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by

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The Home Selling Problem: Theory and Evidence†

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Abstract

This paper formulates and solves the problem of a homeowner who wants to sell her house for the maximum possible price net of transactions costs (including real estate commissions). The optimal selling strategy consists of an initial list price with subsequent weekly decisions on how much to adjust the list price until the home is sold or withdrawn from the market. The solution also yields a sequence of reservation prices that determine whether the homeowner should accept offers from potential buyers who arrive stochastically over time with an expected arrival rate that is a decreasing function of the list price. We estimate the model using a rich data set of complete transaction histories for 780 residential properties in England introduced by Merlo and Ortalo-Magné (2004). For each home in the sample, the data include all listing price changes and all offers made on the home between initial listing and the final sale agreement. The estimated model fits observed list price dynamics and other key features of the data well. In particular, we show that a very small "menu cost" of changing the listing price (estimated to equal 10 thousandths of 1% of the house value, or approximately £10 for a home worth £100,000), is sufficient to explain the high degree of "stickiness" of listing prices observed in the data.

Keywords: housing, bargaining, sticky prices, optimal selling strategy, dynamic programming.
JEL classification: H5

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1 Introduction

Buying and selling a home is one of the most important financial decisions most individuals make during their lifetime. Home equity is typically the biggest single component of the overall wealth of a household, and given the highly leveraged situation that most households are in (where mortgage debt is a high fraction of the overall value of the home), the outcome of the home selling process can have very serious consequences for their financial well-being.

Given its importance, we would expect *a priori* that households have strong incentives to be forward-looking and behave rationally when they sell their home. In particular, it seems reasonable to model the household's objective as trying to maximize the expected gains from selling their home net of transactions costs.

Surprisingly, dynamic rational models of the "home selling problem" have been understudied both theoretically and, most notably, empirically. In pioneering work, Salant (1991) formulated and solved for the optimal selling strategy of a risk neutral seller using dynamic programming. Salant's model involves an initial choice by the household whether to use a real estate agent to help sell their home, versus deciding to save on the high commissions charged by most real estate agencies and follow a "for sale by owner" selling strategy. Under either of these options, the seller must also choose a list price each period the home is up for sale, and whether to accept an offer for the home when one arrives, or to wait and hope that a higher offer will arrive in the near future. Salant showed that the optimal solution generally involves a strictly monotonically declining sequence of list prices, and that it is typically optimal to begin selling the home by owner, but if no acceptable offers have arrived within a specified interval of time, the seller should retain a real estate agent. Under some circumstances, the optimal list price can jump up at the time the seller switches to the real estate agency, but list prices decline thereafter. To our knowledge, the implications of Salant's theoretical analysis have not been investigated empirically.

Horowitz (1992) was the first attempt to empirically estimate a dynamic model of the home seller's problem. Unlike Salant, who considered an environment with a finite horizon, Horowitz adopted an infinite-horizon stationary search framework, and characterized the optimal (time-invariant) list and reservation prices of the seller. Horowitz's model implies that the duration to sale of a house is geometrically distributed, and he estimated his model using data on the list price, sale price and duration to sale for a sample of 1196 homes sold in Baltimore, Maryland in 1978.

Horowitz concluded that his econometric model "gives predictions of sale prices that are considerably more accurate than those of a standard hedonic price regression" (p. 126). He also noted that his model "explains why sellers may not be willing to reduce their list prices even after their houses have remained unsold for long periods of time" (p.126). The latter conclusion, however, is unwarranted because time invariance of list and reservation prices are inherent features of Horowitz's stationary search framework. Hence, his model is logically incapable of addressing the issue of what is the optimal sequence of list price choices by a seller over time (and in particular whether list prices should decline or remain constant over time). Further, his data set does not appear to contain any information on changes in the list price between when a home was initially listed and when it was finally sold.[1]

It seems that the question of whether optimal list prices should or should not decline over time can only be addressed in a non-stationary, finite-horizon framework such as Salant's, or else in a stationary infinite-horizon framework that includes variables such as duration since initial listing, or duration since previous offer, as state variables.[2] Also, it is quite evident that any progress in the specification and estimation of plausible dynamic models of the home selling problem critically hinges on the availability of richer micro data containing detailed information on the history of relevant events (e.g., list price revisions and offers received) during the home selling process.

