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“The Valley of Death for New Energy Technologies”

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Abstract

It is often claimed that scarce financing prevents promising new energy technologies from attaining commercial viability. We examine this issue using a dynamic intertemporal model of the displacement of fossil fuel energy technologies by non-fossil alternatives. Our model highlights the fact that since capital used to produce energy services from fossil fuels is a sunk cost, it will continue to be used so long as the price of energy covers merely short-run operating costs. Until fossil fuels are abandoned, the price of energy is insufficient to cover even the operating costs of renewable energy production, let alone provide a competitive return on the capital employed. The full long-run costs of renewable energy production are not covered until some time after fossil fuels are abandoned.

Keywords: Energy innovation, energy transition, valley of death

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1 Introduction

Fossil fuels currently supply more than 90% of the world's primary energy, while more than 8% comes from nuclear power and hydroelectricity. Large-scale energy production from non-hydroelectric renewable sources such as wind, solar, geothermal or biofuels has recently become technologically feasible. Nevertheless, such sources provide only a tiny fraction of the world's energy, and even then mainly as a result of subsidies. It is often said that many promising new energy technologies have perished in a "valley of death" between discovery and commercial viability.

According to Markham et al. (2010), the phrase "valley of death" was first used in 1995 to refer to the challenges of transferring agricultural technologies to Third-World countries. In one of the earliest papers to use the phrase to describe barriers to commercializing new technologies, Frank et al. (1996) applied it to environmental remediation technologies. They claim that taking technologies beyond the basic research stage is not usually considered to be a government role. However, the private sector appears reluctant to finance mass production of new technologies that have not yet been implemented even if they have been shown to be effective. Frank et al. contrasted environmental remediation technologies to the pharmaceutical industry. They suggest that the latter might not suffer from a valley of death because "Government has funded medical research across the continuum of technology development, from basic R&D, through human clinical trials, to supporting health care for the needy." They also point to the involvement of large, well-capitalized private sector firms in all stages of new product development in the pharmaceutical industry.

A Report by the US House of Representatives Committee on Science (Sensenbrenner, 1998) also identified a valley of death for new technology developments as "a widening gap between federally-funded basic research and industry-funded applied research and development." The committee suggested partnerships between universities and firms as a means of spanning the valley, but cautioned against direct funding from the federal government, arguing that the government lacked sufficient resources for such a task and the attempt would draw funds away from basic research.

Markham et al. (2010) provide a simple illustration of the notion, repeated as Figure 1. They claim that while adequate resources for new technology development are available during the basic research phase, available resources often drop precipitously once the basic research has been completed. If an idea makes it through the valley of death to prove commercial viability, however, once again ample resources are available to take the idea to market.

One of the earliest authors to apply the valley of death notion to energy technologies, Norberg-

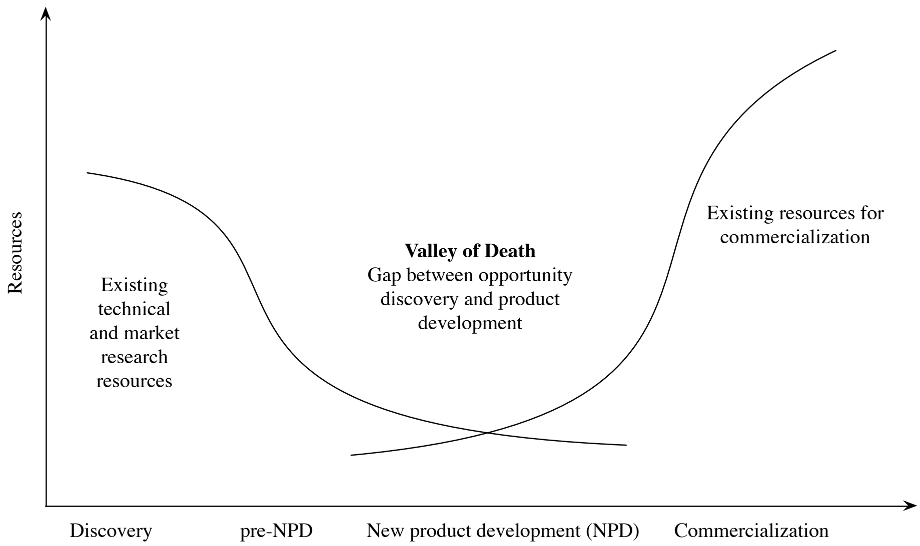

Figure 1: Illustration of Valley of Death Concept after Markham et al. (2010)

Bohm (2000), noted that the phrase is meant to reflect the common experience that many new technologies "die" before being successfully commercialized. She argued that, "For technologies such as power plants, which may be standardized but not mass produced, the initial plant is much more expensive than the 5th or 10th plant" and that early versions may not survive "an extended period of negative net cumulative cash flow." Furthermore, she suggested that generating technologies may find it difficult to capture a market niche because the homogeneity of electricity as a product meant that it was not possible to use "quality improvements . . . to charge higher prices to the lead adopters."[1]

Murphy and Edwards (2003) also discussed renewable energy technologies. They identified high risk as the main barrier to commercialization. In turn, they ascribed the risk to five factors. First, the developer is likely to know more about the technology than private sector investors, suppliers, or strategic partners, making the latter hesitant to invest for fear of being exploited. Second, there is uncertainty about whether the new product can successfully compete, or how the firm's rivals will respond and what they may be developing in secret. Third, government support of R&D paradoxically may increase uncertainty about market size, customer benefits and profitability because these have not been critical to success of the venture to date. Fourth, continued government support beyond the R&D stage, such as in the form of subsidies or tax credits, may be uncertain. Finally, many of the firm's assets – such as trade secrets, patents and key human resources – are

[1]Nevertheless, a perceived lower environmental footprint for renewable energy allows electricity distributors in many countries to sell power said to be "generated by renewable sources" at a premium.

hard to evaluate and cannot be used as collateral for loans.

Bürer and Wüstenhagen (2009) asked a principal or senior fund manager in 60 venture capital or private equity fund management firms to rate the effectiveness of different policies in stimulating new energy technology investments. They grouped policies into "push" and "pull" categories. Push policies included more funding for R&D, investment subsidies for manufacturing facilities, grants to install equipment, tax breaks for entrepreneurs and, the most favored in this category, government grants for demonstration plants. Feed-in tariffs were the most favored pull policy, and were preferred to *all* other policies, including renewable portfolio standards and tradable green certificates. By contrast, Azar and Sandén (2011) criticize feed-in tariffs on the grounds that they often support only specific identified renewable electricity generating technologies. As a result, "very promising but currently expensive technologies will not gain the support that is needed to come down in cost." Azar and Sandén also suggest that the fundamental impediment to the development of new energy technology is an inability to capture benefits external to the firm, including some benefits of learning-by-doing, learning-by-using and network externalities. Failure to capture (some of) the external benefits then limits the ability of new technology firms to attain the economies of sale that could lead to lower costs.

While the above authors could be read as suggesting that renewable technologies are already competitive but inefficiently excluded from the market by a scarcity of funding, Zindler and Locklin (2010) claim that, "Much more work remains to drive down costs so that renewables can truly compete with and beat their fossil rivals on cost." They describe the task of moving technology from R&D to commerciality as "arduous." Zindler and Locklin report that "participants [in their interviews with more than 60 thought leaders in 10 countries] actually identified two critical locations where a shortfall of capital often comes into play. The first occurs early in a technology's development, just as it is ready to exit the lab. The second occurs later, when much more substantial levels of capital availability are needed to prove viability at commercial scale." Zindler and Locklin claimed that government energy technology laboratories supplemented by grants to private firms could alleviate the first challenge. However, the second financing gap was judged to be more intractable. Many interviewees felt that while banks and other financial institutions are "not structurally positioned to back large-scale projects deploying new technologies" venture capital firms "have high technology risk tolerance but relatively limited capital, and they demand short-to-medium returns."

Beard et al. (2009) takes the most literal interpretation of a "valley" in the funds available for new product development, as illustrated in Figure 1. As the authors say, while "several explanations for the valley of death have been proffered ... none provides a mechanism that clearly explains the non-linear ... 'valley' characteristic of the phenomenon ... [that is] a shortfall of funding at an intermediate stage that is more systematic and profound than the shortfall to either side of the intermediate stage." In effect, Beard et al. interpret the vertical axis in Figure 1 as the *proportion* of projects funded at each stage of the process (or the *probability* that any one project at that stage will receive funding). Specifically, they consider a project that takes 3 stages of R&D expenditure. At stage i of the project, there is a probability of success P_i, and an opportunity cost of funds tied up in the project I_i. The project pays off a private value V only if it progresses to stage 3. Assuming that a project has made it to stage 2, a risk neutral entrepreneur will undertake the final stage only if $P_3 V > I_3$. Similarly, a project that has progressed through stage 1 will enter stage 2 only if $V > I_3/P_3 + I_2/(P_2 P_3)$. Finally, a project will be begun in stage 1 only if $V > I_3/P_3 + I_2/(P_2 P_3) + I_1/(P_1 P_2 P_3)$. Since each of the terms on the right side of the inequalities is positive, a successful project at either stage 1 or stage 2 will automatically be taken to the next stage. From these inequalities, if an exogenous factor makes stage 2 investment less attractive by raising I_2 or reducing P_2, it will also make stage 1 investment less attractive in the same way. It cannot result in a "valley" of death in stage 2 *relative to* stage 1. However, a "valley" can occur if decisions made in stage 1 proceed in part independently of what is likely to happen in the later stages. In particular, subsidies in stage 1 can result in more output of stage 1 projects than the private sector is willing to finance in stage 2, where the original decision criterion remains in force. "So, while pathologies such as risk, uncertainty, appropriation problems and so forth are present at intermediate stages of an innovation process, a non-economic actor operating at early stages is required for there to be a valley of death."[2] In contrast to most of the papers reviewed above, which assumed, either explicitly or implicitly, that the valley of death reflects a misallocation of resources, Beard et al. note that the existence of a valley need not imply an inefficient outcome. Stage 1 research is likely to involve more basic science with more spillover benefits for other endeavors than the more applied research at later stages. More of the fruits of stage 1 research thus may not be appropriable and efficiency may require subsidizing stage 1 R&D but not the later stages. The resulting "valley" would not indicate any market failure.

Weyant (2011) is the only paper we found that raises the concern that research subsidies may be inefficient. He observes that "technologies that are not likely to be remotely economically com-

[2]Beard et al. elaborate on this model in several respects, but the details are not relevant for our discussion.

petitive (or maybe not even technically feasible) at commercial scale have often nonetheless been pursued at great expense, through a pernicious combination of political self-seeking and technological over optimism."

With regard to the valley of death phenomenon, Weyant argues that companies backed by venture capital do not generally focus on basic R&D since venture capitalists are less willing to assume technological than institutional risks. He notes that while large energy companies could fund some of the R&D, regulations may reduce the ability to recover costs and investors may react negatively to higher perceived risk. The inability to appropriate some of the benefits of R&D will also constrain private investors. Another issue that Weyant highlights arises from the work of Schumpeter (1942) and especially his concept of "creative destruction" whereby new technologies make old ones obsolete. Weyant suggests that existing firms in energy industries may have strong incentives to delay this process if their current technologies are more profitable than the new ones. He also suggests that the likelihood of this happening is greater if the industries are oligopolistic or imperfectly regulated.

This paper makes several contributions to the literature on the difficulties that new energy technologies face in competing with entrenched fossil fuel industries. First, we look at the issue in the context of a dynamic intertemporal model of the displacement of fossil fuel energy technologies by non-fossil alternatives. The model distinguishes between investment in energy industry R&D and investment in the physical capital required to produce energy services. Since the physical capital used to supply energy services from fossil fuels has to be replaced by a different set of capital facilities used to supply energy services from non-fossil sources, our model highlights the "creative destruction" element mentioned by Weyant (2011).

Second, we allow for progress in the fossil fuel technologies as well as the alternatives. The unconventional oil and gas revolution is just the latest in a long line of technological improvements in the fossil fuel producing or consuming industries. These technological changes offset what would otherwise be a rising cost of energy services produced from fossil fuels, and make it harder for non-fossil alternatives to compete.

The third novel feature of our analysis in the context of the literature on the valley of death in developing new energy technologies is that we do not assume that cost reductions result from explicit R&D alone. Instead, following much of the empirical literature investigating technological progress in the renewable energy sector, we assume learning-by-doing also contributes to the accumulation of knowledge about the renewable technology.

Finally, following previous authors, we associate the early stage of the process of displacing old energy technologies with new ones as consisting largely of R&D expenditure. However, we associate the "commercialization" phase with the need to build physical capital in order to supply energy services using the alternative technology. Our model highlights the fact that, in order to ensure uninterrupted supply of energy services, investment in new energy technologies is required before fossil fuels are abandoned. However, capital used to produce energy services from fossil fuels is a sunk cost, so it will continue to be used so long as the price of energy is sufficient to cover short-run operating costs. Thus, from the time that investment in capital used to produce energy services from fossil fuels ceases until the time fossil fuels are abandoned, the operating cost of fossil fuel production sets the energy price. Furthermore, at the switchover point, the price of energy just matches the operating costs of renewable energy production. Prior to that time, the price of energy is insufficient to cover even the operating costs of renewable energy production, let alone provide a competitive rate of return to the capital employed. In fact, we show that the full long-run costs of renewable energy (including a competitive rate of return on capital) are not covered until some time after fossil fuels are abandoned.

