can be negative. In equilibrium party i proposes public good spending

$$(\kappa_{i1}^*(g_0), \gamma_{i1}^*(g_0)) \in \underset{(k_1,g_1)\in Z | F_j(k_1,g_1) \geq F_j(0,g_0)}{\arg\max} F_i(k_1, g_1). \tag{A10}$$

In each of these problems, the objective function is continuous and the constraint set is compact for any $g_0 \in \mathbb{R}_+$. Continuity of F_i and f_i has been noted above and compactness, noting that V_i is bounded, follows from the analogous argument to the one made for the second period. Hence for any $g_0 \in \mathbb{R}_+$, a solution to each of the optimization problems exists, so that $\kappa_{i1}^*(g_0)$ and $\gamma_{i1}^*(g_0)$ are well defined. This concludes the proof of the existence of equilibrium σ^*.

We now prove parts 1 and 2, characterizing second-period κ_{i2}^* and γ_{i2}^*. For part 1, let Z be a budgetary institution that allows mandatory spending. Notice that for any $(k_2, g_2) \in Z$ with $k_2 + g_2 = x_2 \notin [\theta_{A2}, \theta_{B2}]$, there exists $(k_2', g_2') \in Z$ with $k_2' + g_2' = x_2' \in [\theta_{A2}, \theta_{B2}]$ such that $u_{A2}(x_2) < u_{A2}(x_2')$ and $u_{B2}(x_2) < u_{B2}(x_2')$. If $\kappa_{i2}^*(g_1) + \gamma_{i2}^*(g_1) \notin [\theta_{A2}, \theta_{B2}]$ for some $i \in \{A, B\}$ and $g_1 \in \mathbb{R}_+$, then there exists another proposal $z_2 = (k_2', g_2') \in Z$ that is accepted under status quo g_1 and makes the proposer strictly better off. Hence $\kappa_{i2}^*(g_1) + \gamma_{i2}^*(g_1) \in [\theta_{A2}, \theta_{B2}]$ for all $i \in \{A, B\}$ and $g_1 \in \mathbb{R}_+$.

For part 2, assume $\theta_{A2} \neq \theta_{B2}$. There are three possible cases.

Case (i): $g_1 < \theta_{A2}$. Because $u_{A2}(g_1) < u_{A2}(\theta_{A2})$ and $u_{B2}(g_1) < u_{B2}(\theta_{A2})$, party A proposes $\kappa_{A2}^*(g_1) + \gamma_{A2}^*(g_1) = \theta_{A2}$. When party B proposes we claim $\kappa_{B2}^*(g_1) + \gamma_{B2}^*(g_1) > \theta_{A2}$. To see this first note that $u_{i2}(k_2 + g_2) < u_{i2}(\theta_{A2})$ for any $k_2 + g_2 < \theta_{A2}$ and $i \in \{A, B\}$.

Furthermore, there exists $\epsilon > 0$ such that $u_{A2}(g_1) < u_{A2}(\theta_{A2} + \epsilon)$, by continuity of u_{A2}, and $u_{B2}(\theta_{A2}) < u_{B2}(\theta_{A2} + \epsilon)$, by $\theta_{A2} < \theta_{B2}$, showing the claim.

Case (ii): $g_1 > \theta_{B2}$. Analogous to case (i), $\kappa^*_{B2}(g_1) + \gamma^*_{B2}(g_1) = \theta_{B2}$ and $\kappa^*_{A2}(g_1) + \gamma^*_{A2}(g_1) < \theta_{B2}$.

Case (iii): $g_1 \in [\theta_{A2}, \theta_{B2}]$. Because $u_{A2}(k_2 + g_2) < u_{A2}(g_1)$ for any $k_2 + g_2 > g_1$ and $u_{B2}(k_2 + g_2) < u_{B2}(g_1)$ for any $k_2 + g_2 < g_1$, party A will never propose or accept any $k_2 + g_2 > g_1$ and party B will never propose or accept any $k_2 + g_2 < g_1$. Hence $\kappa^*_{A2}(g_1) + \gamma^*_{A2}(g_1) = \kappa^*_{B2}(g_1) + \gamma^*_{B2}(g_1) = g_1$. ■

A2.2 Proof of Proposition 5

Take any equilibrium σ^*. The proposition rules out $\theta_{At} = \theta_{Bt}$ for some t, so assume $\theta_{At} \neq \theta_{Bt}$ for all t throughout. Recall that by Proposition 4 part 2, the second-period public good spending, given initial status quo g_0 and proposing party $i \in \{A, B\}$ in the first period, satisfies either $\gamma^*_{A2}(\gamma^*_{i1}(g_0)) \neq \gamma^*_{B2}(\gamma^*_{i1}(g_0))$ or $\gamma^*_{A2}(\gamma^*_{i1}(g_0)) = \gamma^*_{B2}(\gamma^*_{i1}(g_0)) = \gamma^*_{i1}(g_0)$. In the former case, $x_1^{\sigma^*}(g_0) = \gamma^*_{i1}(g_0)$ and at least one of the equilibrium allocations $\{x_1^{\sigma^*}, \gamma^*_{A2}(\gamma^*_{i1}(g_0))\}$ and $\{x_1^{\sigma^*}, \gamma^*_{B2}(\gamma^*_{i1}(g_0))\}$ is dynamically Pareto inefficient by Lemma 1. Hence if σ^* is dynamically Pareto efficient equilibrium given g_0, the equilibrium allocation $\mathbf{x}^{\sigma^*}(g_0)$ satisfies $x_1^{\sigma^*}(g_0) = x_2^{\sigma^*}(g_0)$.

To see part 1, consider the $\theta_{B1} < \theta_{A2}$ case. When $\theta_{B2} < \theta_{A1}$ the argument is similar and omitted. Assume that σ^* is dynamically Pareto efficient equilibrium given g_0. Then $\mathbf{x}^{\sigma^*}(g_0)$ with $x_1^{\sigma^*}(g_0) = x_2^{\sigma^*}(g_0)$ is dynamically Pareto efficient allocation. Now, by Proposition 2 part 1, any dynamically Pareto efficient allocation $\mathbf{x}^*$ satisfies $x_1^* \in [\theta_{A1}, \theta_{B1}]$ and $x_2^* \in [\theta_{A2}, \theta_{B2}]$, which, combined with $\theta_{B1} < \theta_{A2}$, implies $x_1^* < x_2^*$, contradicting dynamic Pareto efficiency of $\mathbf{x}^{\sigma^*}(g_0)$.

Lemma A2. *Suppose $u_{it}(x_t)$ is regular with increasing returns for all $i \in \{A, B\}$. If $\theta_{it'} > \theta_{it''}$ for all $i \in \{A, B\}$, then any dynamically Pareto efficient allocation $\mathbf{x}^*$ satisfies $x^*_{t'} > x^*_{t''}$.*

Proof. Since $\mathbf{x}^*$ is dynamically Pareto efficient, it follows that for some $\alpha \in [0, 1]$, $\mathbf{x}^*$ is a

solution to the following maximization problem

$$\max_{\mathbf{x}\in\mathbb{R}_+^T} \alpha U_A(\mathbf{x}) + (1-\alpha)U_B(\mathbf{x}),$$

which implies that x_t^* is a solution to the following maximization problem

$$\max_{x_t\in\mathbb{R}_+} \alpha u_A(x_t,\theta_{At}) + (1-\alpha)u_B(x_t,\theta_{Bt})$$

for all t.