The model presented in this paper is motivated by the empirical findings of Merlo and Ortalo-Magné (2004), who introduced a novel data set that to our knowledge provides the first opportunity to study the home selling problem in considerable detail. Merlo and Ortalo-Magné's study is based on a panel data of complete transaction histories of 780 residential properties that were sold via a real estate agency in England between June 1995 and April 1998. For each home in the sample, the data include all listing price changes and all offers made on the home between initial listing and the final sale agreement (i.e., the data include all rejected offers on each home, if any, as well as the accepted offer that lead to its sale). Merlo and Ortalo-Magné characterized a number of key stylized facts pertaining to the sequence of events that occur within individual property transaction histories, and discussed the limitations of existing theories of a home seller's behavior in explaining the data.

The dynamic model of the home selling problem we propose and estimate takes advantage of the

[1] Also note that Horowitz's estimated model explains little of the observed variation in time from listing to sale. Carrillo (2012) specified and estimated an equilibrium search model of the housing market which fits the observed variation in time from listing to sale well. However, Carillo's model is also stationary and hence unable to explain observed changes in the listing price through time.

[2] However, once one includes a state variable such as duration since initial listing, the seller's problem automatically becomes a non-stationary dynamic programming problem that is essentially equivalent to Salant's formulation.

richness of this data set and incorporates several realistic features of the home selling process into a finite-horizon, dynamic programming model of the behavior of the seller of a residential property. We take the decision to sell a house via a real estate agency as a given, and consider the decisions of which price to list the house at initially, how to revise this price over time, whether or not to accept offers that are received, and whether to withdraw the house if insufficiently attractive offers are realized.[3] To make these decisions the seller forms expectations about the probability a potential buyer will arrive and make an initial offer, the probability she will make additional offers if any of her offers are rejected, and the level of each of these offers. These expectations are revised over time based on the realized event history.

In this paper, we do not explicitly model the behavior of buyers and the bargaining game that leads to the sale of a house. Typically, when a potential buyer arrives and makes an initial offer for the home, it is just the first move in a *bargaining subgame* where the buyer and the seller negotiate over the sale price. This negotiation may either lead to a transaction, when the buyer and seller reach an agreement over the terms of the sale, or end with the buyer leaving the bargaining table when no mutually agreeable deal can be reached. Rather than modeling this situation as a bargaining model with two-sided incomplete information (where the buyer and the seller each possess private information about their own idiosyncratic valuation of the home), we capture the key features of this environment by specifying a simplified model of buyers' bidding behavior. In particular, we assume that if a potential buyer arrives, she makes up to n consecutive offers which are drawn from bids distributions that depend, among other things, on the list price and the amount of time the house has been on the market.[4] The seller can either accept or reject each offer, but after any rejection there is a positive probability the buyer "walks" (i.e. she decides not to make a further offer and move on and search for other properties instead).[5]

[3] One aspect that we do not model in this paper is the seller's decision whether to use a real estate agent, something that was a key focus of Salant's analysis. While we agree that this is a very interesting and important issue, it is one that we cannot say much about empirically, since Merlo and Ortalo-Magné's data set only includes properties that were listed and sold via a real estate agent.

[4] In our empirical work, we assume that $n = 3$, which is the maximum number of offers made by a potential buyer on the same house observed in the data.

[5] As is well known, game-theoretic models of bargaining with two-sided incomplete information typically admit multiple equilibria — and often a continuum of them (e.g., Muthoo (1999)). Furthermore, there are no general results in the literature that characterize the full set of equilibria for such games, and adopting an arbitrary equilibrium selection rule seems a rather unappealing alternative. We avoid these problems by treating buyers as *bidding automata* using simple piecewise linear bidding functions with exogenously specified random termination in the bargaining process. It should be noted, however, that such bidding functions could be derived endogenously in the unique equilibrium of a bargaining game with one-sided incomplete information, where the buyer is uninformed about the seller's valuation, but the buyer's valuation of the house is common knowledge. Our specification also accommodates the possibility of "auctions", i.e., situations where multiple buyers are bidding simultaneously for a home, and offers may exceed the list price.