We stress that the paths of investment, technological progress and energy price that we calculate in the model are efficient (they solve the Pareto optimum problem). Whether those paths could be implemented in a competitive equilibrium is an entirely different matter. In particular, a private firm investing in capital to supply renewable energy services would have to accept an energy price that is below the long-run cost of supplying renewable energy for a long time. Conceptually, the return from reducing costs through promoting technological change ought to pay for some of the costs of investing in renewable energy technology and productive capital. In practice, however, some of the benefits to R&D may be external to the firm and therefore fail to yield appropriable returns. This could exacerbate the problems of early investment in the new technologies. Alternatively, if government were to subsidize R&D into alternative energy technologies, it could lead to an overhang of "first stage" research projects as discussed by Beard et al. (2009).[3]

As noted above, some literature on the valley of death phenomenon has observed that it does not appear to be present in sectors such as pharmaceuticals or information technology. One reason these industries may differ from the energy sector is that patents and copyrights might be more effective at enforcing property rights in the pharmaceutical and IT industries. However, our analysis suggests

[3]To investigate this further, one would need to examine equilibrium in an explicit decentralized model with various assumptions about the appropriability of the benefits of R&D and different amounts of subsidization of research.

another reason. Once a new drug has been discovered or invented, or a new piece of software has been created, these items often can be reproduced at very low marginal cost. In the energy sector, however, very large capital investments are required after the R&D phase in order to supply energy services using new energy technologies. Our model points to potential difficulties in financing these investments in productive capital in the face of competition from fossil fuels.

Our model abstracts from some real world complicating factors. Imperfect substitutability between energy from fossil and non-fossil sources allows non-fossil sources to be more competitive under special circumstances, such as solar panels in remote locations. The model also abstracts from the fact that a significant part of current energy supply comes from hydroelectricity and nuclear instead of fossil fuels. To accommodate these sources, we could add a third type of energy producing capital that is used with both fossil fuels and renewables. This would add considerable complexity to the model, but would not change the problems associated with replacing the "fossil fuel only" part of the current energy supply system.

The other literature that our paper is related to is intertemporal optimization models of economic growth and energy use. We follow most closely Hartley et al. (2013) in adopting the same underlying economic growth model, the same way of allowing technological change to affect the cost of supplying fossil fuel energy, and the same way of allowing both learning-by-doing and explicit investment in R&D to affect the cost of renewable energy. However, that paper does not have investment in physical capital needed to supply energy services, and so cannot address the issues examined in this paper. Restricting physical capital accumulation to the capital needed to produce final output simplifies the model in Hartley et al. relative to this paper.[4]

Our analysis also is related to a paper by Chakravorty, Roumasset, and Tse (1997). They also consider a model with substitution between energy sources, improvements in extraction, and a declining cost of renewable energy. They find that if historical rates of cost reductions in renewables continue, a transition to renewable energy will occur before over 90% of the world's coal is used. In contrast to their paper, we generate an endogenous transition to renewable energy and allow for explicit investment in physical capital stocks. Unlike Chakravorty et. al., we do not study the implications of energy use for environmental externalities and we do not conduct policy experiments.

Other papers in the literature have also examined the transition from fossil fuels to renewable technologies in a dynamic intertemporal modeling framework. In particular, Acemoglu et al.

[4]It is nevertheless reassuring for the central results of that paper that the more complicated model in this paper also produces "an endogenous energy crisis" around the time of the transition between energy sources.

(2009) study a growth model that takes into consideration the environmental impact of operating "dirty" technologies. They examine the effects of policies that tax innovation and production in the dirty sectors. Their paper focuses on long run growth and sustainability and abstracts from the endogenous evolution of R&D expenditures or the need to invest in physical capital in order to supply energy services.

2 The Model

We model economic activity in continuous time, indexed by t. The state variables, the controls, and the technology variables are functions of time. We shall usually simplify notation, however, by omitting time as an explicit argument.

2.1 Goods and services production and consumption

There is a single consumption good in the economy. Letting c denote per capita consumption, in common with much of the growth literature we assume that the lifetime utility function of the representative agent is the constant relative risk averse form:

$$U = \max \int_0^\infty e^{-\beta\tau} \frac{c(\tau)^{1-\gamma}}{1-\gamma}\, d\tau \tag{1}$$

where $e^{-\beta\tau}$ is the discount factor and γ is the coefficient of relative risk aversion.[5]

Per capita output y can be produced using per capita capital k and per capita energy e as inputs. Ignoring for the moment the required energy input, we assume that output depends linearly on k. Effectively, this allows technological progress to expand labor input through investment in human capital even if hours and number of employees remain fixed. Hence, the marginal product of capital does not decline as k accumulates. Capital depreciates at the rate δ, while investment in new capital is denoted by i:

$$\dot{k} = i - \delta k \tag{2}$$

Energy is also an essential input to production. We assume for simplicity that there is no substitution between energy and non-energy inputs in producing y,[6] allowing the production function

[5] More precisely, since there is no uncertainty in our model, γ relates to intertemporal elasticity of substitution.

[6] We have examined an extension of the model that allows for investment in end-use energy efficiency. This requires an additional state, and corresponding co-state, variable and increases the number of regimes in the solution. While being more realistic, it does not add much to the issues under discussion.

to be written $y = \min\{Ak, e\}$. Since it is costly to produce both k and e, however, it will be optimal at all times to have $y = Ak = e$. We further assume that per capita energy input $e = Fk$ is required where F can be interpreted as the fuel intensity of the capital stock.[7]

2.2 Energy production

Energy can be provided by two different technologies that also require capital investments to produce useful output. One, with capital stock denoted k_R, mines fossil fuels that are depletable and converts them into useable energy products using, for example, refineries and power stations. The other, with capital stock denoted k_B, is a backstop or renewable technology where the energy source itself is "harvested" from the environment using the capital equipment, so there is no resource depletion, although there are operating and maintenance costs. Once energy-producing capital is in place it cannot be converted from one type to the other. However, energy from fossil fuels and renewable sources are perfect substitutes for producing goods output. Total energy input into goods production is $e = R + B$ where R equals the energy produced using k_R and B the energy produced using k_B. We also assume linear production functions for the energy producing industries, $R = Gk_R$ and $B = Hk_B$. The two-energy producing capital stocks are accumulated via investment i_B and i_R and depreciate at the same rate δ as capital k used to produce final output:[8]

$$\dot{k}_B = i_B - \delta k_B \tag{3}$$

$$\dot{k}_R = i_R - \delta k_R \tag{4}$$

We define units of fossil fuel resources so that operating one unit of k_R requires a fixed input of one unit of fossil fuel resource. Since k_R is measured in per capita terms, so also will be the fossil fuel input, implying that population growth will also increase the total amount of fossil fuel resources that are mined. Letting Q denote the (exogenous) population and labor supply, and $0 \leq \rho_R \leq 1$ the utilization variable[9] for k_R, the total fossil fuel used will be $\rho_R Q k_R$, and the total

[7] The usual energy identity relates energy input e, such as gallons of fuel, to utilization u per unit of capital, such as miles per vehicle, times the capital stock k, such as the number of vehicles, divided by energy efficiency E, such as miles per gallon. The fuel intensity F then is u/E. Although we usually refer to F as "end-use energy efficiency" it should be understood that fuel intensity of final production can change for reasons other than improvements in energy efficiency as usually understood. For example, Medlock (2010) emphasizes that changes in the composition of production, for example the shift to services, reduce energy intensity as an economy grows.

[8] Although different types of capital could depreciate at different rates, the data we use to calibrate the model provides only a single rate of depreciation for capital. Having just one rate also simplifies the analysis somewhat.

[9] Since capital depreciates exponentially once investment in k_R ceases, a positive amount of k_R will always remain. Mining and conversion costs can then be avoided by choosing $\rho_R = 0$ and ceasing to use k_R to provide energy.

quantity of resources mined to date, S will satisfy:

$$\dot{S} = \rho_R Q k_R \tag{5}$$

For simplicity, we assume that Q grows at the constant rate π, that is, $\dot{Q} = \pi Q$.

We model the cost of fossil fuel production as consisting of two components. This is somewhat analogous to the distinction Venables (2012) draws between costs of extraction on the intensive margin on the one hand, versus costs of new field development, or expansion on the extensive margin, on the other hand. However, we simplify by assuming that, for a given value of S, resources can be extracted at a constant marginal (and average) cost μ. Depletion (an increase in S) raises that cost over time,[10] but technological progress in mining production and fossil energy use can offset the cost increases.[11] The state of technical knowledge in energy services production from fossil fuels is encapsulated in a variable N, which is not assumed to depreciate over time, and where the chosen investment n leads to an accumulation of N:

$$\dot{N} = n \tag{6}$$

Investment n could be associated with bringing new fields into production as emphasized by Venables (2012). However, we have in mind longer-run processes, such as new technologies that enable exploitation of new categories of resources (shale gas and oil, deepwater or pre-salt deposits, oil sands, methane hydrates or underground gasification of deep coal), or increase the efficiency with which fossil fuel is used to provide useful energy services.

While the total feasible technically recoverable fossil energy resource $\bar{S}$ is vast we assume that in the absence of investment in N the maximum recoverable resource S_0 is far smaller. Specifically, we assume that if N were to remain at zero, the marginal mining and conversion costs will be $\mu(S,0)\rho_R k_R$, where $\mu(S,0)$ is increasing and convex in S and unbounded as $S \uparrow S_0$. However, if investment N were to increase without bound, the upper bound on S (where $\mu(S,N) \uparrow \infty$) would

[10] Heal (1976) introduced the idea of an increasing marginal cost of extraction to show that the optimal price of an exhaustible resource begins above marginal cost, and falls toward it over time. This claim is rigorously proved in Oren and Powell (1985). See also Solow and Wan (1976).

[11] Investment to slow increases in mining costs can also be interpreted as investment in the efficiency of fossil fuel use that allows the same energy services to be produced using less input of primary fossil fuel resource.

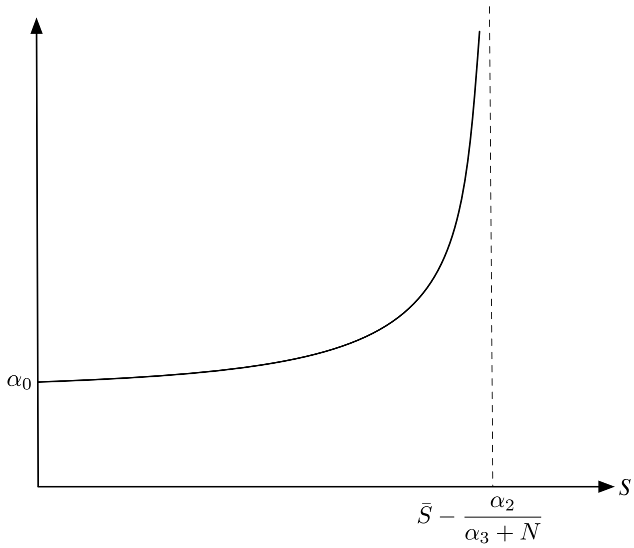

Figure 2: Unit cost of mining fossil fuels $\mu(S, N)$

converge to $\bar{S}$. Figure 2 illustrates a simple functional form that incorporates these assumptions:

$$\mu(S, N) = \alpha_0 + \frac{\alpha_1}{\bar{S} - S - \alpha_2/(\alpha_3 + N)} = \alpha_0 + \frac{\alpha_1(\alpha_3 + N)}{(\bar{S} - S)(\alpha_3 + N) - \alpha_2} \tag{7}$$

The terms $\alpha_0, \alpha_1, \alpha_2$ and α_3 in (7) are parameters, and we can associate the afore-mentioned S_0 with $\bar{S} - \alpha_2/\alpha_3$. Investment in N expands the temporary capacity limit, and the flat portion of the marginal cost curve, to the right, extending the competitiveness of fossil fuels.[12]

By differentiating $\mu(S, N)$ one can show that $\partial\mu/\partial S > 0$ and $\partial\mu/\partial N < 0$, so depletion raises fossil fuel energy costs while investment in N lowers them. Also, $\partial^2\mu/\partial S^2 > 0$, so depletion increases cost at an increasing rate, while $\partial^2\mu/\partial N^2 > 0$, so investment in N decreases costs at a decreasing rate. Investment in N also delays the increase in fossil fuel energy costs accompanying increased exploitation, that is, $\partial^2\mu/\partial S\partial N < 0$. However, since $\partial\mu/\partial N \to 0$ as $N \to \infty$, it will become uneconomic at some point to invest further in reducing the costs of fossil fuel energy. The costs of depletion will then swamp improvements in mining technology and conversion efficiency. Fossil fuel resources will be abandoned before $S = \bar{S}$ as rising costs make renewable energy technologies more attractive. For mining to cease at that time, the utilization ρ_R of k_R has to fall to zero and remain at zero thereafter. Also, once fossil fuel use ceases, S, N and μ will remain constant.

[12] Learning-by-doing in the accumulation of N would increase the productivity of investments n. Depletion can be viewed as "inverse learning-by-doing," since cumulative past production raises current costs. Investment in N offsets this process in the case of fossil fuels, whereas investment in R&D reinforces the cost-reducing effects of learning-by-doing for renewables.

The production of energy from renewable sources requires investment in k_B and incurs operating and maintenance (O&M) costs. Specifically, for renewable energy capital k_B, we assume that renewable energy production is given by $\rho_B H k_B$ and O&M costs by $m\rho_B k_B$, where $0 \leq \rho_B \leq 1$ is the utilization rate of k_B.

We allow technological progress to increase H, and hence reduce the amount of capital k_B required to yield a given level of energy output B. Explicitly, we assume that the accumulation of knowledge that leads to a change in H follows a two-factor learning process whereby increases in the stock of knowledge require the construction of k_B in addition to direct R&D expenditure j:[13]

$$\dot{H} = \begin{cases} b k_B^{\psi} j^{\alpha-\psi} & \text{if } H \leq \bar{H}, \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where $\psi < \alpha < 1$, so there are decreasing returns to investing in renewable energy efficiency. The parameter ψ determines the relative contribution from experience versus explicit investment in research to the accumulation of H. Klaassen et. al. (2005)[14] derive robust estimates suggesting that direct R&D is roughly twice as productive for reducing costs in wind turbine farms as is learning-by-doing. Hence, we assume that $\psi = \alpha/3$. The coefficient b relating investment in knowledge to the resulting technological progress is analogous to the coefficient A in the production function for final output. Finally, we also assume that H cannot exceed an upper limit $\bar{H}$.[15]

2.3 The resource constraint

Goods output is consumed, used to produce energy, or invested in k, k_R, k_B, N or H. This leads to a resource constraint (in per capita terms):

$$c + i + i_R + n + i_B + j + \mu(S, N)\rho_R k_R + m\rho_B k_B = Ak \tag{9}$$

[13]Since ρ_B is absent from (8), learning does not require energy output to be produced. In addition, we assume that accumulated capacity k_B influences growth in H, not just new investments i_B.