Let $f(x_t,\theta_{At},\theta_{Bt}) = \alpha u_A(x_t,\theta_{At}) + (1-\alpha)u_B(x_t,\theta_{Bt})$. Since $\frac{\partial u_i}{\partial x_t}$ is strictly increasing in θ_{it} for all i, we have that $\frac{\partial f}{\partial x_t}$ is strictly increasing in θ_{it} for all i. It follows from standard monotone comparative statics results (for example, Edlin and Shannon, 1998, Theorem 3) that if $\theta_{it'} > \theta_{it''}$ for all $i \in \{A,B\}$, then $x_{t'}^* > x_{t''}^*$. ■

An identical argument to part 1 can be used to prove part 2. By Lemma A2, we know that any dynamically Pareto efficient allocation $\mathbf{x}^*$ satisfies $x_1^* \neq x_2^*$, whereas σ^*, if it constitutes dynamically Pareto efficient equilibrium given g_0, gives rise to equilibrium allocation $\mathbf{x}^{\sigma^*}(g_0)$ with $x_1^{\sigma^*}(g_0) = x_2^{\sigma^*}(g_0)$.

Lemma A3. *Suppose $u_{it}(x_t) = -(|x_t - \theta_{it}|)^r$ where $r > 1$. Then an allocation $\mathbf{x}$ is dynamically Pareto efficient if and only if $\mathbf{x} = \alpha\boldsymbol{\theta}_i + (1-\alpha)\boldsymbol{\theta}_j$ for any $\alpha \in [0,1]$.*

Proof. Fix $i,j \in \{A,B\}$ with $i \neq j$ in (DSP). For any $\overline{U} > U_j(\boldsymbol{\theta}_j)$ the solution to (DSP) does not exist so assume $\overline{U} \leq U_j(\boldsymbol{\theta}_j)$.

Suppose $\boldsymbol{\theta}_i = \boldsymbol{\theta}_j$. Then for any $\overline{U} \leq U_j(\boldsymbol{\theta}_j)$ there exists unique solution to (DSP), $\mathbf{x}^* = \boldsymbol{\theta}_i$, and $\mathbf{x}^* = \alpha\boldsymbol{\theta}_i + (1-\alpha)\boldsymbol{\theta}_j$ for any $\alpha \in [0,1]$.

Suppose $\boldsymbol{\theta}_i \neq \boldsymbol{\theta}_j$. For $\overline{U} = U_j(\boldsymbol{\theta}_j)$, the solution is $\mathbf{x}^* = \boldsymbol{\theta}_j$ and for any $\overline{U} \leq U_j(\boldsymbol{\theta}_i)$, the solution is $\mathbf{x}^* = \boldsymbol{\theta}_i$, to that $\mathbf{x}^* = \alpha\boldsymbol{\theta}_i + (1-\alpha)\boldsymbol{\theta}_j$ for $\alpha \in \{0,1\}$. What remains is to consider the case when $\overline{U} \in (U_j(\boldsymbol{\theta}_i), U_j(\boldsymbol{\theta}_j))$.

For the rest of the proof, suppose $\overline{U} \in (U_j(\boldsymbol{\theta}_i), U_j(\boldsymbol{\theta}_j))$ and $\boldsymbol{\theta}_i \neq \boldsymbol{\theta}_j$. By Proposition 2 part 1, we can drop the $\mathbf{x} \in \mathbb{R}_+^T$ constrain from (DSP) since it will be satisfied. Lagrangian of the modified problem is $L(\mathbf{x},\lambda) = U_i(\mathbf{x}) - \lambda[-U_j(\mathbf{x}) + \overline{U}]$. By Takayama (1974, Theorem 1.D.2), and since there exists $\mathbf{x}$ such that $U_j(\mathbf{x}) > \overline{U}$, $\mathbf{x} = \boldsymbol{\theta}_j$, $\frac{\partial L(\mathbf{x}^*,\lambda^*)}{\partial x_t} = 0$ with $\lambda^* \geq 0$ for all t is both sufficient and necessary for $\mathbf{x}^*$.

Assume $i = A$. If $i = B$, the argument is similar and omitted. Because $x_t^* \in [\theta_{At}, \theta_{Bt}]$ for all t, $\frac{\partial L(\mathbf{x}^*,\lambda^*)}{\partial x_t}\frac{1}{r} = -(x_t^* - \theta_{At})^{r-1} + \lambda^*(\theta_{Bt} - x_t^*)^{r-1}$ so that $\frac{\partial L(\mathbf{x}^*,\lambda^*)}{\partial x_t} = 0$ is equivalent to $-(x_t^* - \theta_{At})^{r-1} + \lambda^*(\theta_{Bt} - x_t^*)^{r-1} = 0$. If $\lambda^* = 0$, we obtain $\mathbf{x}^* = \boldsymbol{\theta}_A$, violating the $U_B(\mathbf{x}) \geq \overline{U}$ constraint, and hence $\lambda^* > 0$. If $x_t^* = \theta_{Bt}$ or $x_t^* = \theta_{At}$, we obtain $x_t^* = \theta_{At}$ or $x_t^* = \theta_{Bt}$ respectively, which is possible if and only if $\theta_{At} = \theta_{Bt}$. Hence for all t such that $\theta_{At} \neq \theta_{Bt}$, $\frac{x_t^* - \theta_{At}}{\theta_{Bt} - x_t^*} = [\lambda^*]^{\frac{1}{r-1}}$ with $\lambda^* > 0$.

Thus $\theta_{Bt} - x_t^* = (\theta_{Bt'} - x_{t'}^*)\frac{\theta_{Bt} - \theta_{At}}{\theta_{Bt'} - \theta_{At'}}$ for all t where $t' \in \{t|\theta_{At} \neq \theta_{Bt}\}$, so that $U_B(\mathbf{x}^*) = \left(\frac{\theta_{Bt'} - x_{t'}^*}{\theta_{Bt'} - \theta_{At}}\right)^r \sum_{t=1}^T \delta^{t-1}(-(\theta_{Bt} - \theta_{At})^r)$. Argument similar to the one used in the proof of Proposition 2 shows that $U_B(\mathbf{x}^*) = \overline{U}$, which rewrites as $\frac{\theta_{Bt'} - x_{t'}^*}{\theta_{Bt'} - \theta_{At}} = \left(\frac{\overline{U}}{U_B(\boldsymbol{\theta}_A)}\right)^{\frac{1}{r}} = \alpha \Leftrightarrow x_{t'}^* = \alpha\theta_{At'} + (1-\alpha)\theta_{Bt'}$. To conclude the proof we note that $\overline{U} \in (U_B(\boldsymbol{\theta}_A), U_B(\boldsymbol{\theta}_B))$ and $U_B(\boldsymbol{\theta}_B) = 0$, and hence $\alpha \in (0,1)$. ■

To see the final result, assume again that σ^* is dynamically Pareto efficient given g_0, so that $x_1^{\sigma^*}(g_0) = x_2^{\sigma^*}(g_0)$. By Lemma A3, $\mathbf{x}$ is dynamically Pareto efficient allocation if and only if $\mathbf{x} = \alpha\boldsymbol{\theta}_A + (1-\alpha)\boldsymbol{\theta}_B$ for any $\alpha \in [0,1]$. This implies any dynamically Pareto efficient allocation $\{x_1^*, x_2^*\}$ can be written as $x_1^*(\alpha) = \alpha\theta_{A1} + (1-\alpha)\theta_{B1}$ and $x_2^*(\alpha) = \alpha\theta_{A2} + (1-\alpha)\theta_{B2}$. Because $x_1^{\sigma^*}(g_0) = x_2^{\sigma^*}(g_0)$ is dynamically Pareto efficient allocation, $x_1^{\sigma^*}(g_0) = x_1^*(\alpha^*)$ and $x_2^{\sigma^*}(g_0) = x_2^*(\alpha^*)$ where α^* solves $x_1^*(\alpha) = x_2^*(\alpha)$. Straightforward algebra shows $\alpha^* = \frac{\theta_{B1} - \theta_{B2}}{\theta_{B1} - \theta_{B2} + \theta_{A2} - \theta_{A1}}$. Because $\text{sgn}\,[\theta_{B1} - \theta_{B2}] = \text{sgn}\,[\theta_{A2} - \theta_{A1}]$ and $\theta_{i1} \neq \theta_{i2}$ for all $i \in \{A, B\}$, $\alpha^* \in (0,1)$. This implies $x_1^*(\alpha^*) \in (\theta_{A1}, \theta_{B1})$, $x_2^*(\alpha^*) \in (\theta_{A2}, \theta_{B2})$ and $x_1^{\sigma^*}(g_0) = x_2^{\sigma^*}(g_0) \in (\theta_{A1}, \theta_{B1}) \cap (\theta_{A2}, \theta_{B2})$. Because α^* is unique and $x_1^{\sigma^*}(g_0) = x_1^*(\alpha^*)$, $\gamma_{A1}^*(g_0) = \gamma_{B1}^*(g_0) \in (\theta_{A1}, \theta_{B1}) \cap (\theta_{A2}, \theta_{B2})$, if σ^* is dynamically Pareto efficient equilibrium given g_0.