While treating buyers as *bidding automata* is obviously a simplification, modeling the offer process as one-sided, where the potential buyer makes offers that the seller can either accept or reject without making counteroffers, is not. Contrary to the standard procedure we are accustomed to in the U.S. as well as many other countries, where the owner of a house for sale can typically respond to a buyer's offer with a counteroffer, and there may be multiple real estate agents representing the various parties involved in the sale process, the negotiating protocol that pertains to the residential properties transactions in the Merlo and Ortalo-Magné English data set is quite different. In England, most residential properties are marketed under sole agency agreement (i.e., a house is listed with a single real estate agency that coordinates all market related activities concerning the house from the time it is listed until it either sells or is withdrawn). Agencies represent the seller only, and a potential buyer who wants to make an offer on a property has to communicate the offer in writing to the agency representing the seller of that property. Upon being notified of the offer, the general practice is for the seller simply to either accept the offer or reject it, in which case the buyer has the option of either submitting a revised offer or terminating the negotiation.[6]

Our model incorporates a fixed "menu cost" of changing the list price. One of the most striking features of the Merlo and Ortalo-Magné data is that housing list prices appear to be highly (though not completely) *sticky*. That is, 77% of the home sellers in the data never changed the initial list price between the time the house was initially listed and when it was sold. List prices were changed only once in 18% of the cases, only twice in 4% of the cases, and only three times in the remaining 1% of the cases observed.[7] Merlo and Ortalo-Magné conclude that "listing price reductions are fairly infrequent; when they occur they are typically large. Listing price revisions appear to be triggered by a lack of offers. The size of the reduction in the listing price is larger the longer a property has been on the market" (p. 214). These features of the data are not specific to England. They are also common in housing markets across the U.S. (e.g., Knight (2002)).

This finding presents a challenge, since the conventional wisdom is that traditional, rational, forward-

[6] Another reason for our simplified treatment of buyers is that the English data set we use contains very limited information on the buyers. While the data allow us to follow the decisions of sellers through time, we have no record of the search and bargaining behavior of individual buyers except for the sequence of offers on a single property. In other words, we know the number, timing, and levels of offers made by the same potential buyer on a property, but we do not know whether the same buyer is also making offers on other properties. We believe that our model may provide a reasonably good approximation to a seller's beliefs in a fluid environment where there is a high degree of heterogeneity in potential buyers, and sellers have a great deal of uncertainty about the buyers' motivations and outside options.

[7] None of the homeowners made more than one change in their initial list price during the first eleven weeks on the market, which is the mean duration between initial listing and the sale of the home in the sample.

looking economic theories are unable to explain extreme price stickiness of this sort, unless there are large menu costs associated with price revisions.[8] While list price changes certainly entail a cost for the seller (e.g., in England, all documents pertaining to the listing needs to be updated — analogously, in the U.S., the new price information must be entered in the Multiple Listing Service data base), this cost is unlikely to be large.

One of the primary contributions of this paper is to show that a *very small* menu cost, amounting to 10 thousandths of 1% of the estimated house value, or approximately £10 for a home worth £$100,000$, is sufficient to generate the high degree of list price stickiness observed in the data with a forward-looking dynamic programming model with risk-neutral sellers who have rational expectations about the ultimate selling price of their homes.

There are several reasons why a very small menu cost yields a high degree of list price stickiness in our model. One reason is that our model assumes that sellers have accurate *ex ante* beliefs about the *financial value* of their homes. That is, we assume sellers have *rational expectations* about the future selling price. In the absence of macro shocks or learning about the financial value of the house, the fact that offers from potential buyers fail to arrive (or not) does not have a significant information content that would cause sellers to revise their beliefs and adjust their list price.