[14]Building on an earlier paper by Kouvaritakis et al. (2000), Klaassen et. al. (2005) estimated a two-factor learning curve model that allowed both capacity expansion (learning-by-doing) and direct public R&D to produce cost reducing innovations for wind turbine farms in Denmark, Germany and the UK. They interpret their results as enhancing the validity of the two-factor learning curve formulation.

[15]While this is a technical assumption to allow the final regime to be solved analytically, in reality the laws of physics would prevent H from increasing forever.

Also, equilibrium in the energy market requires[16]

$$Fk = \rho_R G k_R + \rho_B H k_B \tag{10}$$

3 The optimization problem

The objective function (1) is maximized subject to the differential constraints (2), (3), (4), (5), (6) and (8) with initial conditions $k(0) = k_0, k_R(0) = k_{R0}, k_B(0) = k_{B0}, H(0) = H_0$ and $S(0) = N(0) = 0$, the resource constraint (9), and the energy market equilibrium condition (10). The control variables are $c, \rho_R, \rho_B, i, i_R, i_B, n$ and j, while the state variables are k, k_R, k_B, N, S and H. Denote the corresponding co-state variables by q, q_R, q_B, ν, σ and η. Let λ be the Lagrange multiplier on the resource constraint and p_e the multiplier on the energy market equilibrium constraint. Use θ_{RL} and θ_{RU} for the Lagrange multipliers on the inequality constraints on ρ_R, and θ_{BL} and θ_{BU} the corresponding multipliers on the inequality constraints on ρ_B. Also use ω to denote the multiplier on the constraint $i \geq 0$, ω_R the multiplier on the constraint $i_R \geq 0$, ω_B the multiplier on the constraint $i_B \geq 0$, ω_N the multiplier on the constraint $n \geq 0$, and ω_H the multiplier on the constraint $j \geq 0$. We can then define the current value Hamiltonian and hence Lagrangian by

$$\begin{aligned}
\mathcal{H} = {} & \frac{c^{1-\gamma}}{1-\gamma} + q(i - \delta k) + q_R(i_R - \delta k_R) + q_B(i_B - \delta k_B) + \nu n + \sigma \rho_R Q k_R \\
& + \eta b k_B^{\psi} j^{\alpha-\psi} + \lambda\Big\{Ak - c - i - i_R - i_B - n - j - \mu(S,N)\rho_R k_R - m\rho_B k_B\Big\} \\
& + p_e\big\{\rho_R G k_R + \rho_B H k_B - Fk\big\} + \theta_{RL}\rho_R + \theta_{RU}(1-\rho_R) + \theta_{BL}\rho_B \\
& + \theta_{BU}(1-\rho_B) + \omega i + \omega_R i_R + \omega_B i_B + \omega_N n + \omega_H j
\end{aligned} \tag{11}$$

The first order conditions for a maximum with respect to the control variables are:

$$\frac{\partial \mathcal{H}}{\partial c} = c^{-\gamma} - \lambda = 0 \tag{12}$$

$$\begin{aligned}
& \frac{\partial \mathcal{H}}{\partial \rho_R} = \sigma Q k_R - \lambda k_R \mu(S,N) + p_e G k_R + \theta_{RL} - \theta_{RU} = 0 \\
& \theta_{RL}\rho_R = 0, \theta_{RL} \geq 0, \rho_R \geq 0, \theta_{RU}(1-\rho_R) = 0, \theta_{RU} \geq 0, \rho_R \leq 1
\end{aligned} \tag{13}$$

[16] We can use (10) to write Ak in terms of the *net* contribution to output from the energy sector:

$$c + i + i_R + i_B + n + f + j = \rho_R k_R\big[\frac{AG}{F} - \mu(S,N)\big] + \rho_B k_B\big(\frac{AH}{F} - m\big)$$

We assume A is large enough, and initial fuel intensity F is low enough, that A/F exceeds $\mu(0,0)/G$ and m/H_0.

$$\frac{\partial \mathcal{H}}{\partial \rho_B} = -\lambda m k_B + p_e H k_B + \theta_{BL} - \theta_{BU} = 0$$
$$\theta_{BL}\rho_B = 0, \theta_{BL} \geq 0, \rho_B \geq 0, \theta_{BU}(1-\rho_B) = 0, \theta_{BU} \geq 0, \rho_B \leq 1 \tag{14}$$

$$\frac{\partial \mathcal{H}}{\partial i} = q - \lambda + \omega = 0; \omega i = 0, \omega \geq 0, i \geq 0 \tag{15}$$

$$\frac{\partial \mathcal{H}}{\partial i_R} = q_R - \lambda + \omega_R = 0; \omega_R i_R = 0, \omega_R \geq 0, i_R \geq 0 \tag{16}$$

$$\frac{\partial \mathcal{H}}{\partial i_B} = q_B - \lambda + \omega_B = 0; \omega_B i_B = 0, \omega_B \geq 0, i_B \geq 0 \tag{17}$$

$$\frac{\partial \mathcal{H}}{\partial n} = \nu - \lambda + \omega_N = 0, \omega_N n = 0, \omega_N \geq 0, n \geq 0 \tag{18}$$

$$\frac{\partial \mathcal{H}}{\partial j} = \eta(\alpha - \psi) b k_B^{\psi} j^{\alpha-\psi-1} - \lambda + \omega_H = 0, \omega_H j = 0, \omega_H \geq 0, j \geq 0 \tag{19}$$

The differential equations for the co-state variables are:

$$\dot{q} = \beta q - \frac{\partial \mathcal{H}}{\partial k} = (\beta + \delta) q - \lambda A + p_e F \tag{20}$$

$$\dot{q}_R = \beta q_R - \frac{\partial \mathcal{H}}{\partial k_R} = (\beta + \delta) q_R - \sigma \rho_R Q + \rho_R \lambda \mu(S, N) - \rho_R p_e G \tag{21}$$

$$\dot{q}_B = \beta q_B - \frac{\partial \mathcal{H}}{\partial k_B} = (\beta + \delta) q_B - \eta \psi b k_B^{\psi-1} j^{\alpha-\psi} + \rho_B \lambda m - \rho_B p_e H \tag{22}$$

$$\dot{\nu} = \beta \nu - \frac{\partial \mathcal{H}}{\partial N} = \beta \nu + \lambda \rho_R k_R \frac{\partial \mu}{\partial N} \tag{23}$$

$$\dot{\sigma} = \beta \sigma - \frac{\partial \mathcal{H}}{\partial S} = \beta \sigma + \lambda \rho_R k_R \frac{\partial \mu}{\partial S} \tag{24}$$

$$\dot{\eta} = \beta \eta - \frac{\partial \mathcal{H}}{\partial H} = \beta \eta - \rho_B p_e k_B \tag{25}$$

We also recover the resource constraint (9), the energy market equilibrium condition (10), and the differential equations for the state variables, (2), (3), (4), (5), (6) and (8).

Let V denote the maximized value of the objective function (1) subject to the constraints. Recalling that $\mathcal{H}$ is the current value and not present value Hamiltonian, from the Hamilton-Jacobi-Bellman equation we have $-V_t = \max e^{-\beta t}\mathcal{H}$. Furthermore, the (current value) co-state variables satisfy $e^{-\beta t} q = \partial V/\partial k \geq 0$, $e^{-\beta t} q_R = \partial V/\partial k_R \geq 0$, $e^{-\beta t} q_B = \partial V/\partial k_B \geq 0$, $e^{-\beta t}\sigma = \partial V/\partial S \leq 0$, $e^{-\beta t}\nu = \partial V/\partial N \geq 0$ and $e^{-\beta t}\eta = \partial V/\partial H \geq 0$. Also note that the utility function ensures that $c > 0$ and hence from (12), $\lambda > 0$.

Equations (13) and (21) together can be interpreted as a version of the peak load problem.

Specifically, $p_eG - (\lambda\mu - \sigma Q)$ is the short run marginal profit of energy supply using fossil fuels. Each unit of k_R delivers G units of energy each valued at p_e. Explicit marginal production costs measured in output units are $\lambda\mu$. In addition there is an implicit depletion cost measured by $-\sigma Q > 0$. If the capacity is non-binding, $\rho_R < 1$ and marginal profits are simply these short run profits. If $\rho_R = 1$, however, $\theta_{RU} = p_eG - (\lambda\mu - \sigma Q) > 0$ measures the "surplus profits" that represent an implicit return to scarce capacity. The differential equation (21) then implies that the shadow value q_R of k_R is equal to the discounted future value of these implicit rents when $\rho_R = 1$, with a discount rate given by the time discount rate β plus the depreciation rate δ of k_R. Equation (16) then implies that when $i_R > 0$ this shadow value equals the marginal costs of investment represented by λ (the marginal utility of consumption) and thus the shadow cost of investing.

In accordance with this interpretation, we will call the energy price:

$$p_e = \frac{\lambda\mu - \sigma Q}{G} \tag{26}$$

the short run cost of fossil energy production. Using (20) and (21) when both $i, i_R > 0$, $\rho_R = 1$ and (16) and (15) imply $\dot{q}_R = \dot{q}$, we also define the long run cost of fossil energy production:

$$p_e = \frac{\lambda(A + \mu) - \sigma Q}{F + G} \tag{27}$$

Equations (14) and (22) have a similar interpretation for the renewable energy supply. The marginal costs in this case are just the explicit marginal costs λm. Thus, the short run cost of renewable energy analogous to (26) is

$$p_e = \frac{\lambda m}{H} \tag{28}$$

Similarly, (17) and (15) with $i, i_B > 0$ imply $\dot{q}_B = \dot{q}$, and yield a long run cost of renewable energy. In this case, however, the differential equation (22) includes an additional implicit long run return to investment in capacity $\eta\psi b k_B^{\psi-1} j^{\alpha-\psi}$ so long as $H < \bar{H}$.[17] This arises because the learning process implies that investing in k_B brings an additional benefit by lowering future production costs and making investment j in explicit R&D more productive. Defining a function $Y \equiv [\eta(\alpha - \psi)b/\lambda]^s$, where $s \equiv 1/(1 + \psi - \alpha) > 1$, the solution for j from (19) can be written $j = Y k_B^{s\psi}$. Using this

[17] Once $H = \bar{H}$, $\eta = e^{\beta t}\partial V/\partial H = 0$ and $j = 0$ even though k_B remains positive.

solution for j, we conclude that when $\rho_B = 1$, the long run cost of renewable energy production is:

$$p_e = \frac{\lambda}{F+H}\left[A + m - \frac{\psi}{\alpha - \psi} k_B^{(\alpha-1)s} Y\right] \tag{29}$$

4 The evolution of the economy

We assume that at $t = 0$ fossil fuels alone are used to supply energy input.[18] Specifically, we assume that at $t = 0$ the price of energy p_e is determined by the long run cost of fossil fuel energy supply (27), and that this exceeds the short run cost of fossil energy (26) but is less than the short run cost of renewable energy $\lambda m/H_0$. Then (13) implies $\rho_R = 1$ while (14) implies $\rho_B = 0$ at $t = 0$.

Although only fossil sources are used to supply energy at $t = 0$, renewable production capacity will be non-zero at $t = 0$. The reason is that, as already noted above, the benefits of learning-by-doing imply there is an additional return $\lambda\psi k_B^{(\alpha-1)s} Y/(\alpha - \psi)$ to investing in k_B. Then since $\eta > 0$ for $H < \bar{H}$ and we have assumed that $1 > \alpha > \psi > 0$, this additional marginal value of k_B becomes unbounded as $k_B \to 0$. Thus, k_B, and also i_B, are strictly positive at $t = 0$. From the assumptions that $i_B > 0$ but $\rho_B = 0$ at $t = 0$, together with $i > 0$ and (15), (17), (20) and (22), the price of energy at $t = 0$ also has to satisfy

$$p_e = \frac{\lambda}{F}\left[A - \frac{\psi}{\alpha - \psi} k_B^{(\alpha-1)s} Y\right] \tag{30}$$

From (13) energy production from fossil fuels will continue with $\rho_R = 1$, and k_R fully utilized, so long as p_e at least equals (26). Even if, counterfactually, investment n in fossil fuel technology could keep μ from rising as a result of depletion (increasing S), increases in H will reduce the short run cost of renewable energy production (28). Eventually, the economy must reach a time T_R when p_e matches (28). For $t > T_R$, further rises in H would reduce p_e below (26), so $\rho_R(t) = 0$. Since further changes in S, N or k_R then have no effect on V, we also have $\sigma = \nu = q_R = 0$ for $t \geq T_R$.

Although $\nu = 0 < \lambda$ at T_R, (18) implies $\nu = \lambda$ while $n > 0$. Furthermore, (23) implies $\dot{\nu} \neq 0$ for $\rho_R > 0$.[19] Solving (23) backwards in time from T_R, ν must rise faster than λ. When $\nu = \lambda$, n will become positive. This will occur at some $T_N < T_R$ with $\nu < \lambda$ and $n = 0$ for $t \in [T_N, T_R]$.

Similarly, $0 = q_R < \lambda = q$ at T_R, and (16) implies $q_R = \lambda = q$ while $i_R > 0$. In contrast to

[18] In practice, non-fossil sources other than hydroelectricity and nuclear power have a small share of primary energy production and generally would be uncompetitive without subsidies or mandates.