We now argue that $f_A(\gamma_{A1}^*(g_0)) = f_A(g_0)$ and $f_B(\gamma_{A1}^*(g_0)) = f_B(g_0)$ for any g_0 such that σ^* is dynamically Pareto efficient given g_0. To see this, note that $f_A(\gamma_{A1}^*(g_0)) \geq f_A(g_0)$ and $f_B(\gamma_{A1}^*(g_0)) \geq f_B(g_0)$, because $\gamma_{A1}^*(g_0)$ is proposed by A and accepted by B under status quo g_0. Now assume that at least one of the inequalities is strict. We claim that $f_A(\gamma_{A1}^*(g_0)) = f_A(g_0)$ and $f_B(\gamma_{A1}^*(g_0)) > f_B(g_0)$ contradicts optimality of A proposing $\gamma_{A1}^*(g_0)$. Similar omitted arguments can be used to rule out the remaining cases. When $f_A(\gamma_{A1}^*(g_0)) = f_A(g_0)$ and $f_B(\gamma_{A1}^*(g_0)) > f_B(g_0)$, we will show that there exists $\epsilon > 0$ such that $f_i(\gamma_{A1}^*(g_0) - \epsilon) >$

$f_i(\gamma^*_{A1}(g_0))$ for all $i \in \{A, B\}$, so that proposing $\gamma^*_{A1}(g_0)$ cannot be optimal for A. To see that such ϵ exists, we note that $\gamma^*_{A1}(g_0) \in (\theta_{A1}, \theta_{B1}) \cap (\theta_{A2}, \theta_{B2})$ and that $f_A(g_1)$ and $f_B(g_1)$ are respectively strictly decreasing and strictly increasing in g_1 on $(\theta_{A1}, \theta_{B1}) \cap (\theta_{A2}, \theta_{B2})$. The claimed properties of f_i for $i \in \{A, B\}$ follow from $f_i(g_1) = u_{i1}(g_1) + \delta V_i(g_1)$, where $V_i(g_1) = u_{i2}(g_1)$ for all $g_1 \in [\theta_{A2}, \theta_{B2}]$ by Proposition 4 part 2.

We know that $f_A(\gamma^*_{A1}(g_0)) = f_A(g_0)$ for any g_0 such that σ^* is dynamically Pareto efficient equilibrium given g_0 and that $\gamma^*_{A1}(g_0) = x^*_1(\alpha^*)$. To finish the proof of the final result, we show that there exists finite set of initial status quos g_0 that solve $f_A(x^*_1(\alpha^*)) = f_A(g_0)$. We use the following set of results.

Lemma A4. *Suppose* $u_{it}(x_t) = -(|x_t - \theta_{it}|)^r$ *where* $r > 1$ *and* $Z = \{0\} \times \mathbb{R}_+$. *Then* $\kappa^*_{i2}(g_1) = 0$ *for all* $i \in \{A, B\}$ *and all* $g_1 \in \mathbb{R}_+$ *and*

$$\gamma^*_{A2}(g_1) = \max\{\theta_{A2}, \min\{g_1, 2\theta_{B2} - g_1\}\}$$
$$\gamma^*_{B2}(g_1) = \min\{\theta_{B2}, \max\{g_1, 2\theta_{A2} - g_1\}\}$$

for all $g_1 \in \mathbb{R}_+$. *Moreover, for all* $g_1 \in \mathbb{R}_+ \setminus \{2\theta_{A2} - \theta_{B2}, \theta_{A2}, \theta_{B2}, 2\theta_{B2} - \theta_{A2}\}$ *and for all* $i \in \{A, B\}$, $V_i''(g_1)$ *exists and* $V_i''(g_1) \leq 0$.

Proof. If $Z = \{0\} \times \mathbb{R}_+$, only mandatory spending programs are allowed and $\kappa^*_{i2}(g_1) = 0$ for all $i \in \{A, B\}$ and all $g_1 \in \mathbb{R}_+$ is immediate. To prove the claimed structure of $\gamma^*_{i2}(g_1)$, the acceptance set of each party is

$$\begin{aligned} A_{A2}(g_1) &= [\min\{g_1, 2\theta_{A2} - g_1\}, \max\{g_1, 2\theta_{A2} - g_1\}] \cap \mathbb{R}_+ \\ A_{B2}(g_1) &= [\min\{g_1, 2\theta_{B2} - g_1\}, \max\{g_1, 2\theta_{B2} - g_1\}] \cap \mathbb{R}_+ \end{aligned} \tag{A11}$$

which follows from the fact that under status quo g_1, party $i \in \{A, B\}$ is willing to accept any policy (weakly) closer to θ_{i2} than g_1. When party $i \in \{A, B\}$ is the proposer and party $j \neq i$ the responder, party i proposes θ_{i2} for any g_1 such that $\theta_{i2} \in A_{j2}(g_1)$. For any g_1 such that $\theta_{i2} \notin A_{j2}(g_1)$, party A proposes the minimal level of public good B is willing to accept, $\min\{g_1, 2\theta_{B2} - g_1\}$, and party B proposes the maximal level of public good A is willing to accept, $\max\{g_1, 2\theta_{A2} - g_1\}$. Because $\theta_{A2} \leq \min\{g_1, 2\theta_{B2} - g_1\}$ when $\theta_{A2} \notin A_{B2}(g_1)$ and $\theta_{A2} \geq \min\{g_1, 2\theta_{B2} - g_1\}$ when $\theta_{A2} \in A_{B2}(g_1)$, we can write $\gamma^*_{A2}(g_1)$ as stated. Similarly,

$\theta_{B2} \geq \max\{g_1, 2\theta_{A2} - g_1\}$ when $\theta_{B2} \notin A_{A2}(g_1)$ and $\theta_{B2} \leq \max\{g_1, 2\theta_{A2} - g_1\}$ when $\theta_{B2} \in A_{A2}(g_1)$ allows us to write $\gamma^*_{B2}(g_1)$ as stated.