A second reason for the price stickiness in our model is that sellers realize that the list price is just a *starting point* for negotiations, and the seller is not committed to selling only at the list price. In general, most offers are less than the list price and subsequent bargaining between the buyer and the seller leads to an increasing sequence of offers until a final transaction price is agreed upon (or the buyer walks away). However, the final transaction price is generally less than the current list price of the home. Thus, most of the real "action" in terms of the realized transaction price occurs during this bargaining process, and the purpose of the list price is mainly to attract potential buyers to the bargaining table. While we do not model the bargaining process explicitly, our empirical framework incorporates the key features of this process, and in particular the fact that when a potential buyer arrives, she may make not just one offer (as it is assumed in the models of Horowitz and Salant alike), but an increasing sequence of offers. Indeed, our estimated model predicts that while list prices are piecewise flat functions of duration on the market (just

[8] For example, Salant's model, which abstracts from menu costs, predicts that list prices should decline monotonically over the period the home is on the market. However, it is well known that the type of non-convexity introduced by a menu cost can generate *regions of inaction* where it is optimal for the seller not to change the list price even though the list price inherited from the previous period is not the optimal forward-looking list price that the seller would choose if there was no cost of changing the list price. The larger the menu cost, the bigger the regions of inaction.

as we observe in the data), the seller's *reservation values* do decline continuously as a function of duration on the market. The combination of the probability of receiving multiple increasing offers from a potential buyer once the potential buyer arrives and declining reservation prices results in significant *actual* price flexibility that is not evident in the list prices.

A final reason for list price stickiness is that while we find that the rate of arrival of offers is a decreasing function of the list price, the estimated relationship between the arrival rate and the list price is fairly inelastic. In effect, it appears that it is a matter of common knowledge that most of the action in terms of determining an actual sale price of a home will occur as a result of a bargaining process, and therefore while we show that the list price is a good predictor of the ultimate transaction price (and indeed, a much more accurate predictor of the transaction price than a hedonic price estimate), once the initial list price is set at the time the house is listed, the apparently highly rational manner in which the initial list price was set largely precludes the need for significant further adjustments over reasonable horizons. Our estimated model predicts only large reductions in the list price for houses that have been on the market for a very long time without having received an acceptable offer, consistent with what we observe in the data.

Our estimated model is also consistent with most of the other key features of the data, including the distributions of times to sale, initial list prices, the overall trajectory of list prices, sale prices and the number of "matches" between a seller and a potential buyer. An interesting finding of our empirical analysis is that houses are generally *overpriced* when they are first listed. In the English housing data the degree of overpricing is not huge: the initial list is on average 5% higher than the ultimate transaction price for the home. However, it is important to point out that our theoretical model could also generate *underpricing* as an optimal seller's behavior. Underpricing can result when the arrival rate of buyers is sufficiently sensitive to the list price, and when there is a significant chance that multiple buyers can arrive at the same time, resulting in an auction situation and potential "bidding war" that tends to drive the final transaction price to a value far higher than the list price.[9]

The rest of the paper is organized as follows. Section 2 describes the data. Section 3 introduces our model of the seller's decision problem. Section 4 describes the simplified model of buyer arrival and

[9] In the data, initial bids and final transaction prices in excess of the list price are observed in approximately 4% of all sales. Our model allows for the possibility of such "overbidding" which results from the fact that in England, the seller has no legal obligation to accept a bid that is greater than or equal to the list price. Previous models, including both Salant's and Horowitz's models, do not allow for the possibility that a bid or transaction price would ever exceed the list price. For a recent theoretical model of the housing market were, in equilibrium, transactions can occur above, below, or at the list price see Albrecht, Gautier and Vroman (2012).

bidding behavior that constitutes the key “belief objects” in the seller’s decision problem that must be estimated to empirically implement our model. Section 5 presents estimation results based on a simulated minimum distance (SMD) estimation method. Section 6 provides some concluding comments and directions for future research.