[19] Since $\partial\mu/\partial N < 0$, (23) implies that ν starts out positive, but declines to zero at T_R, and since $\partial\mu/\partial S > 0$, (24) implies that σ starts out negative and rises to zero at T_R.

ν, however, $\dot{q}_R$ can equal zero if $\rho_R > 0$, specifically when p_e equals (26). We return to this point below. Define T_Q as the first time that $i_R = 0$ so that $i_R > 0, \forall t < T_Q$.

We next discuss the use of fossil capacity, that is, the value of ρ_R for $t < T_R$. By the definition of T_R as the first time fossil fuels are no longer used, $\rho_R(T_R) = 0$ and $\rho_R(t) > 0$ for $t < T_R$. Since $k_R > 0$, the first order condition (13) implies $[\sigma Q - \lambda\mu + p_e G]\rho_R \geq 0$ for $t < T_R$ and is zero if $\rho_R < 1$ or p_e equals the short-run fossil fuel cost (26). In particular, for $t < \min(T_N, T_Q)$, $i_R, n > 0$ and from (16) and (18), $q_R = \lambda = \nu$. Thus, $\dot{q}_R = \dot{\lambda} = \dot{\nu}$, and since ν is positive and strictly decreasing for $t < T_R$, $q_R > 0$ and $\dot{q}_R < 0$ for $t \leq \min(T_N, T_Q)$ and hence, from (21), $\sigma Q - \lambda\mu + p_e G > (\beta + \delta)q_R/\rho_R > 0$ and thus $\rho_R = 1$.

Focusing next on the transition to renewable energy, the energy market equilibrium condition (10) will require $\rho_B > 0$ at T_R to ensure that energy input to final production is maintained as fossil fuel is abandoned. Specifically, since $k_R, k_B > 0$, if $1 \geq \rho_R, \rho_B > 0$, ρ_B must be given by:

$$\rho_B = \frac{Fk - \rho_R G k_R}{H k_B} \tag{31}$$

Thus, if $\rho_R \downarrow 0$ continuously on some interval $(\hat{t}, T_R)$, (31) requires $\rho_B \uparrow 1$ continuously over the same interval. But then $0 < \rho_R, \rho_B < 1$ on $(\hat{t}, T_R)$, and the price of energy would have to equal the two short run costs (26) and (28) over that interval. However, the short-run cost of fossil fuel $\mu - Q\sigma/\lambda \to \mu$ as $t \to T_R$ and the absence of investment in N and the effects of continuing depletion cause μ to increase rapidly. On the other hand, increases in H would imply mG/H is decreasing. We thus arrive at a contradiction, implying we must have $\rho_R = 1$ until T_R, when it jumps discontinuously to zero.

Given this behavior of ρ_R, (31) then implies that, for $t < T_R$, $\rho_B < 1$. Since ρ_B jumps from a value less than 1 to equal 1 at T_R, (14) implies that the price of energy at T_R must equal the short-run renewable energy cost (28) at T_R. However, for $t < T_R$, H must be smaller than it is at T_R and thus the short-run real cost of renewable energy must be higher than it is at T_R. On the other hand, the real cost of energy as determined by fossil fuel production must be rising as $t \uparrow T_R$. Hence, (14) implies $\rho_B = 0$ for $t < T_R$. Thus, we have a "bang-bang" solution for energy capital utilization whereby $\rho_R = 1$ and $\rho_B = 0$ for $t < T_R$ and then $\rho_R = 0$ and $\rho_B = 1$ for $t > T_R$.

Once investment in k_R ceases at T_Q, k_R will decline at the rate δ. Then since $\rho_R = 1$ and $\rho_B = 0$, and the depreciation rate on k also is δ, energy market equilibrium (10) will require $i = 0$ and $\dot{k} = -\delta k$ for $t \in [T_Q, T_R]$. Thus, energy market equilibrium will be just exactly satisfied by

$\rho_R = 1$. Then with $q_R = 0$ at T_R, and p_e equalling the short-run cost of fossil energy production (26) for $t \in [T_Q, T_R]$, solving (21) backwards in time from T_R, we conclude that $q_R = 0, \forall t \in (T_Q, T_R]$.[20]

With both $i_B > 0$ and $\rho_B = 0$ for $t < T_R$, (17), (19), and (22) imply that $\dot{\lambda} = \dot{q}_B$ satisifes

$$\dot{\lambda} = (\beta + \delta)\lambda - \eta\psi b k_B^{(\alpha-1)s} Y \tag{32}$$

On the other hand, for $t > T_R$, $i > 0$ and (15) imply $q = \lambda$ and thus from (21)

$$\dot{\lambda} = (\beta + \delta - A)\lambda + p_e F \tag{33}$$

In particular, if $i_B > 0$ at T_R, these two values for $\dot{\lambda}$ would be equal and the price of energy at T_R would be given by (30). However, the price of energy at T_R must also equal the short-run cost of renewable energy production (28). Hence, we would have

$$\frac{m}{H} = \frac{A}{F} - \frac{\psi}{(\alpha - \psi)F} k_B^{(\alpha-1)s} Y \tag{34}$$

For the parameter values we will consider later, however, $A/F = 1$ and the second term on the right hand side of (34) is close to zero. Thus, we would have $H \approx m$ at T_R. But we also have $H > m$ at $t = 0$, and H is increasing over time while m is constant. We conclude that we cannot have $i_B > 0$ at T_R and there must an interval immediately after T_R when $i_B = 0$. In effect, the need to replace k_R at T_R leads to over-investment (from an ex-post perspective) in k_B at T_R and there is a pause from investing in k_B while the price of energy rises from the short-run cost of renewable energy production (28) to the the long-run cost (29).

We summarize the above discussion in Figure 3. The upper part of Figure 3 shows the different investment regimes, while the lower part shows the different regimes of energy production and use. The economy passes through five regimes before entering the final regime where an analytical solution is possible. The differential equations that determine the evolution of the state and co-state variables in each regime are derived and discussed in an appendix.

The most striking feature of the solution is that even though investment in new energy technologies is required before fossil fuels are abandoned at T_R, the price of energy is insufficient to cover the operating costs of renewable energy for $t < T_R$. Furthermore, the full long-run costs of renewable energy are not covered until $T_B > T_R$.

[20] Note q_R will be left continuous (and differentiable) as $t \uparrow T_Q$, but will jump discontinuously to zero at T_Q and remain zero $\forall t \geq T_Q$.

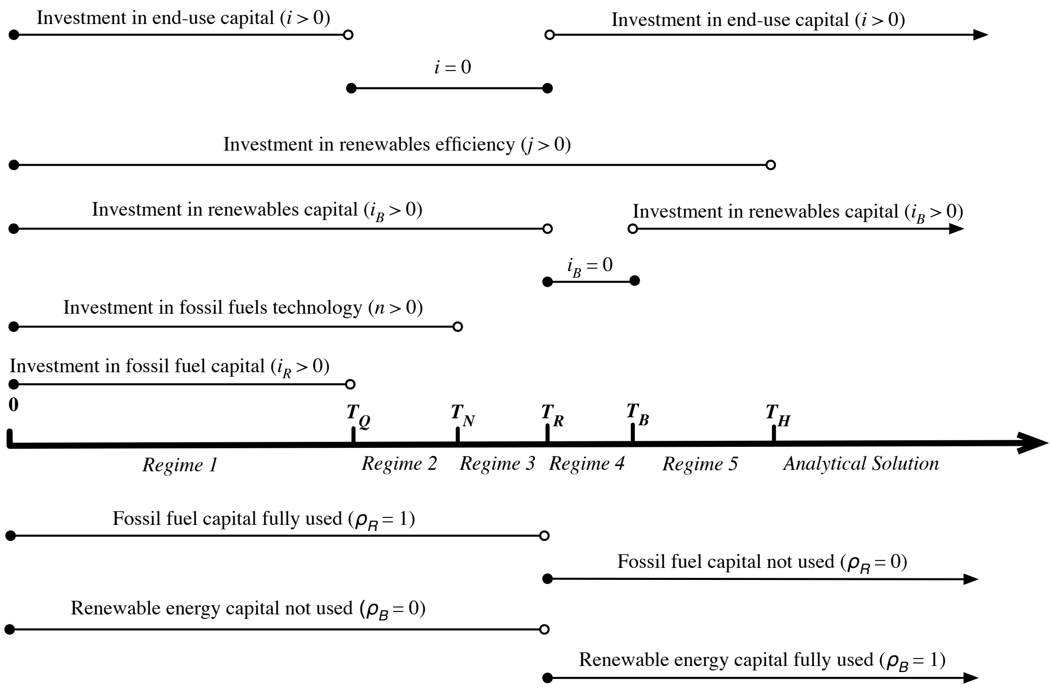

Figure 3: A schematic representation of the different regimes

5 Calibration

To solve the model, we need to specify numerical values for the parameters. As far as possible, we choose values so variables at $t = 0$ are consistent with observations from the world economy.

By definition, we start the economy with $S = N = 0$ and with $Q = Q_0$ and $H = H_0$. For convenience, we take the current population $Q_0 = 1$ and effectively measure future population as multiples of the current level. We will assume that the population growth rate is 1%.[21]

In line with standard assumptions made to calibrate growth models, we assume $\beta = 0.05$. From previous analyses of macroeconomic and financial data, we would expect the coefficient of relative risk aversion γ to lie between 1 and 10, but there is no strong consensus on what the value should be. The per capita growth rate of the economy depends inversely on γ. However, larger values of γ also extend the time it takes the economy to enter the terminal regime. We set $\gamma = 5$ as a compromise between these considerations.

To calibrate values for the initial production, capital stocks and energy quantities we used data from the US *Energy Information Administration* (EIA),[22] the *Survey of Energy Resources*

[21] This is consistent with a simple extrapolation of recent world growth rates reported by the Food And Agriculture Organization of the United Nations, http://faostat.fao.org/site/550/default.aspx

[22] International data is available at http://www.eia.doe.gov/emeu/international/contents.html

2007 produced by the *World Energy Council* (WEC),[23] and *The GTAP 8 Data Base* produced by the *Center for Global Trade Analysis* in the Department of Agricultural Economics, Purdue University.[24] The last mentioned data source is useful for our purposes because it provides a consistent set of international accounts that also take account of energy flows. We also use the GTAP depreciation rate δ on capital of 4%.

National accounts include government spending in GDP, which does not appear in the model. Since a substantial amount of government spending, such as that on health care or education, is directly substitutable with private consumption spending, we classified government spending as part of consumption.

Converting the GTAP data base estimates of the total capital stock to units of GDP, we obtain $k + k_R = 2.6551$. Similarly, we identify investment in physical capital in the GTAP data as $i + i_R$ in the model. Rescaling units so output equals 1, we conclude that $i + i_R = 0.2299$.

The GTAP data gives firm purchases of the capital resource endowment (and other factors of production) by sector. We identify the "energy sector" to include production of the primary fuels (coal, oil and natural gas), plus the energy commodity transformation sectors of refining, electricity generation and natural gas distribution, plus the transportation services sector.[25] After doing so, we find that 12.35%, or 0.3278 of the capital would be k_R while $k = 2.3273$. Defining units so that output equals 1 implies that $A = 1/k = 0.4297$. Similarly, choosing units of energy service inputs into final production so $R = Gk_R = 1$ implies $G = 1/k_R = 3.0506$. The energy market equilibrium condition then requires $Fk = Gk_R = 1$, so we must also have $F = A = 0.4297$.

Next we calibrate the initial marginal cost of producing energy. From the resource constraint, the difference between total output and $i + i_R$, namely 0.7701, would equal $c + \mu k_R + n$, which is all classified as *consumption* expenditure in the GTAP data. We then associate $\mu k_R + n$ with the share of consumption falling on the output of the energy sector as defined above and c with the remainder. The result is $c = 0.7140$ and $\mu k_R + n = 0.0561$.

The *International Energy Agency* (IEA) publishes statistics[26] on energy sector investment in R&D. The expenditure is categorized into: energy efficiency (around 18% of the total from 2005–

[23]This is available at http://www.worldenergy.org/publications/survey_of_energy_resources_2007/default.asp The data are estimates as of the end of 2008.

[24]Information on this can be found at https://www.gtap.agecon.purdue.edu/databases/v8/default.asp The GTAP 8 data extracted below pertains to data for 2007.

[25]Nuclear, hydroelectric, wind and solar energy, currently supply some electricity, and some transportation is powered by electricity and biofuels, but as an approximation we are assuming that fossil fuels supply all energy services at $t = 0$, that is, $\rho_B(0) = 0$. Using the remaining calibrated values, we also find that the equations linking the capital stocks at $t = 0$ imply that $k_B(0)$ and $i_B(0)$ are positive but negligible.

[26]The data is available at http://www.iea.org/stats/rd.asp.

2010), fossil fuels (14%), renewable energy sources (15%), nuclear (33%), hydrogen and fuel cells (6%), other power and storage technologies (5%), and other cross-cutting technologies and basic research (8%). Much of the explicit R&D expenditure on renewable, nuclear, hydrogen and fuel cells, and cross-cutting technologies and basic research would be part of the variable j in the model. Like the use of renewables to produce energy, a substantial amount of this expenditure would not exist without extensive government support and we take $j(0)$ to be negligible as implied by the model solution for the remaining calibrated values. Items in the other power and storage technologies category relate mainly to electrical system improvements that are not specific to renewable energy sources and thus could be grouped with R&D into fossil fuels as part of n. The IEA also reports real GDP for the economies in their panel allowing the R&D expenditures to be expressed as a proportion of GDP. Taking the average ratio from 2005–2010 for all countries in the sample, we find that the R&D part of n would be just 0.00644% of GDP. However, the countries in the sample are mostly higher income countries from the OECD, which may invest a higher proportion of GDP in energy sector R&D than the remaining countries. Using the GTAP data, the countries in the IEA sample supplied about 71.7% of total GDP versus 28.3% for the remaining countries. If we assume the proportion of GDP invested in energy sector R&D in the remaining countries is on average one fourth that of the included countries, we would conclude that, for the world as a whole, the R&D part of n would be 0.0051% of GDP.