To prove that $V_i''(g_1)$ exists and $V_i''(g_1) \leq 0$ for any $i \in \{A, B\}$ and any status quo $g_1 \in \mathbb{R}_+ \setminus \{2\theta_{A2} - \theta_{B2}, \theta_{A2}, \theta_{B2}, 2\theta_{B2} - \theta_{A2}\}$, denote $Q_1 = (0, \max\{0, 2\theta_{A2} - \theta_{B2}\})$, $Q_2 = (\max\{0, 2\theta_{A2} - \theta_{B2}\}, \theta_{A2})$, $Q_3 = (\theta_{A2}, \theta_{B2})$, $Q_4 = (\theta_{B2}, 2\theta_{B2} - \theta_{A2})$ and $Q_5 = (2\theta_{B2} - \theta_{A2}, \infty)$. Note that some of these intervals need not exists, for example when $2\theta_{A2} - \theta_{B2} \leq 0$ or $\theta_{A2} = 0$ or $\theta_{A2} = \theta_{B2}$. Inspection of expression for $\gamma^*_{A2}(g_1)$ shows that it is constant in g_1 on $Q_1 \cup Q_2 \cup Q_5$, equals g_1 on Q_3 and equals $2\theta_{B2} - g_1$ on Q_4. Similarly, $\gamma^*_{B2}(g_1)$ is constant in g_1 on $Q_1 \cup Q_4 \cup Q_5$, equals $2\theta_{A2} - g_1$ on Q_2 and equals g_1 on Q_3. Direct substitution into $V_i(g_1) = \sum_{j\in\{A,B\}} p_j u_{i2}(\kappa^*_{j2}(g_1) + \gamma^*_{j2}(g_1))$ then gives

$$V_i'(g_1) = \begin{cases} -p_B u_{i2}'(2\theta_{A2} - g_1) & \text{if } \ g_1 \in Q_2 \\ u_{i2}'(g_1) & \text{if } \ g_1 \in Q_3 \\ -p_A u_{i2}'(2\theta_{B2} - g_1) & \text{if } \ g_1 \in Q_4 \\ 0 & \text{if } \ g_1 \in Q_1 \cup Q_5. \end{cases} \tag{A12}$$

Clearly $V_i''(g_1)$ exists and, by strict concavity of the stage utilities, $V_i''(g_1) \leq 0$ for all $i \in \{A, B\}$ and for all $g_1 \in Q_k$ for $k \in \{1, \ldots, 5\}$. Noting $\mathbb{R}_+ \setminus \{2\theta_{A2} - \theta_{B2}, \theta_{A2}, \theta_{B2}, 2\theta_{B2} - \theta_{A2}\} = \cup_{k\in\{1,\ldots,5\}} Q_k$ concludes the proof of the lemma. ■

Returning to the proof of the proposition, we need to show that there exists a finite set of initial status quos g_0 that solve $f_A(x_1^*(\alpha^*)) = f_A(g_0)$. By Lemma A4, $f_A''(g_0) = u_{A1}''(g_0) + \delta V_A''(g_0) < 0$ for any $g_0 \in \cup_{k\in\{1,\ldots,5\}} Q_k$. Hence $f_A(x_1^*(\alpha^*)) = f_A(g_0)$ has at most two solutions on Q_k for each $k \in \{1, \ldots, 5\}$, because $f_A(g_0)$ is strictly concave on Q_k. Since $\mathbb{R}_+ \setminus \cup_{k\in\{1,\ldots,5\}} Q_k$ includes at most finite set of points $\{2\theta_{A2} - \theta_{B2}, \theta_{A2}, \theta_{B2}, 2\theta_{B2} - \theta_{A2}\}$, set of solutions of $f_A(x_1^*(\alpha^*)) = f_A(g_0)$ on $\mathbb{R}_+$ is finite. ■

A2.3 Proof of Proposition 6

The strategy of the proof is to show that if $Z = \{0\} \times \mathbb{R}_+$ and $u_{it}(x_t) = -(x_t - \theta_{it})^2$ for all $i \in \{A, B\}$ and all t, then $\frac{\theta_{A1} - \theta_{A2}}{\theta_{B2} - \theta_{A2}} \in (0, \psi(\delta, p_A))$ implies that $f_A(g_1)$ has unique global maximum at $g_A^* < \theta_{A1}$ and $\frac{\theta_{B2} - \theta_{B1}}{\theta_{B2} - \theta_{A2}} \in (0, \psi(\delta, p_B))$ implies that $f_B(g_1)$ has unique global

maximum at $g_B^* > \theta_{B1}$. For initial status quo $g_0 = g_i^*$ for some $i \in \{A, B\}$, this implies that the only proposal i is willing to accept is $(0, g_i^*)$ and we know, by Proposition 1, that $0 + g_i^* \notin [\theta_{A1}, \theta_{B1}]$ is statically Pareto inefficient. That the inefficiency extends to a set $\mathcal{I}$ of non-zero measure follows from continuity of f_i shown in the proof of Proposition 4 and from g_i^* begin unique maximizer of f_i.

We will show that, under the remaining conditions of the proposition, $f_A(g_1)$ has unique global maximum with the claimed properties. Proof for f_B is analogous and omitted in the sake of space. Because $\frac{\theta_{A1}-\theta_{A2}}{\theta_{B2}-\theta_{A2}} \in (0, \psi(\delta, p_A))$ cannot hold when $\theta_{A1} \leq \theta_{A2}$ or when $\theta_{A2} = \theta_{B2}$ when $\psi(\delta, p_A)$ is finite, throughout the proof assume $\theta_{A1} > \theta_{A2}$ and $\theta_{A2} < \theta_{B2}$. These assumptions imply, denoting $\rho_A = \frac{\theta_{A1}-\theta_{A2}}{\theta_{B2}-\theta_{A2}}$, $\rho_A \in (0, \infty)$.

From the proof of Proposition 4 we know $f_A(g_1)$ is continuous function of g_1 on $\mathbb{R}_+$. From the proof of Lemma A4, we know that if $Z = \{0\} \times \mathbb{R}_+$ and $u_{it}(x_t) = -(x_t - \theta_{it})^2$ for all $i \in \{A, B\}$ and all t, then V_A is continuously differentiable on $\mathbb{R}_+ \setminus \{2\theta_{A2} - \theta_{B2}, \theta_{A2}, \theta_{B2}, 2\theta_{B2} - \theta_{A2}\}$. Inspection of (A12) in the proof of Lemma A4 shows that $V_A(g_1)$ is increasing in g_1 on $[0, \theta_{A2}]$. Because $f_A(g_1) = u_{A1}(g_1) + \delta V_A(g_1)$ and $\theta_{A1} > \theta_{A2}$, $g_A^* > \theta_{A2}$ for any $g_A^* \in \arg\max_{g_1 \in \mathbb{R}_+} f_A(g_1)$. Using the same Q_k intervals as in the proof of Lemma A4, $Q_3 = (\theta_{A2}, \theta_{B2})$, $Q_4 = (\theta_{B2}, 2\theta_{B2} - \theta_{A2})$ $Q_5 = (2\theta_{B2} - \theta_{A2}, \infty)$, this implies $g_A^* \in (\{\theta_{B2}, 2\theta_{B2} - \theta_{A2}\}) \cup (\cup_{k \in \{3,4,5\}} Q_k)$.

From (A12) and $\theta_{A2} < \theta_{B2}$, we have

$$\begin{gathered} \lim_{g_1 \to \theta_{B2}^-} V_A'(g_1) = u_{A2}'(\theta_{B2}) < 0 < \lim_{g_1 \to \theta_{B2}^+} V_A'(g_1) = -p_A u_{A2}'(\theta_{B2}) \\ \lim_{g_1 \to (2\theta_{B2} - \theta_{A2})^-} V_A'(g_1) = 0 = \lim_{g_1 \to (2\theta_{B2} - \theta_{A2})^+} V_A'(g_1) \end{gathered} \tag{A13}$$

so that $\theta_{B2} \neq g_A^*$ and $f_A'(g_A^*) = 0$, where the latter condition is also sufficient for g_A^* because f_A is piecewise strictly concave by Lemma A4. Solving $f_A'(g_1) = 0$ for different intervals Q_k gives

$$g_{A,3}^* = \tfrac{\theta_{A1} + \delta\theta_{A2}}{1+\delta} \qquad g_{A,4}^* = \tfrac{\theta_{A1} + \delta p_A(2\theta_{B2} - \theta_{A2})}{1 + \delta p_A} \qquad g_{A,5}^* = \theta_{A1} \tag{A14}$$

where, for $k \in \{3, 4, 5\}$, $g_{A,k}^*$ is local maximum of f_A if $g_{A,k}^* \in Q_k$. $g_{A,k}^* \in Q_k$ rewrites as $\rho_A \in (0, 1+\delta)$ for $k = 3$, $\rho_A \in (1 - \delta p_A, 2)$ for $k = 4$ and $\rho_A \in (2, \infty)$ for $k = 5$. The remaining

possibility, $f'_A(2\theta_{B2} - \theta_{A2}) = 0$, implies $u'_{A1}(2\theta_{B2} - \theta_{A2}) = 0$ and rewrites as $\rho_A = 2$.