2 The English Housing Data

This section provides a brief overview of the English housing data analyzed by Merlo and Ortalo-Magné (2004), reviewing the legal environment, the overall housing market, and the way the real estate agency operates in the parts of England where the data were gathered.[10]

In England, most residential properties are marketed under sole agency agreement. This means that a property is listed with a single real estate agency that coordinates all market related activities concerning that property from the time it is listed until it either sells or is withdrawn. Agencies represent the seller only. Listing a property with an agency entails publishing a sheet of property characteristics and a listing price. Although not legally binding, the listing price is generally understood as a price the seller is committed to accept.

The listing price may be revised at any time at the discretion of the seller. If the seller chooses to revise the listing price, she communicates the decision to the agent, who then adjusts the price on the posted property sheet and reprints any property detail sheets in stock with the cost charged to the seller. Hence, changing the listing price only entails a small cost for the seller.

Potential buyers search by visiting local real estate agencies and viewing properties. A match between the seller and a potential buyer occurs when the potential buyer makes an offer. Within a match, the general practice is for the seller to either accept or reject offers. In the event the seller rejects an offer, the potential buyer either makes another offer or walks away. If agreement occurs, both parties engage the administrative procedure leading to the exchange of contracts and the completion of the transaction. This procedure typically lasts three to eight weeks. During this period, among other things, the buyer applies for mortgage and has the property surveyed. Each party may cancel the sale agreement up to the exchange of contracts.

[10] We refer the reader to Merlo and Ortalo-Magné (2004) for a more in depth analysis, but we do lay out here the key features of the data that we attempt to account for in this analysis.

For each property it represents, the agency keeps a file containing a detailed description of the property, its listing price, and a record of listing price changes, offers, and terms of the sale agreement, as required by law. The information contained in each individual file is also recorded on the accounting register that is used by each agency to report to the head office. Although all visits of a property by potential buyers are arranged by the listing agency, recording showings is not required either by the head office or by law. However, individual agencies may require their agents to collect this information for internal management purposes.

The data set we use in our research was obtained from the sales records of four real estate agencies in England. These agencies are all part of Halifax Estate Agencies Limited, one of the largest network of real estate agents in England. Three of these agencies operate in the Greater London metropolitan area, one in South Yorkshire. Our sample consists of 780 complete transaction histories of properties listed and sold between June 1995 and April 1998 under sole agency agreement. Each entry in our data was validated by checking the consistency of the records in the accounting register and in the individual files.

Each observation contains the property's characteristics as shown on the information sheet published by the agency at the time of initial listing, the listing price and the date of the listing. If any listing price change occurs, we observe its date and the new price. Each match is described by the date of the first offer by a potential buyer and the sequence of buyer's offers within the match. When a match is successful, we observe the sale agreed price and the date of agreement which terminate the history. In addition, for the properties listed with one of our Greater London agencies (which account for about a fourth of the observations in our sample), we observe the complete history of showings. Since events are typically recorded by agents within the week of their occurrence, we use the week as our unit of measure of time.

The main features of the data can be summarized as follows. First, listing price reductions are fairly infrequent; when they occur they are typically large. Listing price revisions appear to be triggered by a lack of offers. The size of the reduction in the listing price is larger the longer a property has been on the market. Second, the level of a first offer relative to the listing price at the time the offer is made is lower the longer the property has been on the market, the more the property is currently over-priced, and if there has been no revision of the listing price. Negotiations typically entail several offers. About a third of all negotiations are unsuccessful (i.e., they end in a separation rather than a sale). The probability of success of a negotiation decreases with the number of previous unsuccessful negotiations. Third, in the vast majority of cases, a property is sold to the first potential buyer who makes an offer on the property (i.e., within the first negotiation), although not necessarily at the first offer. The vast majority of sellers whose first negotiation is unsuccessful end up selling at a higher price, but a few end up accepting a lower offer. The higher the number of negotiations between initial listing and sale agreement, the higher the sale price.