The variable n in the model should also include expenditures to expand fossil fuel reserves through exploration and investment in new mines. The EIA reports exploration and development expenditures by oil and natural gas firms.[27] Over the period 2003–2009, these averaged more than seventy-two times the total fossil fuel R&D expenditure recorded by the IEA. The Australian Bureau of Statistics reports[28] exploration expenditures by Australian coal mining firms, which according to EIA data produce around 20% of the total coal output from those countries included in the IEA R&D expenditure data set. Multiplying the recent annual Australian coal exploration expenditure by five, we arrive at a figure that is only about 3% of the oil and gas exploration and development expenditure reported by the EIA. Adding these exploration expenditures to the previously calculated fossil fuel R&D spending, we arrive at a total value for n of 0.31% of GDP. Expressing this in units defined so output equals 1, we have $n = 0.0031$.

[27] The data is available at http://www.eia.gov/finance/performanceprofiles/ as Table 15.

[28] See publication 8412.0 – "Mineral and Petroleum Exploration, Australia" available at http://www.abs.gov.au and searching by catalog number.

Subtracting n from the previously calculated $\mu k_R + n = 0.0561$, we obtain $\mu k_R = 0.0530$,[29] and using the previously obtained k_R we then obtain $\mu = 0.1614$.[30] After we set the initial values of S and N to zero and R to 1 (by defining energy units), the initial value for μ also would imply

$$0.1614 = \alpha_0 + \frac{\alpha_1}{\bar{S} - \alpha_2/\alpha_3} \tag{35}$$

Next, we evaluate $\bar{S}$ in the same units as R. The EIA web site gives world wide production of oil in 2007 of 178.596 quads (where one quad equals 10^{15} BTU), of natural gas 107.391 quads and of coal 133.367 quads. Summing these gives a total of 419.354 quads, which we will take as our measure of one unit of R.

To obtain an estimate of total fossil fuel resources $\bar{S}$ in the same units, we used data from the WEC and the *US Geological Survey* (USGS). The millions of tonnes of coal, millions of barrels of oil, extra heavy oil, natural bitumen and oil shale and trillions of cubic feet of conventional and unconventional natural gas were converted to quads using conversion factors available at the EIA. The result is 70.282 quintillion BTU of coal, 72.122 quintillion BTU of conventional and unconventional oil and 13.821 quintillion BTU of conventional and unconventional natural gas. These resources are nevertheless relatively small compared to estimates of the volume of methane hydrates that may be available. Although experiments have been conducted to test methods of exploiting methane hydrates, a commercially viable process is yet to be demonstrated. Partly as a result, resource estimates vary widely. According to the *National Energy Technology Laboratory* (NETL),[31] the USGS has estimated potential resources of about 200,000 trillion cubic feet in the United States alone. According to Timothy Collett of the USGS,[32] current estimates of the worldwide resource in place are about 700,000 trillion cubic feet of methane. The latter figure would be equivalent to 718.900 quintillion BTU. Adding this to the previous total of oil, natural gas and coal resources yields a value for $\bar{S} = 875.125$ quintillion BTU or around 2086.8425 in terms of the energy units defined so that $R = 1$.

Using $\bar{S}$, (35) will give us one equation in the four parameters $\alpha_i, i = 0, \ldots, 3$. The quantity $\bar{S} - \alpha_2/\alpha_3$ represents the limit of total fossil fuels that could be extracted in the absence of any

[29] By comparison, the EIA Annual Energy Review gives energy expenditures as amounting to around 5% of gross output in the U.S.

[30] Observe that, since we have defined units so $R = 1$, this value of μ implies fossil fuels yield positive net output at $t = 0$, that is, $AG/F_0 - \mu = G - \mu = 2.8892 > 0$.

[31] http://www.netl.doe.gov/technologies/oil-gas/FutureSupply/MethaneHydrates/about-hydrates/estimates.htm

[32] http://www.netl.doe.gov/kmd/cds/disk10/collett.pdf

investment n. Thus, we associate $\bar{S} - \alpha_2/\alpha_3$ with current producing reserves of fossil fuel.[33] We deduce these from current production and decline rates.[34] A report from *Cambridge Energy Research Associates* (CERA, 2009),[35] for example, gives weighted average decline rates for oil production from existing fields of around 4.5% per year. However, this figure is dominated by a small number of "giant" fields and that, "the average decline rate for fields that were actually in the decline phase was 7.5%, but this number falls to 6.1% when the numbers are production weighted." As an approximation, we shall use 6% as a decline rate for oil fields. Using United States natural gas production and reserve figures as a guide, we find that natural gas decline rates are closer to 8% per year. The United States data on coal mine decline rates approximate 6% per year. In accordance with these figures, we assume the ratio of fossil fuel production to proved and connected reserves equals the share weighted average of these figures, namely $(178.60 * 0.06 + 107.391 * 0.08 + 133.367 * 0.06)/419.354 = 0.0651$. Thus, in terms of the energy units defined so that $R = 1$, the initial value of producing reserves $\bar{S} - \alpha_2/\alpha_3$ would equal 1/0.0651=15.3559. Using the previously calculated value for $\bar{S}$, this leads to $\alpha_2/\alpha_3 = 2071.4867$.

We obtain another equation by examining investment in expanding fossil fuel production at $t = 0$. As noted above, we calibrated the initial value of $n = 0.0031$. We assume that this level of investment at $t = 0$ is sufficient to increase producing reserves by a percentage amount equal to the average annual growth over 2004-10 of around 2.43%.[36] In other words, we assume that the investment $n = 0.0031$ increases producing reserves to 15.729, that is, $\alpha_2/(\alpha_3 + 0.0031) = 2071.1135$. Then using $\alpha_2/\alpha_3 = 2071.4867$, we find

$$\alpha_3 = \frac{0.0097 \cdot 2071.1135}{2071.4866 - 2071.1135} \approx 17.0411 \tag{36}$$

The previously calculated value for α_2/α_3 then implies $\alpha_2 \approx 35300$. We thus have determined the μ function up to one degree of freedom. Once we specify either α_0 or α_1, (35) along with the previously determined $\bar{S}$ and α_2/α_3 will give us the remaining α_i parameter. We chose α_1 in an attempt[37] to ensure that $\lambda(0) = c(0)^{-\gamma}$ at $t = 0$.

[33] Current official reserves are not the relevant measure since many of these are not available for production without further investment, denoted n in the model.

[34] Note that the rate of exploitation of fossil fuels is determined endogenously in the model. We are using data on decline rates *solely* to arrive at an estimate of current producing reserves.

[35] "The Future of Global Oil Supply: Understanding the Building Blocks," Special Report by Peter Jackson, Senior Director, IHS Cambridge Energy Research Associates, Cambridge, MA.

[36] These calculations are again based on data from the EIA.

[37] The model is highly non-linear making it difficult and time consuming to solve. Each time α_1 is changed, the model needs to be solved many times over to find solution paths with values at $t = 0$ that approximate initial values

Turning next to technological progress in renewable efficiency, we rely on empirical estimates based on experience with subsidized installations of wind turbines and solar panels to set ψ and α. In a study of wind turbines, Coulomb and Neuhoff (2006) found values corresponding to the parameter ψ in our model of 0.158 and 0.197. Grübler and Messner (1998) found a value for $\psi = .36$ using data on solar panels, while van Bentham et. al. (2008) report several studies finding approximate values of $\psi = 0.322$ for solar panels. We will take $\psi = 0.25$. Klaassen et. al. (2005) estimated a model that allowed for both learning-by-doing and direct R&D. Although they assume the capital cost is multiplicative in total R&D and cumulative capacity, while we assume the *change in knowledge* is multiplicative in new R&D and cumulative capacity, we take their parameter estimates as a guide. They find direct R&D is roughly twice as productive for reducing costs as is learning-by-doing. Consequently, we assume that $\alpha = 0.75$.

Finally, we need to establish values for the initial $H(0)$ and final $\bar{H}$ values of the productivity of k_B in producing energy services, and the operating and maintenance costs m for renewable energy production. Although a substantial amount of current primary energy consumption is direct rather than indirect through the consumption of electricity,[38] we focus on the relative costs of producing electricity using fossil and non-fossil sources. One justification may be that electric vehicles represent the main way of using non-fossil fuels in the transportation sector. Electricity may also be the main alternative to fossil fuels for indoor space heating, heating water and other residential and commercial uses.

We focus on natural gas and coal generated electricity for the fossil fuel cost and nuclear,[39] wind and pumped storage for non-fossil generation. Not all locations have suitable geography for pumped storage, however, so a certain fraction of capacity needed to provide ancillary services will have to take more expensive forms such as batteries, flywheels, or compressed air.

In the *Annual Energy Outlook, 2010* the EIA gives indicative costs for different types of generation capacity and heat rates for the natural gas and coal plants.[40] We used these cost estimates

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of the state variables. We therefore could obtain only an approximate match for $c(0)$.

[38]Data from the EIA for the US shows that around 28% of primary energy is consumed in transportation, 20% in industry and another 10% in residential and commercial activities. The remainder is used to generate electricity and allocated as primary energy to users based on their consumption of electricity.

[39]Although nuclear fuels also are mined, the energy content of known uranium, thorium and other fissionable material is huge. The available fuel supply can also be extended using breeder reactors, while if electricity production from nuclear fusion is ever perfected, the fuel supply would, for all practical purposes, be inexhaustible. We can also use nuclear power costs as a proxy for the costs of unconventional geothermal energy based on "hot rocks".

[40]The heat rate for nuclear plants comes from the average realized heat rate in the US in 2010 as reported in Table 5.3 of the EIA publication *Electric Power Annual.*

along with fuel prices for 2009,[41] indicative load factors,[42] and assumed operating lives of 40 years for the coal, nuclear and pumped storage plants, 30 years for the single and combined cycle turbines, and 25 years for the wind generators[43] to calculate costs of electricity generation for the different types of plants. We separated these costs into those that would be part of investment in the GTAP data (capital costs) and those that would be part of "consumption" (operating and maintenance expenditures). The equivalent annual capital cost (EAC) of capital per MW of capacity was calculated based on an assumed annual real required rate of return of 7.5%.

The annual capital costs *for the system* are calculated as a capacity-weighted average of the EAC for each type of plant.[44] Similarly, the annual O&M costs for the system are calculated as a capacity-weighted share of the fixed O&M costs plus an output-weighted sum of the combined variable O&M and fuel costs.[45] Finally, the ratio of the annual capital costs for the non-fossil

[41]The natural gas and coal fuel prices were obtained from Table 3.5 of the *Electric Power Annual*. The uranium price is the 2009-2011 average monthly price of U_3O_8 per pound obtained from the IndexMundi web site available at http://www.indexmundi.com/commodities/?commodity=uranium&months=60 divided by the average energy content of U_3O_8, namely 180 MMBTU per pound obtained from the TradeTech web site http://www.uranium.info/unit_conversion_table.php.

[42]The load factors for the coal and nuclear plants were obtained by dividing net generation from Table 1.1 of *Electric Power Annual* by net summer capacity from Table 1.1.A of the same publication and then averaging the result for 2007-2010. Performing the same calculation for natural gas fired plants produces an average capacity factor of 0.2396. However, this would cover combined cycle and conventional steam plants fired by natural gas, which are operated as base or intermediate load, and combustion turbines, which are operated as peaking plants at a very low load factor. A technology brief from the IEA, available at http://www.iea-etsap.org/web/E-TechDS/PDF/E02-gas_fired_power-GS-AD-gct.pdf, claims that typical international values for load factors of combined cycle plants range from 0.2–0.6, while corresponding values for combustion turbines range from 0.1–0.2. We have assumed that combined cycle plants are operated at the top of the IEA range (0.6) and combustion turbines at the low end of the IEA range (0.1). We also assumed that the load factor for pumped storage would equal the load factor (0.1) of natural gas combustion turbines in the fossil fuel world and that the cost of other forms of storage would be double the cost of pumped storage. Using EIA data on annual electricity generation and installed generation capacity by country and type, available at http://www.eia.gov/countries/data.cfm, we found that the average load factor for wind generators for the world as a whole in 2009 was 0.20.

[43]The *Environmental Protection Agency* National Electric Energy Data System (NEEDS) database, available at http://www.epa.gov/airmarkt/progsregs/epa-ipm/past-modeling.html#needs, gives an average age of US coal-fired generators of 38 years, but many plants will be far from the end of their useful life. The conventional oil or gas-fired steam plants had an average age of 44 years. The average age of the nuclear plants was 24 years, but many more of these plants would still be a long way from retirement. The average age of the combustion turbines in the NEEDS database was 27 years, while the combined cycle plants averaged just 13 years, but combined cycle is a relatively new technology. The average age of the pumped storage plants in the NEEDS database was also 30 years. Finally, the wind generators in the NEEDS database were also constructed recently, so we do not have a good indication of how long they may last. However, several sources on the internet gave a design life of 20 years for wind turbines, while the maximum estimated lifespan we found was 30 years.

[44]We weighted the EAC/MW for each technology by the share of that technology in the overall generating capacity. Allowing for the share of combined cycle plants to be somewhat higher in the future to reflect the higher efficiency of those plants, we assumed that combustion turbines would constitute 10% of total capacity (and be used only for 10% of total hours) and then divided the remaining required capacity equally between coal and natural gas combined cycle plants. For the non-fossil world, we increased the share of backup storage to 20% of capacity to account for the intermittency of wind power. The intermittency of wind generation also makes it unlikely that network stability could be maintained with such sources constituting more than 30% of system wide capacity. We therefore set the wind capacity at 30% and nuclear at 50% in the non-fossil world.