This implies that if $\rho_A \geq 2$, f_A has unique global maximum at $g^*_A = \theta_{A1}$. If $\rho_A \in (0, 2)$, f_A has global maximum either at $g^*_{A,3}$ or at $g^*_{A,4}$. Since, $g^*_{A,3} < \theta_{A1}$ if $g^*_{A,3} \in Q_3$ and $g^*_{A,4} > \theta_{A1}$ if $g^*_{A,4} \in Q_k$, we need to show that the condition ensuring that $g^*_{A,3}$ is unique global maximum of f_A is $\rho_A \in (0, \psi(\delta, p_A))$. Direct evaluation, $g^*_{A,3} \in [\theta_{A2}, \theta_{B2}]$ with Proposition 4 part 2 and some algebra give

$$\begin{aligned} f_A(g^*_{A,3}) &= -\tfrac{\delta}{1+\delta}(\theta_{A1} - \theta_{A2})^2 \\ f_A(g^*_{A,4}) &= -\tfrac{\delta p_A}{1+\delta p_A}(2(\theta_{B2} - \theta_{A2}) - (\theta_{A1} - \theta_{A2}))^2 \\ &\quad - \delta(1 - p_A)(\theta_{B2} - \theta_{A2})^2. \end{aligned} \tag{A15}$$

Further algebra shows that $f_A(g^*_{A,3}) > f_A(g^*_{A,4})$ implies $\rho_A < \psi(\delta, p_A)$, where

$$\psi(\delta, p) = (1 + \delta)\left[1 + \tfrac{1+p}{1-p}\left(\sqrt{\tfrac{1+\delta p}{1+\delta}} - 1\right)\right]. \tag{A16}$$

What remains is to show $\psi(\delta, p) \in (1, 1+\delta)$ (and hence $\psi(\delta, p) > 1$), $\frac{\partial \psi(\delta,p)}{\partial \delta} > 0$, $\frac{\partial \psi(\delta,p)}{\partial p} < 0$ for any $(\delta, p) \in (0, 1] \times (0, 1)$. $\psi(\delta, p) < 1 + \delta$ is immediate as $\frac{1+\delta p}{1+\delta} < 1$ for any $(\delta, p) \in (0, 1] \times (0, 1)$. $\psi(\delta, p) > 1$ for any $(\delta, p) \in (0, 1] \times (0, 1)$ follows from $\lim_{\delta \to 0^+} \psi(\delta, p) = 1$ for any $p \in (0, 1)$, obtained by direct evaluation, and $\lim_{p \to 1^-} \psi(\delta, p) = 1$ for any $\delta \in (0, 1]$, obtained by application of L'Hopital's rule, if we prove $\frac{\partial \psi(\delta,p)}{\partial \delta} > 0$ and $\frac{\partial \psi(\delta,p)}{\partial p} < 0$ for any $(\delta, p) \in (0, 1] \times (0, 1)$. The partial derivatives are

$$\begin{aligned} \tfrac{\partial \psi(\delta,p)}{\partial \delta} &= \tfrac{(1+p)((1-p)+2p(1+\delta)) - 4p(1+\delta)\sqrt{\frac{1+\delta p}{1+\delta}}}{2(1-p)(1+\delta)\sqrt{\frac{1+\delta p}{1+\delta}}} \\ \tfrac{\partial \psi(\delta,p)}{\partial p} &= \tfrac{4(1+\delta p) + \delta((1-p)(1+p)) - 4(1+\delta)\sqrt{\frac{1+\delta p}{1+\delta}}}{2(1-p)^2\sqrt{\frac{1+\delta p}{1+\delta}}}. \end{aligned} \tag{A17}$$

The denominator in each expression is strictly positive for any $(\delta, p) \in (0, 1] \times (0, 1)$. Conditions for the numerators to have the claimed signs are

$$\begin{aligned} (1 - p)^2[(1 + p)^2 + 4p(1 + \delta)(1 + \delta p)] > 0 \\ \delta(1 - p)^2[8(1 + \delta p) - \delta(1 + p)^2] > 0 \end{aligned} \tag{A18}$$

and clearly hold for any $(\delta, p) \in (0, 1] \times (0, 1)$. ■

A2.4 Proof of Proposition 7

We first prove part 1. When $\theta_{A2} = \theta_{B2}$ by Proposition 4 part 1, $\kappa^*_{i2}(g_1) + \gamma^*_{i2}(g_1) = \theta_{A2}$ for all $i \in \{A, B\}$ and all $g_1 \in \mathbb{R}_+$. The continuation value of each party $i \in \{A, B\}$ is thus $V_i(g_1) = u_{i2}(\theta_{A2})$ for all $g_1 \in \mathbb{R}_+$. Proposing party $i \in \{A, B\}$ in the first period under initial status quo g_0 thus makes a proposal that solves $\max_{(0,g_1)\in Z} u_{i1}(g_1) + \delta u_{i2}(\theta_{A2})$ subject to acceptance constraint by the responder $j \neq i$, $u_{j1}(g_1) + \delta u_{j2}(\theta_{A2}) \geq u_{j1}(g_0) + \delta u_{j2}(\theta_{A2})$. This problem is identical, safe for the constants, to (SSP). $\kappa^*_{i1}(g_0) + \gamma^*_{i1}(g_0)$ for any $g_0 \in \mathbb{R}_+$ is thus statically Pareto efficient allocation in period 1. Because θ_{A2} is statically Pareto efficient allocation in period 2, by Lemma 2, $\{x_1^{\sigma^*}(g_0), x_2^{\sigma^*}(g_0)\}$ with $x_1^{\sigma^*}(g_0) = \kappa^*_{i1}(g_0) + \gamma^*_{i1}(g_0)$ for any $i \in \{A, B\}$ and $x_2^{\sigma^*}(g_0) = \theta_{A2}$ is statically Pareto efficient allocation in period t for all t and hence is dynamically Pareto efficient. Hence equilibrium σ^* is dynamically Pareto efficient for any initial status quo $g_0 \in \mathbb{R}_+$.

We begin the proof of part 2 with the following lemma.

Lemma A5. *Suppose $u_{it}(x_t)$ is regular for all $i \in \{A, B\}$ and θ_{it} is constant in t for all $i \in \{A, B\}$. Then an allocation $\mathbf{x}$ is dynamically Pareto efficient if and only if $\mathbf{x} = \{\tilde{x}\}_{t=1}^T$ and $\tilde{x} \in [\theta_{A1}, \theta_{B1}]$.*

Proof. We first prove the only if part of the lemma: if $u_{it}(x_t) = u_i(x_t, \theta_{it})$ and θ_{it} is constant in t for all $i \in \{A, B\}$, then any dynamically Pareto efficient allocation $\mathbf{x}^*$ satisfies $\mathbf{x}^* = \{\tilde{x}\}_{t=1}^T$ with $\tilde{x} \in [\theta_{A1}, \theta_{B1}]$.