Figure 2.1 illustrates two typical observations in the data set. We have plotted list prices over the full duration from initial listing until sale as a ratio of the initial listing price. The dots plot the first offer and the squares are the second offers received in a match. The stars plot the final accepted transaction prices. Thus, the seller of property 1046 in the left hand panel of figure 2.1 experienced 3 separate matches. The first occurred in the fourth week that the property was listed, and the seller rejected the first bid by a bidder equal to 95% of the list price. The buyer "walked" after the seller rejected the offer. The next match occurred on the sixth week on he market. The seller once again rejected this second prospective buyer's first bid, which was only 93% of the list price. However this time the bidder did not walk after this first rejection, but responded with a second higher offer equal to 95% of the list price. However when the seller rejected this second higher offer, the second bidder also walked. The third match occurred in the 11th week the home was on the market. The seller accepted this third bidder's opening offer, equal to 98% of the list price. Note that there were no changes in the initial list price during the 11 weeks this property was on the market.

The right hand panel plots a case where there was a decrease in the list price by 5% in the fourth week this property was on the market. After this price decrease another 5 weeks elapsed before the first offer was made on this home, equal to 90% of the initial list price. The seller rejected this offer and the buyer made a second offer equal to 91% of the initial list price. The seller rejected this second offer too, prompting the buyer to make a final offer equal to 94.5% of the initial list price which the seller accepted.

Figure 2.2 plots the number of observations in the data set and the mean and median list prices as a function of the total number of weeks on the market. The left hand panel plots the number of observations (unsold homes remaining to be sold) as a function of duration since initial listing. For example only 54 of the 780 observations remain unsold after 30 weeks on the market, so over 93% of the properties listed by this agency sell within this time frame. If we compute the ratio of first offers received to the number of remaining unsold properties, we get a crude estimate of the offer arrival rate (a more refined model and estimate of this rate and its dependence on the list price will be presented subsequently). There is an 11% arrival rate in the first week a home is listed, meaning that approximately 11% of all properties will receive

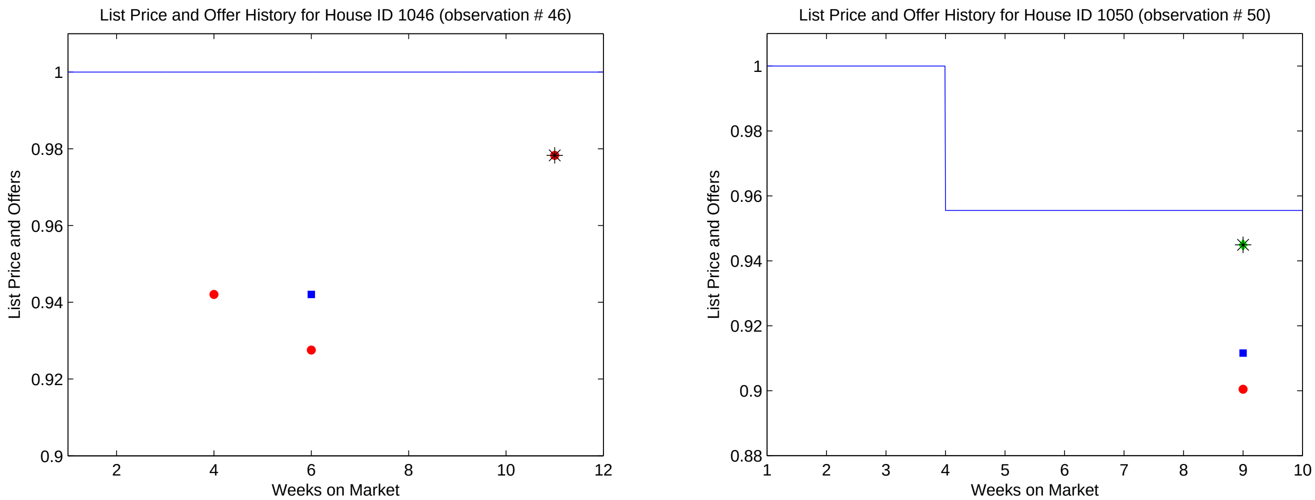

Figure 2.1 Selected Observations from the London Housing Data

one or more offers in the first week after the home is listed with the real estate agency. The arrival rate increases to approximately 15% in weeks 2 to 6, then it decreases to approximately 12% in weeks 7 to 12, and then drops to about 10% thereafter, although it is harder to estimate arrival rates for longer durations given the declining number of remaining unsold properties.