[45]The latter were converted to an annual basis by multiplying by the number of hours in a year that each type of

system to the annual capital costs for the fossil-based system was 9.9807. The ratio of the annual O&M costs for the two generating systems was 0.4948.[46]

We already calculated above that the energy output to capital ratio for fossil fuels as $G = 3.0506$. We therefore assumed that the corresponding initial energy output to capital ratio for non-fossil fuels was $H = 3.0506/9.9807 \approx 0.3056$. Similarly, since we calculated above that the initial value for $\mu = 0.1614$, we take the value of $m = 0.1614 * 0.4948 \approx 0.0799$.

Finally, we need to specify a final limiting value for H and the coefficient b in the renewable energy technological change function. We arbitrarily assumed that H could be increased by a factor of 4 to $\bar{H} \approx 1.2224$. Then the final ratio $m/\bar{H} \approx 0.0653$ would approximate the current ratio for fossil fuels $\mu/G \approx 0.0529$. Once the values of $\bar{H}$ and m have been set, the long run per capita growth rate can also be calculated as 4.14%. We could not find a suitable data source to calibrate b, so we arbitrarily set it to 0.006 to obtain an approximate 100 years until the renewable sources attain their final long-run efficiency level.

6 Results

Figure 4 graphs the solution paths for the state variables while Figure 5 graphs the paths for the control variables.[47] The critical times for transition between the various regimes are $T_Q = 73.5248, T_N = 79.0248, T_R = 80.0424, T_B = 85.7931$ and $T_H = 99$. Thus, fossil fuels are abandoned after about 80 years, but investment in k_R ceases after about 73.5 years. Investment in fossil fuel technology N continues for another 5.5 years, ceasing after slightly more than 79 years. The solution for S implies that slightly over 77% of the initial stock of fossil fuel resources are exploited. Investment in fossil fuel technology N does not play a large role until $t \approx 50$.

A striking feature of the optimal paths of the control variables is their non-linearity and somewhat volatile character – especially investment in final capital k and renewable energy producing capital k_B. Some of thes fluctuations in investments appear to be to facilitate investment n and j in the two R&D variables. There is a large spike in investment n as investment in k_R ceases, with a smaller jump in n occurring right before n drops off quickly to zero. Similarly, investment i_B in

plant would be operated, namely the load factor, times 8760.

[46]Details of these calculations are available from the authors.

[47]The solution discussed here has $k(0) = 2.3285, H(0) = 0.3070, S(0) = 0.00004$ and $N(0) = -0.0924$ compared to target values of 2.3273, 0.3056, 0 and 0. The initial value of S is closest to its target because S responds most sensitively to changes in the values used to determine the solution to the model's differential equations. The initial value of N is least sensitive, making it hardest to match. The highly non-linear nature of the solutions prevented us using an automatic solution procedure to find the best initial values to hit the required target values. Small changes in the initial values easily move the differential equations to regions where they cannot be solved.

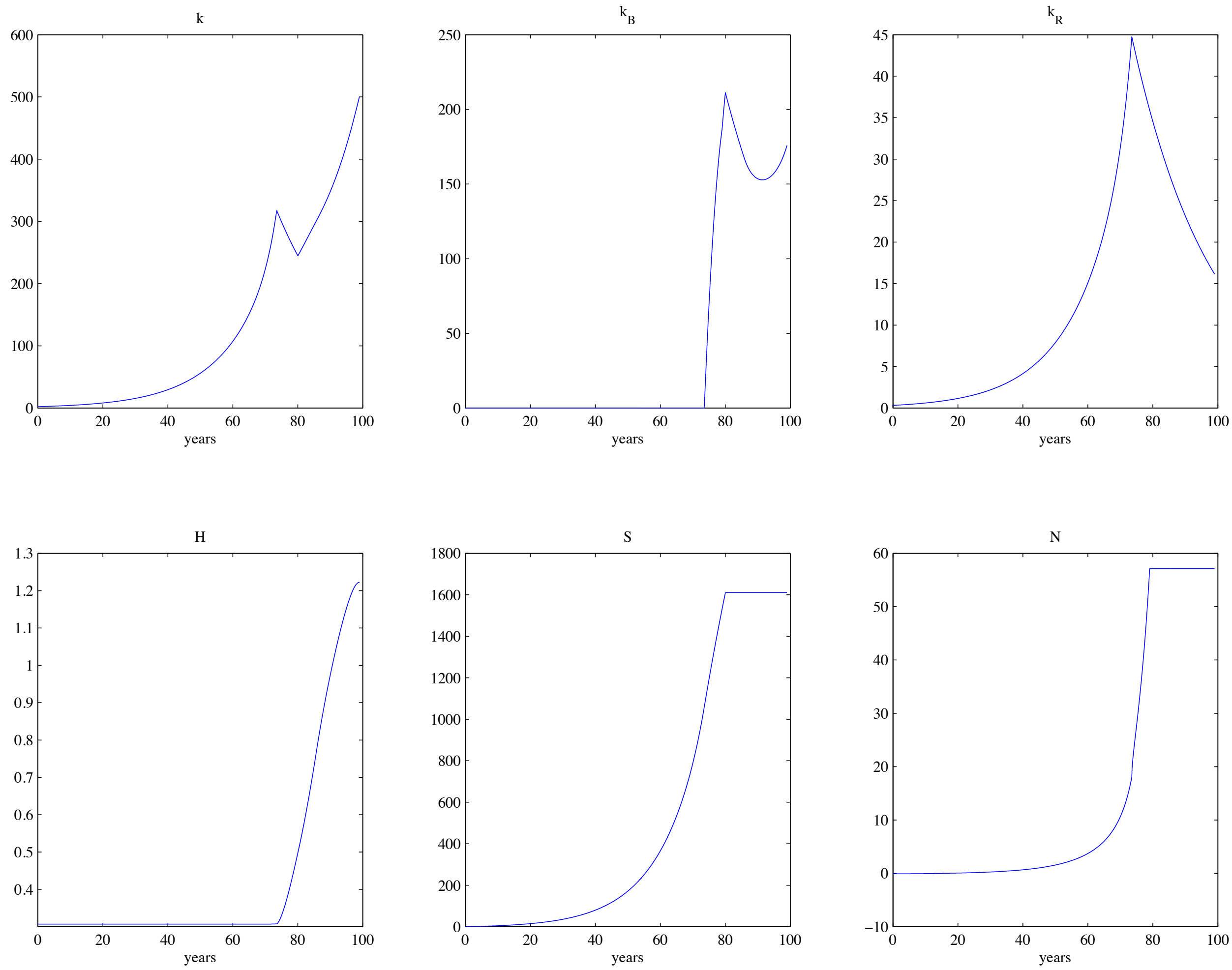

Figure 4: Calculated paths of the state variables

k_B and j in H do not take off until investment in k_R ceases at T_Q. The need to replace fossil fuel energy production at T_Q means that there has to be considerable investment in k_B prior to T_Q. In addition, the rapid increase in H after T_Q means that less k_B is needed to supply a given level of energy services. The result is excess k_B at T_R and, as illustrated in the third graph in Figure 5, investment i_B in k_B is therefore zero for about five years (from T_R until T_B) and k_B declines over this period. Similarly, k declines along with k_R between T_Q and T_R as both i and i_R are zero over this interval. The fossil fuel capital stock continues to decline, but remains positive, for $t > T_R$, although it is not used to supply energy services after T_R since renewable sources are cheaper.

The economy also is severely disrupted by the switch from using fossil fuels to renewables to supply energy services. In particular, per capita consumption actually declines for nearly 7 years (between T_Q and T_R). Part of the explanation is that the real price of energy rises to a peak at T_R

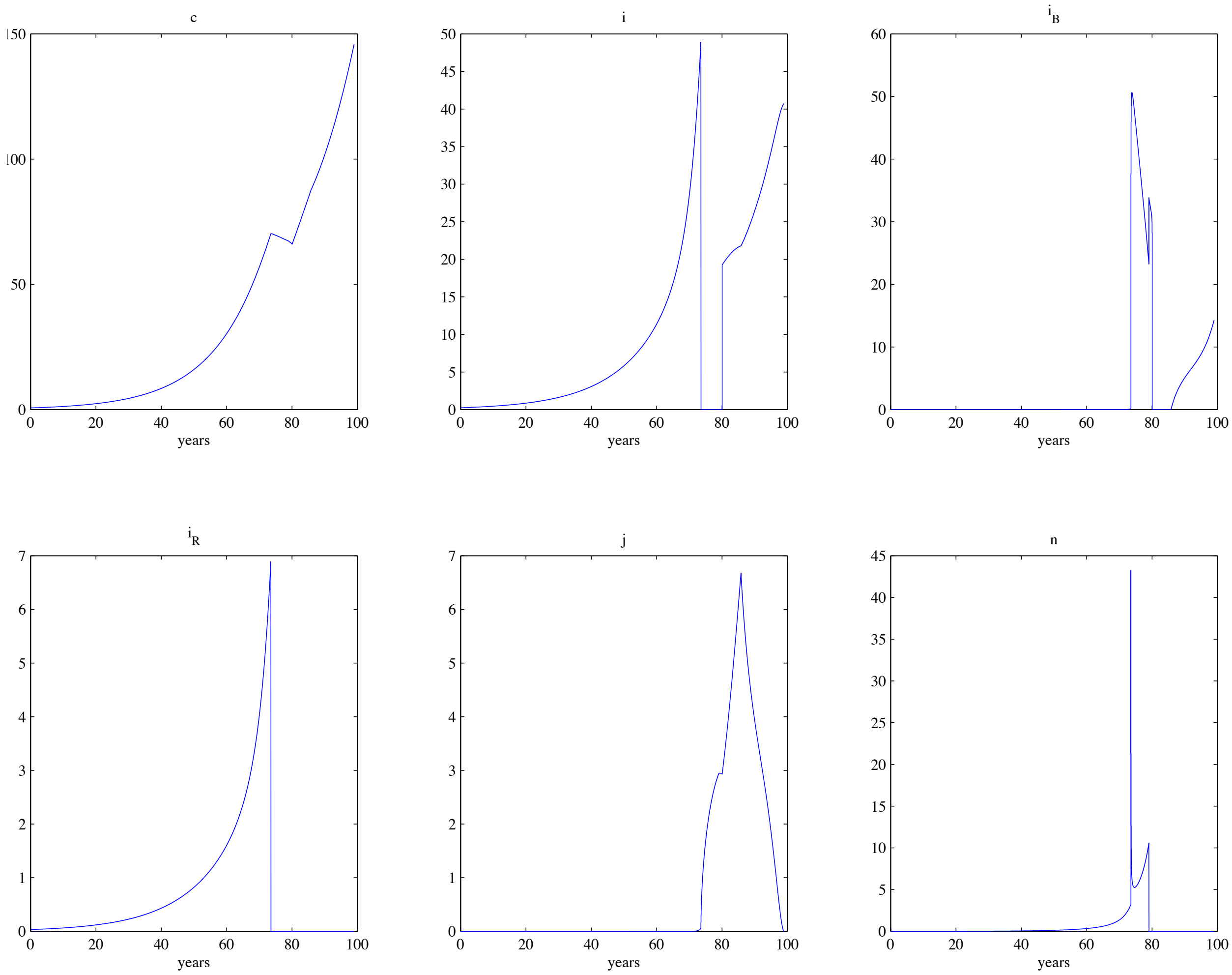

Figure 5: Calculated paths of the control variables

and the need to spend on energy takes resources away from consumption and investment. Around the same time, the growth in energy R&D in the form of increases in both n and j also reduces the resources available for consumption and investment in k.

Figure 6 graphs the solution paths for the main[48] co-state variables. The shadow price ν of investing in N is equal to λ while $n > 0$ (that is, between $t = 0$ and T_N) after which point it declines quickly to zero and remains there. The shadow price σ of the fossil fuel resource mined to date S is negative until fossil fuels are abandoned at T_R, at which point σ becomes zero and remains at zero thereafter. The negative shadow price reflects that assumption that increased mining raises future costs. The real (or utility) value of the shadow price σ/λ declines continuously (increases in absolute value) until T_N, after which the increase to zero is swift. Finally, the shadow price η

[48]A graph of the small interval over which q_B differs from λ has been omitted. Also, $q_R = \lambda$ until T_Q, at which point it declines to zero and remains at zero thereafter.