If $\mathbf{x}^* = \boldsymbol{\theta}_A$, or $\mathbf{x}^* = \boldsymbol{\theta}_B$, then the result follows immediately. Until we prove the only if part, suppose $\mathbf{x}^* \neq \boldsymbol{\theta}_A$ and $\mathbf{x}^* \neq \boldsymbol{\theta}_B$.

Suppose $\theta_{At} = \theta_{Bt}$. By Proposition 2 part 1, $x_t^* = \theta_{it}$. If θ_{it} is constant in t for all $i \in \{A, B\}$, then $\theta_{At'} = \theta_{Bt'}$ for any $t' \neq t$ and by Proposition 2 part 1, $x_{t'}^* = \theta_{it'}$, which in turn implies that $x_{t'}^* = x_t^*$.

Suppose that $\theta_{At} \neq \theta_{Bt}$, then by Proposition 2 part 2, we have, for any t and $t' \neq t$,

$$\frac{u'_{At}(x_t^*)}{u'_{Bt}(x_t^*)} = \frac{u'_{At'}(x_{t'}^*)}{u'_{Bt'}(x_{t'}^*)}.$$

Since $u_{it}(x_t) = u_i(x_t, \theta_{it})$, this implies that

$$\frac{\partial u_A(x_t^*, \theta_{At})/\partial x}{\partial u_B(x_t^*, \theta_{Bt})/\partial x} = \frac{\partial u_A(x_{t'}^*, \theta_{At'})/\partial x}{\partial u_B(x_{t'}^*, \theta_{Bt'})/\partial x}.$$

As shown in the proof of Lemma 1, the solution to $-\frac{u'_{At}(x)}{u'_{Bt}(x)} = \lambda^*$ is unique for any $\lambda^* > 0$ and t. Therefore the solution to $-\frac{\partial u_A(x,\theta_{At})/\partial x}{\partial u_B(x,\theta_{Bt})/\partial x} = \lambda^*$ is unique for any pair of $(\theta_{At}, \theta_{Bt})$. Since $\theta_{it} = \theta_{it'}$ for all $i \in \{A, B\}$, it follows that we have $x_{t'}^* = x_t^*$.

We now prove the if part of the lemma: if $u_{it}(x_t) = u_i(x_t, \theta_{it})$ and θ_{it} is constant in t for all $i \in \{A, B\}$, then any $\mathbf{x} = \{\tilde{x}\}_{t=1}^T$ with $\tilde{x} \in [\theta_{A1}, \theta_{B1}]$ is dynamically Pareto efficient allocation.

If θ_{it} is constant in t for all $i \in \{A, B\}$, then from the previous part $\mathbf{x}^* = \{\tilde{x}\}_{t=1}^T$ for some $\tilde{x}$. We can thus rewrite (DSP) as $\max_{\tilde{x} \in \mathbb{R}_+} u_i(\tilde{x}, \theta_{i1}) \sum_{t=1}^T \delta^{t-1}$ subject to $u_j(\tilde{x}, \theta_{j1}) \sum_{t=1}^T \delta^{t-1} \geq \overline{U}$, which is equivalent to (SSP). By Proposition 1, if $\tilde{x} \in [\theta_{A1}, \theta_{B1}]$, then $\tilde{x}$ solves (SSP). ■

Take any equilibrium σ^*. We show that the equilibrium allocation $\mathbf{x}^{\sigma^*}(g_0)$ satisfies $x_1^{\sigma^*}(g_0) = x_2^{\sigma^*}(g_0) \in [\theta_{A1}, \theta_{B1}]$ for any $g_0 \in \mathbb{R}_+$, which, by Lemma A5, implies that σ^* is dynamically Pareto efficient for any $g_0 \in \mathbb{R}_+$. To prove that $x_1^{\sigma^*}(g_0) = x_2^{\sigma^*}(g_0) \in [\theta_{A1}, \theta_{B1}]$ for any $g_0 \in \mathbb{R}_+$, it suffices to show that $\gamma_{i1}^*(g_0) \in [\theta_{A1}, \theta_{B1}]$ for any $g_0 \in \mathbb{R}_+$ and any $i \in \{A, B\}$. When only mandatory spending programs are allowed, $\kappa_{it}^*(g_{t-1}) = 0$ for any $i \in \{A, B\}$ and any t, so that $x_1^{\sigma^*}(g_0) = \gamma_{i1}^*(g_0)$ and $x_2^{\sigma^*}(g_0) = \gamma_{j2}^*(\gamma_{i1}^*(g_0))$ for some $i, j \in \{A, B\}$. When $\theta_{A2} = \theta_{B2}$ (so that $\theta_{A1} = \theta_{B1}$), $\gamma_{i1}^*(g_0) \in [\theta_{A1}, \theta_{B1}]$ rewrites as $\gamma_{i1}^*(g_0) = \theta_{A1}$ and, by Proposition 4 part 1, $\gamma_{j2}^*(\gamma_{i1}^*(g_0)) = \theta_{A2}$. When $\theta_{A2} \neq \theta_{B2}$, $\gamma_{i1}^*(g_0) \in [\theta_{A1}, \theta_{B1}]$ implies, by Proposition 4 part 2, $\gamma_{j2}^*(\gamma_{i1}^*(g_0)) = \gamma_{i1}^*(g_0)$.

We now show that $\gamma_{i1}^*(g_0) \in [\theta_{A1}, \theta_{B1}]$ for any $g_0 \in \mathbb{R}_+$ and any $i \in \{A, B\}$. Suppose there exists $i \in \{A, B\}$ and $g_0 \in \mathbb{R}_+$ such that $\gamma_{i1}^*(g_0) \notin [\theta_{A1}, \theta_{B1}]$. We claim that this contradicts γ_{i1}^* being part of equilibrium σ^*. The desired contradiction follows from the fact that if $\theta_{A1} = \theta_{A2}$, $\theta_{B1} = \theta_{B2}$ and $u_{it}(x_t) = u_i(x_t, \theta_{it})$, then for any $g_1 \in \mathbb{R}_+ \setminus [\theta_{A2}, \theta_{B2}]$, there exists $\tilde{g}(g_1) \in [\theta_{A2}, \theta_{B2}]$ such that $f_A(g_1) < f_A(\tilde{g}(g_1))$ and $f_B(g_1) < f_B(\tilde{g}(g_1))$.

To see that $\tilde{g}(g_1)$ exists for any $g_1 \in \mathbb{R}_+ \setminus [\theta_{A2}, \theta_{B2}]$ and has the claimed properties, let $\tilde{g}(g_1) = p_A(\kappa_{A2}^*(g_1) + \gamma_{A2}^*(g_1)) + p_B(\kappa_{B2}^*(g_1) + \gamma_{B2}^*(g_1))$. When $\theta_{A2} = \theta_{B2}$, $\tilde{g}(g_0) = \theta_{A2}$ by Proposition 4 part 1 and $f_A(g_1) < f_A(\tilde{g}(g_1))$ and $f_B(g_1) < f_B(\tilde{g}(g_1))$ for any $g_1 \in \mathbb{R}_+ \setminus \{\theta_{A2}\}$

are immediate. When $\theta_{A2} \neq \theta_{B2}$, we consider the case when $g_1 < \theta_{A2}$. When $g_1 > \theta_{B2}$ the argument is similar and omitted. From the proof of Proposition 4 we know that if $g_1 < \theta_{A2}$, then $\kappa^*_{A2}(g_1) + \gamma^*_{A2}(g_1) = \theta_{A2}$ and $\kappa^*_{B2}(g_1) + \gamma^*_{B2}(g_1) > \theta_{A2}$. Because of the restriction to mandatory spending only, this rewrites as $\gamma^*_{A2}(g_1) = \theta_{A2}$ and $\gamma^*_{B2}(g_1) > \theta_{A2}$. We also have $\gamma^*_{B2}(g_1) \leq \theta_{B2}$ by Proposition 4 part 1. From $\gamma^*_{A2}(g_1) = \theta_{A2} < \gamma^*_{B2}(g_1) \in (\theta_{A2}, \theta_{B2}]$ for any $g_1 < \theta_{A2}$, $\tilde{g}(g_1) \in (\theta_{A2}, \gamma^*_{B2}(g_1)) \subseteq [\theta_{A2}, \theta_{B2}]$ for any $g_1 < \theta_{A2}$. This implies, for any $g_1 < \theta_{A2}$, $u_{B2}(g_1) < u_{B2}(\tilde{g}(g_1))$ and $u_{A2}(g_1) \leq u_{A2}(\gamma^*_{B2}(g_1)) < u_{A2}(\tilde{g}(g_1))$, where the weak inequality follows from A's acceptance of $\gamma^*_{B2}(g_1)$ under status quo g_1 and the remaining inequalities follow from $\tilde{g}(g_1) \in (\theta_{A2}, \gamma^*_{B2}(g_1))$. Since $u_{it}(x_t) = u_i(x_t, \theta_{it})$ and $\theta_{i1} = \theta_{i2}$ for all $i \in \{A, B\}$, $u_{i1}(g_1) < u_{i1}(\tilde{g}(g_1))$ for any $i \in \{A, B\}$ and any $g_1 < \theta_{A2}$.

What remains is to show that $V_i(g_1) < V_i(\tilde{g}(g_1))$ for any $i \in \{A, B\}$ and any $g_1 < \theta_{A2}$. The inequality rewrites as

$$p_A u_{i2}(\theta_{A2}) + p_B u_{i2}(\gamma^*_{B2}(g_1)) < u_{i2}(\tilde{g}(g_1)) = u_{i2}(p_A \theta_{A2} + p_B \gamma^*_{B2}(g_1)). \tag{A19}$$

The left hand side simply substitutes into definition of V_i. The right hand side follows from Proposition 4 part 2 and $\tilde{g}(g_1) \in [\theta_{A2}, \theta_{B2}]$. Note that the left hand side is expected utility of a lottery over spending levels θ_{A2} and $\gamma^*_{B2}(g_1)$, while the right hand side is utility of expected spending under the same lottery. By strict concavity of u_{i2} for $i \in \{A, B\}$, the inequality holds and hence $V_i(g_1) < V_i(\tilde{g}(g_1))$ for any $g_1 < \theta_2$. ■

A3 State-contingent mandatory spending

A3.1 Proof of Proposition 10

To prove part 1, by way of contradiction, suppose $x^*_t \neq x^*_{t'}$ for some $t \neq t'$. Then there exists $s \in S$ such that $x^*_t(s) \neq x^*_{t'}(s)$. Suppose $x^*_t(s) < x^*_{t'}(s)$. For $x^*_t(s) > x^*_{t'}(s)$, the argument is similar and will not be repeated. We now show that there exists $x' \in (x^*_t(s), x^*_{t'}(s))$ such that

$$\delta^{t-1} u_i(x^*_t(s), s) + \delta^{t'-1} u_i(x^*_{t'}(s), s) < (\delta^{t-1} + \delta^{t'-1}) u_i(x', s) \tag{A20}$$

for all $i \in \{1,2\}$, so that $\mathbf{x}^* = \{x_t^*\}_{t=1}^T$ cannot be solution to (DSP-S). From strict concavity of u_i, $\alpha u_i(x_t^*(s),s)+(1-\alpha)u_i(x_{t'}^*(s),s) < u_i(\alpha x_t^*(s)+(1-\alpha)x_{t'}^*(s))$ for any $\alpha \in (0,1)$. Setting $\alpha = \frac{\delta^{t-1}}{\delta^{t-1}+\delta^{t'-1}} \in (0,1)$ and $x' = \alpha x_t^*(s)+(1-\alpha)x_{t'}^*(s)$, shows that the desired inequality holds.

Next we prove part 2 by considering possible values of $\overline{U}$. Fix $i,j \in \{A,B\}$ with $i \neq j$. For any $\overline{U} > \sum_{t=1}^T \delta^{t-1}\mathbb{E}_s[u_j(\theta_{js},s)]$, the solution does not exist, so assume $\overline{U} \leq \sum_{t=1}^T \delta^{t-1}\mathbb{E}_s[u_j(\theta_{js},s)]$.

For $\overline{U} = \sum_{t=1}^T \delta^{t-1}\mathbb{E}_s[u_j(\theta_{js},s)]$, the solution to (DSP-S) is $x_t^* = \theta_{js}$ for all t and $s \in S$ and for any $\overline{U} \leq \sum_{t=1}^T \delta^{t-1}\mathbb{E}_s[u_j(\theta_{is},s)]$, the solution to (DSP-S) is $x_t^* = \theta_{is}$ for all t and $s \in S$. What remains is the case when $\overline{U} \in (\sum_{t=1}^T \delta^{t-1}\mathbb{E}_s[u_j(\theta_{is},s)], \sum_{t=1}^T \delta^{t-1}\mathbb{E}_s[u_j(\theta_{js},s)])$. Constructing Lagrangian for (DSP-S), the first order necessary condition with respect to $x_t(s)$ for any t and $s \in S$ writes $\delta^{t-1}u_i'(x_t^*(s),s)+\lambda^*\delta^{t-1}u_j'(x_t^*(s),s) = 0$ for some $\lambda^* \in (0,\infty)$, which simplifies to $-\frac{u_i'(x_t^*(s),s)}{u_j'(x_t^*(s),s)} = \lambda^*$. ∎

A3.2 Proof of Proposition 11

Suppose the state in period 1 is s_1. Consider the following problems:

$$\begin{aligned} \max_{\{x_t: S\to\mathbb{R}_+\}_{t=1}^T} \; & u_i(x_1(s_1),s_1) + \sum_{t=2}^T \delta^{t-1}\mathbb{E}_s[u_i(x_t(s),s)] \\ \text{s.t. } & u_j(x_1(s_1),s_1) + \sum_{t=2}^T \delta^{t-1}\mathbb{E}_s[u_j(x_t(s),s)] \geq \overline{U}' \end{aligned} \tag{DSP-S'}$$

$$\begin{aligned} \max_{\{x_t^A, x_t^B: S\to\mathbb{R}_+\}_{t=1}^T} \; & u_i(x_1^i(s_1),s_1) + \sum_{t=2}^T \delta^{t-1}\mathbb{E}_s[p_A u_i(x_t^A(s),s) + p_B u_i(x_t^B(s),s)] \\ \text{s.t. } & u_j(x_1^i(s_1),s_1) + \sum_{t=2}^T \delta^{t-1}\mathbb{E}_s[p_A u_j(x_t^A(s),s) + p_B u_j(x_t^B(s),s)] \geq \overline{U}' \end{aligned} \tag{DSP-S"}$$

Since u_A and u_B are strictly concave in x for all s, clearly any solution to (DSP-S") satisfies $x_t^A(s) = x_t^B(s)$ for all t,s. So we can just consider (DSP-S').

Lemma A6. *If* $\mathbf{x}$ *is a solution to* (DSP-S')*, then it satisfies:*

1. *For any* $t,t' \geq 2$, $x_t = x_{t'}$. *Moreover,* $x_1(s_1) = x_t(s_1)$ *for* $t \geq 2$.

2. For all $s \in S$ and all $t \geq 2$ either

$$-\frac{u_i'(x_t(s), s)}{u_j'(x_t(s), s)} = \lambda$$

for some $\lambda > 0$, or $x_t(s) = \theta_{As}$ or $x_t(s) = \theta_{Bs}$.

The proof of Lemma A6 is immediate from the proof of Proposition 10. We then have the following result.