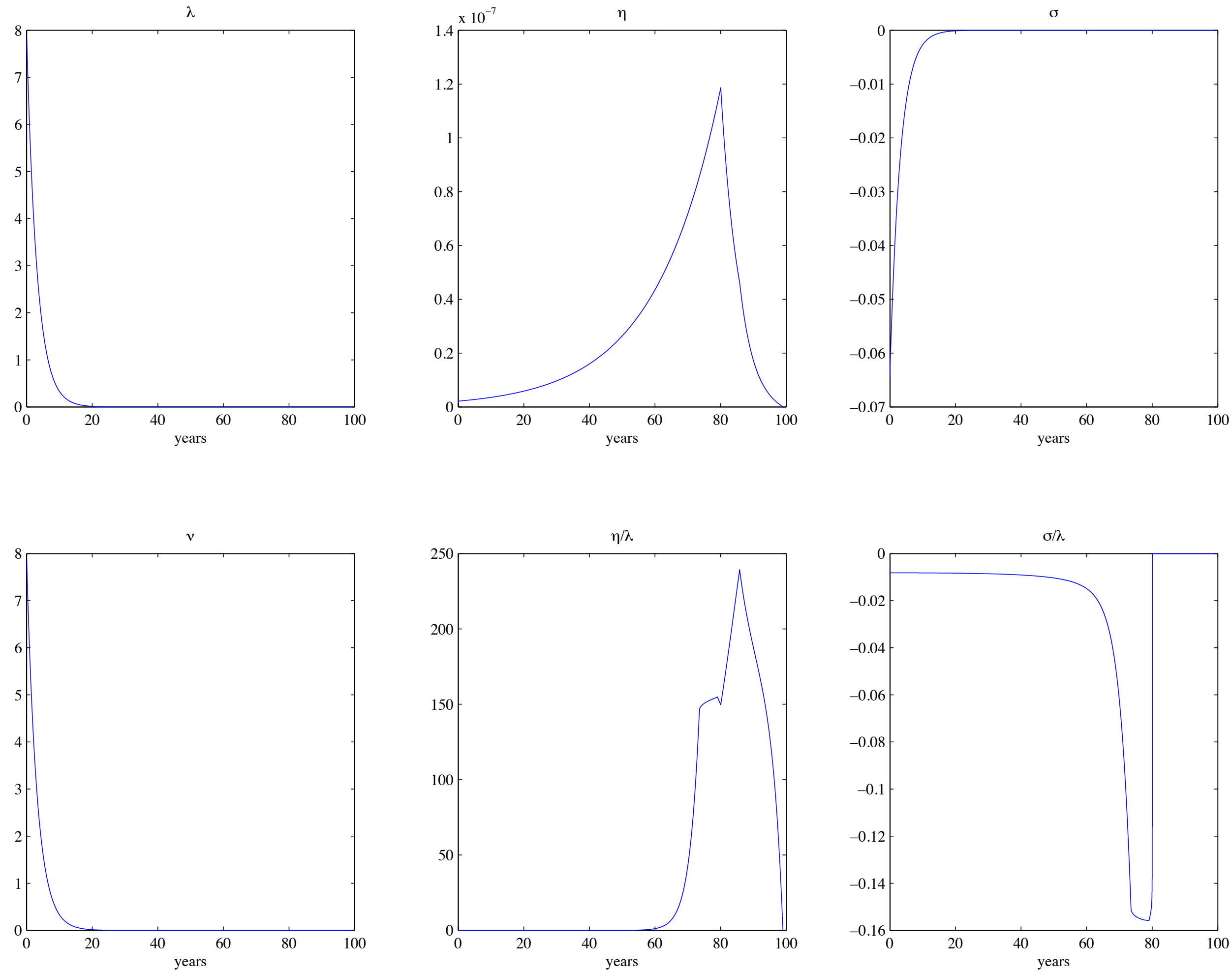

Figure 6: Calculated paths of the co-state variables

of H increases to a peak at T_R and then declines to zero at T_H when additional gains in H are no longer possible. This implies that the incentive to invest in H follows the same pattern. The ratio η/λ looks much more similar to the graph of investment j in H. However, it does start to increase earlier than j (after 60 years) and the "bump" immediately prior to T_R is longer lasting and more noticeable in η/λ than in j.

7 Concluding remarks

We have presented an intertemporal optimizing growth model where energy inputs essential to final production can be supplied by either fossil fuels or renewable sources. Initially, the fossil fuels are much cheaper and supply all the energy. However, the fossil fuels are eventually displaced (at T_R) as depletion raises their cost, while R&D and learning-by-doing allows the cost of renewable sources to decline over time. Although technical progress in fossil energy production can offset the cost increases from depletion and delay the displacement of fossil fuels by renewables, it cannot forestall the transition indefinitely.

Physical capital is needed to turn fossil fuel resources into useful energy services or to "harvest" the non-fossil energy sources (such as sunlight, wind, or geothermal resources) and make them available as energy that can be used in final production. We assume that capital used to provide energy services from fossil fuels k_R cannot be re-purposed as k_B to be used for renewable energy. An important consequence is that the economy has to invest in k_B prior to T_R so that renewables are ready to take over from fossil fuels at T_R.

For $t \leq T_R$, the cost of fossil fuels sets the price of energy, which thus is rising for some time from $t < T_R$ until T_R. Also, the short-run cost of renewable energy, which exceeds the price of energy at $t = 0$, is falling over time as a result of R&D investment and learning-by-doing. At T_R, the short-run cost of producing energy from the two sources is equal. Thus, prior to T_R, the short-run cost of renewable energy exceeds the price of energy.

For most of the time prior to T_R, there is virtually no investment in k_B. The gains from learning-by-doing imply that some investment in k_B and renewable R&D is optimal in order to hasten the transition and lower the costs of energy supply from renewables at the transition. Furthermore, for a few years immediately before the transition, a burst of investment in renewable capacity is needed to ensure continuity in the supply of energy services. At the same time, however, k_B is not used to supply energy services until the transition occurs. Prior to the transition, the energy price is not sufficient to cover even the operating cost of k_B.

In the numerical example, renewable sources take over from fossil fuels after about 80 years, when slightly over 77% of the initial stock of fossil fuel resources have been exploited. Investment in k_B begins in earnest after about 74 years. Energy prices are not sufficient to cover the full costs of producing renewable energy (including a competitive return on capital) until after almost 86 years. While these specific transition dates depend on the chosen parameter values, the qualitative characteristics of the solution were shown to hold much more generally.

We emphasize that the investment paths calculated in the model are part of the efficient outcome. Whether all the benefits of such investments would be fully appropriable to private entrepreneurs and allow a competitive equilibrium to support the Pareto optimum is, of course, an entirely different matter. Our model may in this sense complement much of the literature explaining the difficulty of establishing new energy technologies as akin to transitioning a "valley of death." That literature has focused on the inability of entrepreneurs to appropriate all of the benefits of early research into alternative energy technologies. Our model has added the additional point that, unlike the pharmaceutical and IT industries, the energy industry requires substantial capital investment beyond the R&D phase in order to commercialize the new technologies. These investments will be difficult to finance while the capital invested in existing technologies is a sunk cost and sets a low energy price as the old technology is phased out. Nevertheless, in the model presented, the subsidies to renewable energy in the initial phase ought not extend to subsidizing energy production on a commercial scale.

8 Appendix: The differential equations applying in each regime

We work through the regimes backwards in time. Figure 3, showing the time line and paths of investment and energy production, may make this discussion easier to follow.

8.1 The long run endogenous growth economy

Beyond T_H, H is constant at $\bar{H}$. The control variables are c, i and i_B, while the state variables are k and k_B. In this regime, the resource constraint simplifies to

$$c + i + i_B + mk_B = Ak \tag{37}$$

while the energy market equilibrium condition becomes

$$Fk = \bar{H}k_B \tag{38}$$

Differentiating (38) and using the assumption that the depreciation rates are identical, we obtain

$$Fi = \bar{H}i_B \tag{39}$$

With both $i, i_B > 0$, (15) and (17) imply $q = \lambda = q_B$. Noting also that $j = 0$ and $\rho_B = 1$, the co-state equations for q and q_B in this regime then imply

$$\dot{\lambda} = (\beta + \delta)\lambda - \lambda A + p_e F = (\beta + \delta)\lambda + \lambda m - p_e \bar{H} \tag{40}$$

In particular, the price of energy is constant at

$$p_e = \frac{A+m}{\bar{H}+F}\lambda \tag{41}$$

while λ satisfies the differential equation

$$\frac{\dot{\lambda}}{\lambda} = \beta + \delta - \frac{A\bar{H} - mF}{\bar{H}+F} \equiv -\bar{A} \tag{42}$$

where $\bar{A}$ is a constant. To get perpetual growth, we must have $c \to \infty$ as $t \to \infty$, which from (12) will require $\lambda \to 0$ and hence $\bar{A} > 0$, that is

$$A > \beta + \delta + \frac{F}{\bar{H}}(\beta + \delta + m) \tag{43}$$

The solution to (42) can be written

$$\lambda = \bar{K}e^{-\bar{A}t} \tag{44}$$

for a constant $\bar{K}$. Using the differential equation for k, (44) and the first order condition (12) for c, resource constraint (37), the constraint on investment (39) and the definition of $\bar{A}$ in (42) we get

$$\dot{k} = (\bar{A}+\beta)k - \frac{\bar{H}\bar{K}^{-1/\gamma}}{\bar{H}+F}e^{\bar{A}t/\gamma} \tag{45}$$

The integrating factor for the differential equation (45) is $e^{-(\bar{A}+\beta)t}$, so the solution can be written

$$k = C_0 e^{(\bar{A}+\beta)t} + \frac{\bar{H}\gamma\bar{K}^{-1/\gamma}}{(\bar{H}+F)[\beta\gamma + \bar{A}(\gamma-1)]}e^{\bar{A}t/\gamma} \tag{46}$$

for another constant C_0. However, the transversality condition requires

$$\lim_{t\to\infty} e^{-\beta t}\lambda k = C_0\bar{K} + \lim_{t\to\infty}\frac{\bar{H}\gamma\bar{K}^{(1-1/\gamma)}}{(\bar{H}+F)[\beta\gamma + \bar{A}(\gamma-1)]}e^{(\bar{A}/\gamma - \bar{A} - \beta)t} = 0 \tag{47}$$

Equation (47) in turn requires

$$\lim_{t\to\infty} e^{(\bar{A}/\gamma - \bar{A} - \beta)t} = 0, \text{ that is, } \bar{A}(1-\gamma) < \beta\gamma \tag{48}$$

and also $C_0 = 0$.[49] Thus, the value of k in the final endogenous growth economy will be given by

$$k = \frac{\bar{H}\gamma\bar{K}^{-1/\gamma}e^{\bar{A}t/\gamma}}{(\bar{H}+F)[\beta\gamma+\bar{A}(\gamma-1)]} \tag{49}$$

with λ (and also q and q_B) given by (44) and where $\bar{K}$ is a constant yet to be determined.

From (38) and (49), the capital stock allocated to renewable energy production in the final endogenous growth economy will be

$$k_B = \frac{F\gamma\bar{K}^{-1/\gamma}e^{\bar{A}t/\gamma}}{(\bar{H}+F)[\beta\gamma+\bar{A}(\gamma-1)]} \tag{50}$$

The growth rate of the final regime will be[50]

$$\frac{\bar{A}}{\gamma} = \frac{1}{\gamma}\left[\frac{A\bar{H}-mF}{\bar{H}+F} - (\beta+\delta)\right] \tag{51}$$

Working backwards in time, the beginning of the final analytical regime at T_H occurs when H attains $\bar{H}$ and $\eta = 0$ (which in turn implies $Y = 0$). The values k, k_B, λ and φ at T_H must match the values at the end of regime 5 since these variables must be continuous across the boundary.

8.2 Regime 5: Fully dynamic renewable regime

Regime 5 will have direct investment in renewable energy production efficiency ($j > 0$) in addition to end-use capital ($i > 0$) and renewable energy production capital ($i_B > 0$). The solution for j from (19) implies technological progress in renewable energy production will satisfy

$$\dot{H} = bk_B^{s\psi}Y^{\alpha-\psi} \tag{52}$$

where the function Y and the constant s were defined in the text.

Using the solutions for c from (12) and j from (19), and $\rho_B = 1$, the resource constraint implies

$$i + i_B = Ak - Yk_B^{s\psi} - mk_B - \lambda^{-1/\gamma} \tag{53}$$

[49]Since $\bar{A} > 0$, the inequality in (48) will be satisfied if $\gamma > 1$, as we assume. If $0 < \gamma < 1$, the inequality in (48) would require $A < \frac{\beta}{1-\gamma} + \delta + \frac{F}{H}(\frac{\beta}{1-\gamma} + \delta + m)$ which would further limit the range of acceptable parameter values.

[50]If there were no need for energy input, the growth rate would be $[A - (\beta+\delta)]/\gamma$. Thus, the need for energy reduces the *gross* productivity of capital A by an amount that depends on the marginal cost of producing energy m.

and the energy market equilibrium condition is

$$Fk = Hk_B \tag{54}$$

Differentiating (54), and using (52), (54), and (19), and the differential equations (2) and (3), we obtain a second equation linking i and i_B

$$Fi - Hi_B = bk_B^{s\psi+1} Y^{\alpha-\psi} \tag{55}$$

Equations (55) and (53) then give us two equations to solve for i and i_B, as illustrated in Figure 7. The differential equations (2) and (3) then yield $\dot{k}$ and $\dot{k}_B$.

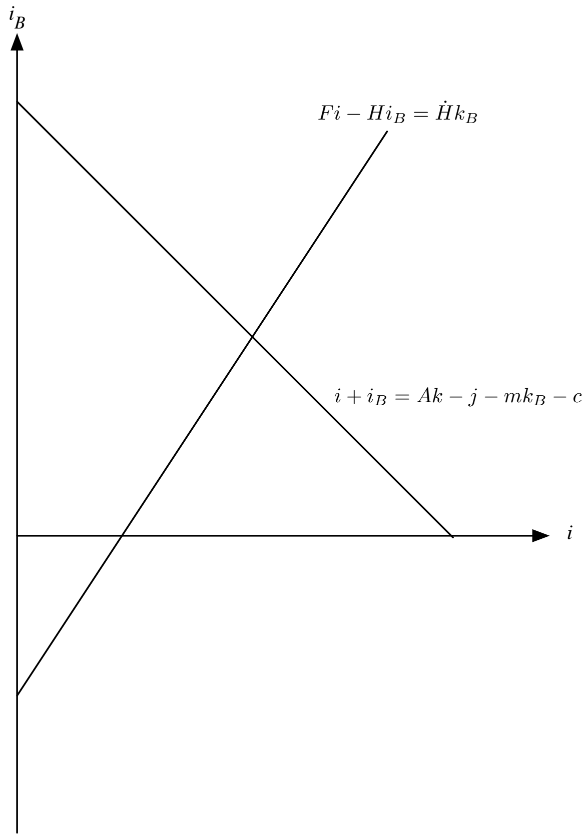

Figure 7: Solving for investments in regime 5

We expect that the incentive to invest in H will tend to decrease over time as $H \uparrow \bar{H}$. Solving backwards in time in regime 5, we therefore expect $\dot{H}/H$ to increase, which will shift the upward sloping line to the right. On the other hand, as we move backwards through time, the resources available to support both investments will tend to decrease, shifting the downward sloping line to the left. Thus, as we solve backwards in time, i_B is likely to decline rapidly and, as argued in the text, the constraint $i_B \geq 0$ will bind (and we enter regime 4) before T_R is reached.