Lemma A7. *If* $\mathbf{x}$ *is a solution to* (DSP-S) *for some* $\overline{U}$*, then it is a solution to* (DSP-S') *for some* $\overline{U}'$*. If* $\mathbf{x}$ *is a solution to* (DSP-S') *for some* $\overline{U}'$ *and it satisfies that* $x_1(s) = x_t(s)$ *for* $t \geq 2$ *and for all* s*, then* $\mathbf{x}$ *is a solution to* (DSP-S) *for some* $\overline{U}$*.*

Proof. Fix $i, j \in \{A, B\}$ with $i \neq j$ and $s_1 \in S$. Note that (DSP-S) with $x_1(s)$ for $s \in S \setminus \{s_1\}$ held constant is equivalent to (DSP-S') and that $x_1(s)$ for $s \in S \setminus \{s_1\}$ do not enter either the objective function or the constraint in (DSP-S'). Hence if $\mathbf{x}$ is a solution to (DSP-S) with $\overline{U}$, $(x_1(s_1), \{x_t : S \to \mathbb{R}_+\}_{t=2}^T)$ is a solution to (DSP-S) with $x_1(s)$ for $s \in S \setminus \{s_1\}$ held constant and hence $\mathbf{x}$ is a solution to (DSP-S') with $\overline{U}' = \overline{U} - \mathbb{E}_{s \in S \setminus \{s_1\}}[u_j(x_1(s), s)]$. Similarly, if $\mathbf{x}$ with $x_1(s) = x_t(s)$ for $t \geq 2$ and for all $s \in X$ solves (DSP-S') with $\overline{U}'$, $(x_1(s_1), \{x_t : S \to \mathbb{R}_+\}_{t=2}^T)$ is a solution to (DSP-S) with $x_1(s)$ for $s \in S \setminus \{s_1\}$ held constant and hence $\mathbf{x}$ is a solution to (DSP-S) with $\overline{U} = \overline{U}' + \mathbb{E}_{s \in S \setminus \{s_1\}}[u_j(x_1(s), s)]$. ■

We prove the proposition by establishing the following two claims. With slight abuse of terminology, we call a spending rule $g \in \mathcal{M}$ dynamically Pareto efficient if $\{g_t\}_{t=1}^T$ with $g_t = g$ for all t is a dynamically Pareto efficient allocation rule.

Lemma A8. *For any* t*, if the status quo* g_{t-1} *is dynamically Pareto efficient, then* $\gamma_{it}(g_{t-1}, s_t) = g_{t-1}$ *for all* s_t *and all* $i \in \{A, B\}$*.*

Proof. Suppose the state in period t is s_t.

For any status quo g_{t-1} in period t, the proposer i's equilibrium continuation payoff is weakly higher than $u_i(g_{t-1}(s_t), s_t) + \sum_{t'=t+1}^T \delta^{t'-t} \mathbb{E}_s[u_i(g_{t-1}(s), s)]$ and the responder j's equilibrium continuation payoff is weakly higher than $u_j(g_{t-1}(s_t), s_t) + \sum_{t'=t+1}^T \delta^{t'-t} \mathbb{E}_s[u_j(g_{t-1}(s), s)]$. To see why this is true, note that for any status quo in any period, a responder accepts a

proposal if the proposal is the same as the status quo, so a proposer can maintain the status quo by proposing it. Hence, proposer i can achieve the payoff above by proposing to maintain the status quo in period t and in future periods continue to propose to maintain the status quo if it is the proposer and rejects any proposal other than the status quo if it is the responder. Similarly, responder j can achieve the payoff above by reject any proposal other than the status quo in period t and in future periods continue to reject any proposal other than the status quo if it is the responder and propose to maintain the status quo if it is the proposer.

Consider proposer i's problem in period t

$$\max_{\{g_t: S \to \mathbb{R}^n_+\}} u_i(g_t(s_t), s_t) + \delta V_{it}(g_t; \sigma^*)$$
$$\text{s.t. } u_j(g_t(s_t), s_t) + \delta V_{jt}(g_t; \sigma^*) \geq u_j(g_{t-1}(s_t), s_t) + \delta V_{jt}(g_{t-1}; \sigma^*).$$

where $V_{it}(g; \sigma^*)$ is the expected discounted utility of party $i \in \{A, B\}$ in period t generated by strategies σ^* when the status quo is g. We have shown above that $u_j(g_{t-1}(s_t), s_t) + \delta V_{jt}(g_{t-1}, \sigma^*) \geq u_j(g_{t-1}(s_t), s_t) + \sum_{t'=t+1}^{T} \delta^{t'-t} \mathbb{E}_s[u_j(g_{t-1}(s), s))]$.

Suppose the solution to the proposer's problem in period t is $g_t^* \neq g_{t-1}$. Then there exist an allocation with $x_t = g_t^*$ and future allocations induced by status quo g_t^* and σ^* such that party i's dynamic payoff is higher than $u_i(g_{t-1}(s_t), s_t) + \sum_{t'=t+1}^{T} \delta^{t'-t} \mathbb{E}_s[u_i(g_{t-1}(s), s)]$ and party j's dynamic payoff is higher than $u_j(g_{t-1}(s_t), s_t) + \sum_{t'=t+1}^{T} \delta^{t'-t} \mathbb{E}_s[u_j(g_{t-1}(s), s))]$. But if g_{t-1} is dynamically Pareto efficient, then having allocation in all periods $t' \geq t$ equal to g_{t-1} is a solution to (DSP-S') with $\overline{U} = u_j(g_{t-1}(s_t), s_t) + \sum_{t'=t+1}^{T} \delta^{t'-t} \mathbb{E}_s[u_j(g_{t-1}(s), s))]$, a contradiction. ■

Lemma A9. *For any initial status quo g_0 and any $s_1 \in S$, the proposer makes a proposal in period 1 that is dynamically Pareto efficient, that is, $\gamma_{i1}(g_0, s_1)$ is dynamically Pareto efficient for all $i \in \{A, B\}$.*

Proof. Fix g_0 and s_1. Let $f_j(g_0, s_1)$ be the responder j's status quo payoff. That is,

$$f_j(g_0, s_1) = u_j(g_0(s_1), s_1) + \delta V_{j1}(g_0; \sigma^*).$$

Let $\overline{U}' = f_j(g_0, s_1)$ and denote the solution to (DSP-S') by $\mathbf{x}(\overline{U}') = (x_1(\overline{U}'), ..., x_T(\overline{U}'))$.

By Lemma A6, $x_t(\overline{U}') = x_{t'}(\overline{U}')$ for any $t, t' \geq 2$. Without loss of generality, suppose $\mathbf{x}(\overline{U}')$ satisfies $x_1(\overline{U}') = x_t(\overline{U}')$ for $t \geq 2$. Note that $x_1(\overline{U}')$ is a dynamically Pareto efficient allocation.

We next show that $\gamma^*_{i1}(g_0, s_1) = x_1(\overline{U}')$. First note that if $\gamma^*_{i1}(g_0, s_1) = x_1(\overline{U}')$, then, since $x_1(\overline{U}')$ is dynamically Pareto efficient, by Lemma A8, the induced equilibrium allocation is $\mathbf{x}(\overline{U}')$. We show by contradiction that $\gamma^*_{i1}(g_0, s_1) = x_1(\overline{U}')$ is the solution to the proposer's problem. Suppose not. Then proposing $\gamma^*_{i1}(g_0, s_1)$ is strictly better than proposing $x_1(\overline{U}')$, that is, proposing $\gamma^*_{i1}(g_0, s_1)$ gives i a strictly higher dynamic payoff while giving j a dynamic payoff at least as high as $f_j(g_0, s_1)$. But since $\mathbf{x}(\overline{U}')$ is a solution to (DSP-S') and hence a solution to (DSP-S"), this is a contradiction. ■