We will again have $q = \lambda = q_B$ and hence we obtain distinct co-state differential equations only for λ and η. Noting that the price of energy p_e will be given by the long run cost of renewable supply (29) and $\rho_B = 1$, the co-state differential equations are $\dot{\eta} = \beta\eta - p_e k_B$ and $\dot{\lambda} = (\beta + \delta - A)\lambda + p_e F$.

8.3 Regime 4: No investment in renewable capacity

Regime 4 involves full use of renewable capacity ($\rho_B = 1$) but no investment in additional capacity ($i_B = 0$ and hence $\dot{k}_B = -\delta k_B$). The lower boundary of regime 4 will be T_R where energy production shifts out of fossil fuels into renewables.

Using the solutions for c from (12) and j from (19), the resource constraint implies:

$$i = Ak - Yk_B^{s\psi} - mk_B - \lambda^{-1/\gamma} \tag{56}$$

However, as in regime 5, the energy market equilibrium condition will also determine a value for i:

$$Fi = bk_B^{s\psi+1}Y^{\alpha-\psi} \tag{57}$$

Equating the two expressions for i from (56) and (57) we obtain

$$bk_B^{s\psi+1}Y^{\alpha-\psi} + FYk_B^{s\psi} - FAk + mFk_B + \lambda^{-1/\gamma}F = 0 \tag{58}$$

Before differentiating (58), we use (20) and (25) to obtain the derivative of η/λ:

$$\frac{d}{dt}\left(\frac{\eta}{\lambda}\right) = -\frac{p_e}{\lambda}\left[k_B + F\frac{\eta}{\lambda}\right] + (A - \delta)\frac{\eta}{\lambda} \tag{59}$$

Also, using the definition of Y, we obtain:

$$\dot{Y} = (s-1)bY^{\alpha-\psi}\frac{d}{dt}\left(\frac{\eta}{\lambda}\right) \tag{60}$$

while we can write the derivative of λ in terms of p_e as

$$\dot{\lambda} = (\beta + \delta - A)\lambda + p_e F \tag{61}$$

The derivative of (58) can then be written in terms of these expressions as:

$$k_B^{s\psi}\left[F\dot{Y} - s\psi\delta FY - b\delta(s\psi+1)k_B Y^{\alpha-\psi} + k_B\frac{\lambda}{\eta}\dot{Y}\right] + F\left[A\delta k - Ai - m\delta k_B - \frac{1}{\gamma}\lambda^{-\frac{1+\gamma}{\gamma}}\dot{\lambda}\right] = 0 \quad (62)$$

which can then be solved for an energy price p_e that will ensure the two expressions for i are equal. Investment i will be given by either (56) or (57). The co-state variable η will evolve according to (23), and q_B will evolve according to (22), with $\rho_B = 1$ in both cases. The co-state q_B will have initial value $q_B = \lambda$ at T_B.

At the lower boundary T_R of regime 4, $q = \lambda$ and $\dot{q} = \dot{\lambda}$ will be given by (20) as also is true for all $t > T_R$. At the same time, $i_B > 0$ in regime 3 and (17) imply $q_B = \lambda$ and $\dot{q}_B = \dot{\lambda}$. Also, $\rho_B = 0$ in regime 3, as it does for all $t < T_R$. Then from (22), $\dot{\lambda}$ will evolve in regime 3 according to

$$\frac{\dot{\lambda}}{\lambda} = \beta + \delta - \frac{\psi}{\alpha-\psi}k_B^{(\alpha-1)s}Y \quad (63)$$

The two expressions (61) and (63) for $\dot{\lambda}$ at T_R imply that p_e will equal the *alternative* long-run cost of renewable energy (30). Furthermore, ρ_R jumps from one to zero, while ρ_B jumps from zero to one, at T_R. Then since (13) implies $\theta_{RL} = \theta_{RU} = 0$ and (14) implies $\theta_{BL} = \theta_{BU} = 0$, p_e also must equal the two short-run costs of energy production, (28) and (26), at T_R. Using the fact that σ converges to zero at T_R, we therefore must have

$$\frac{p_e}{\lambda} = \frac{\mu}{G} = \frac{m}{H} \quad (64)$$

Equation (64) can be used to determine not only T_R but also the value of S at T_R once $N(T_R) = \bar{N}$ has been specified. Also, given that k_B and H are known at T_R when solving backwards in time, energy market equilibrium will determine a limiting value for k_R at T_R:

$$k_R(T_R) = \frac{H(T_R)}{G}k_B(T_R) \quad (65)$$

8.4 Regime 3: Only fossil fuels used, $i_R = n = 0$

In this regime, only fossil fuels are used to produce energy ($\rho_R = 1, \rho_B = 0$). However, we have $i_R = 0$, so k_R declines according to $\dot{k}_R = -\delta k_R$. We also have $n = 0$, so N remains fixed at $\bar{N}$.

Using the solutions for c and j, the resource constraint can be written

$$i + i_B = Ak - Yk_B^{s\psi} - \mu k_R - \lambda^{-1/\gamma} \tag{66}$$

The energy market equilibrium condition now becomes $Fk = Gk_R$. However, since $i_R = 0$ and F and G are constant, differentiation implies we now must also have $i = 0$. Then (66) implies $i_B = Ak - Yk_B^{s\psi} - \mu k_R - \lambda^{-1/\gamma}$. In addition, since $i = 0$ we cannot conclude that $\lambda = q$ in this regime. However, from (17) and $i_B > 0$, we will have $q_B = \lambda$ and hence from (22), $\rho_B = 0$ and (19), λ will evolve according to (63).

Throughout this regime (and also in regime 2) the demand for energy Fk exactly matches the available capacity for fossil fuel energy production Gk_R and p_e will be given by the short-run cost of fossil fuel (26). The co-state variables q_R, ν, σ and η will evolve according to (21), (23), (24) and (25) with p_e given by (26), $\rho_R = 1$ and $\rho_B = 0$. In particular, throughout regime 3 (and also regime 2), $\dot{q}_R = (\beta + \delta)q_R$, and since $q_R = 0$ at T_R we must have $q_R = 0$ throughout regime 3 (and also regime 2). In addition, for all $t < T_R$, η will evolve according to

$$\dot{\eta} = \beta\eta \tag{67}$$

Thus, η will be strictly greater than zero and increasing exponentially for $t < T_R$, while for $t > T_R$ it must ultimately decrease to zero at T_H. In other words, the incentive to invest in renewable energy efficiency improvements increases as $t \to T_R$, but ultimately must decline to zero as $t \to T_H$. Similarly, (24) with $\rho_R = 1$ implies

$$\dot{\sigma} = \beta\sigma + \lambda k_R \frac{\partial \mu}{\partial S} \tag{68}$$

while (23) with $\rho_R = 1$ implies

$$\dot{\nu} = \beta\nu + \lambda k_R \frac{\partial \mu}{\partial N} \tag{69}$$

The lower boundary T_N of regime 3 will be where $\nu = \lambda$.

8.5 Regime 2: Only fossil fuels used, investment in N but not k_R

For $T \in [T_Q, T_N]$, we again have $\rho_R = 1$ and $\rho_B = 0$. We also still have $i_R = 0$, so k_R again declines according to $\dot{k}_R = -\delta k_R$. As in regime 3, energy market equilibrium will then imply that $i = 0$ and $\dot{k} = -\delta k$. However, we have positive investments i_B, j and n.

Using the solutions for c and j, and the conclusion $i = 0 = i_R$, the resource constraint implies

$$i_B + n = Ak - Yk_B^{s\psi} - \mu k_R - \lambda^{-1/\gamma} \tag{70}$$

Since $n > 0$ for all $t \leq T_N$, we also have $\nu = \lambda$ and hence $\dot{\nu} = \dot{\lambda}$. From $i_B > 0$ and (17), $\lambda = q_B$, and using $\rho_B = 0$ and (22), we again deduce that λ evolves according to (63). Then, using also (23) and $\rho_R = 1$, we obtain:

$$k_R \frac{\partial \mu}{\partial N} = \delta - \frac{\psi}{\alpha - \psi} k_B^{(\alpha-1)s} Y \tag{71}$$

Then (noting that $(\alpha - 1)s = \psi s - 1$ and $i_R = 0$) the derivative of (71) can be written as:

$$\begin{aligned} &-\delta \frac{\partial \mu}{\partial N} k_R + k_R \frac{\partial^2 \mu}{\partial N^2} n + Q k_R^2 \frac{\partial^2 \mu}{\partial S \partial N} + \\ &\frac{\psi s k_B^{\psi s - 2}}{\alpha - \psi} \left[(\alpha - 1) Y (i_B - \delta k_B) + (\alpha - \psi) b k_B Y^{\alpha - \psi} \frac{d}{dt} \left(\frac{\eta}{\lambda} \right) \right] = 0 \end{aligned} \tag{72}$$

where (25), $\rho_B = 0$, (63) and (71) yield the derivative of η/λ in this regime:

$$\frac{d}{dt}\left(\frac{\eta}{\lambda}\right) = -\frac{\eta}{\lambda} \frac{\partial \mu}{\partial N} k_R \tag{73}$$

The two equations (70) and (72) can then be solved for the two investments i_B and n.

Using $\nu = \lambda$ we find that $\dot{\lambda}/\lambda$ will now satisfy a much simpler equation

$$\frac{\dot{\lambda}}{\lambda} = \beta + k_R \frac{\partial \mu}{\partial N} \tag{74}$$

The differential equations for the co-state variables η and σ will be (67) and (68) as in regime 3.

Finally, since p_e is given by (26) as in regime 3, $\dot{q}R = (\beta + \delta) q_R$ and hence q_R again remains zero throughout the regime. However, at the lower boundary of regime 2, we must have $q_R = \lambda > 0$ since $i_R > 0$ throughout regime 1 and falls to zero only at T_Q. The co-state variable q_R must therefore be left continuous and differentiable for all $t < T_Q$, but jump discontinuously to zero at T_Q when investment in k_R ceases. The lower boundary T_Q of regime 2 cannot be calculated endogenously and becomes an additional value that has to be set to attain the initial conditions.

8.6 Regime 1: Investment in both k_R and k_B but only fossil fuel is used

Recall that we assume that renewable capital is not used to produce any energy at $t = 0$ ($\rho_B = 0$). All energy investments i_R, i_B, n and j are, however, positive.

Using the solutions for c and j, the resource constraint can now be written

$$i + i_B + i_R + n = Ak - \mu k_R - Y k_B^{s\psi} - \lambda^{-1/\gamma} \tag{75}$$

Once again, the energy market equilibrium condition can be differentiated to yield

$$Fi - Gi_R = 0 \tag{76}$$

and hence $i = Gi_R/F$. A third equation involving the investments can again be obtained from (71). Since $i_R > 0$, however, (72) is modified to an equation involving i_R, i_B and n:

$$\begin{aligned} &\frac{\partial \mu}{\partial N} i_R - \delta \frac{\partial \mu}{\partial N} k_R + k_R \frac{\partial^2 \mu}{\partial N^2} n + Q k_R^2 \frac{\partial^2 \mu}{\partial S \partial N} + \\ &\frac{\psi s k_B^{\psi s - 2}}{\alpha - \psi} \left[(\alpha - 1) Y (i_B - \delta k_B) + (\alpha - \psi) b k_B Y^{\alpha - \psi} \frac{d}{dt} \left(\frac{\eta}{\lambda}\right) \right] = 0 \end{aligned} \tag{77}$$

where the derivative of η/λ is once again given by (73). The fourth equation involving investments arises from the fact that with both $i_R, i_B > 0$ the p_e has to equal both (27) and (30):

$$\mu - \frac{\sigma}{\lambda} Q - \frac{AG}{F} + \frac{\psi k_B^{(\alpha-1)s} Y (F + G)}{(\alpha - \psi) F} = 0 \tag{78}$$

Then differentiating (78) we obtain:

$$\begin{aligned} &\frac{\partial \mu}{\partial N} n + \frac{\sigma Q}{\lambda} \left[\frac{\partial \mu}{\partial N} k_R - \pi \right] + \\ &\frac{\psi s (F + G) k_B^{\psi s - 2}}{(\alpha - \psi) F} \left[(\alpha - 1) Y (i_B - \delta k_B) + (\alpha - \psi) b k_B Y^{\alpha - \psi} \frac{d}{dt} \left(\frac{\eta}{\lambda}\right) \right] = 0 \end{aligned} \tag{79}$$

where we have used $\nu = \lambda$ and thus (23) and (24) with $\rho_R = 1$ to obtain:

$$\frac{d}{dt} \left(\frac{\sigma}{\lambda}\right) = \left[\frac{\partial \mu}{\partial S} - \frac{\sigma}{\lambda} \frac{\partial \mu}{\partial N} \right] k_R \tag{80}$$

The four equations (75), (76), (77) and (79) can then be solved for i, i_R, i_B and n.

The differential equations governing the evolution of the co-state variables will again have

$\rho_R = 1$, $\rho_B = 0$ and p_e given by (30). In particular, $\dot{\lambda}/\lambda$ (with $q_R = q_B = \lambda$) will satisfy the simpler equation (74), while $\dot{\eta}$ and $\dot{\sigma}$ will again satisfy (67) and (68) respectively.

8.7 Initial and terminal conditions

At $t = 0$, there are three initial conditions for the physical capital stocks $k(0)$, $k_R(0)$ and $k_B(0)$, an initial value for renewable energy productivity $H(0)$ and, by definition, the initial values of S and N should be zero. However, active investment in N, k_R and k_B imposes two constraints on the relative magnitudes of these state variables, leaving only four independent targets. We will take these to be $k(0), H(0)$ and $N(0) = S(0) = 0$. Thus, we need to set four initial values for the differential equations that can then be varied to ensure that we hit these targets. The solution in the final analytical regime depends on an unknown constant $\bar{K}$, while we also need to specify values for T_H, N at T_R, and the time T_Q when investment in fossil fuel capital k_R ceases.

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