The right hand panel of figure 2.2 plots the mean and median list prices of all unsold homes as a function of the duration on the market. We have normalized the list prices by dividing by the predicted sale price from a hedonic price regression using the extensive set of housing characteristics that are available in the data set (e.g. location of home, square meters of floor space, number of baths, bedrooms, and so forth). However, the results are approximately the same when we normalize using the *actual* transaction prices instead of the regression predictions: this is a consequence of the fact that the hedonic regression provides a very accurate prediction of actual transaction prices.

We see from the right panel of figure 2.2 that initially houses are listed at an average of a 5% premium above their ultimate selling prices, and there is an obvious downward slope in both the mean and median list prices as a function of duration on the market. However the slope is not very pronounced: even after 25 weeks on the market the list price has only declined by 5%, so that at this point list prices are approximately equal to the *ex ante* expected selling prices. The apparently continuously downward slope in mean and median list prices is misleading in the sense that, as we noted from figure 2.1, individual list price trajectories are piecewise flat with discontinuous jumps on the dates where price reductions occur.

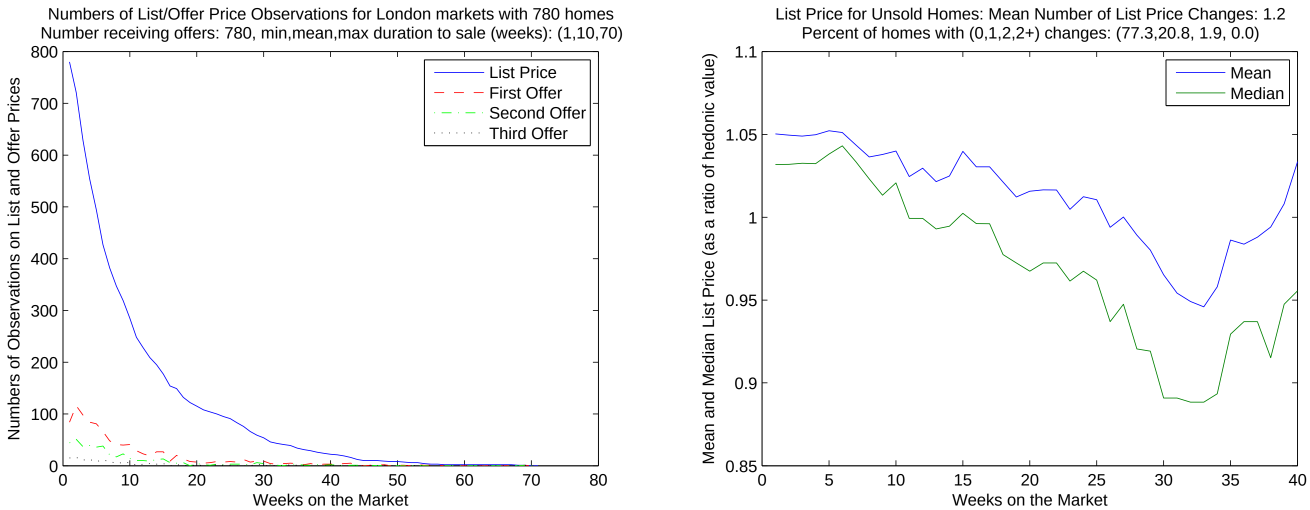

Figure 2.2 Number of Observations and List Prices by Week on Market

Averaging over these piecewise flat list price trajectories creates an illusion that list prices are continuously declining as a function of duration on the market, but we emphasize again that the individual observations do *not* have this property.

Figure 2.3 plots the distribution of sales prices (once again normalized as a ratio to the predicted transaction price) and the distribution of duration to sale. The left hand panel of figure 2.3 plots the distribution of sales price ratios. There are two different distributions shown: the relatively more dispersed one is the distribution of ratios of sale price to the hedonic prediction of sales price, and the relatively more concentrated one is the distribution of the ratio of sales price to the initial list price, multiplied by 1.05 (this latter factor is the average markup of the initial list price over the ultimate transaction price, as noted above). Both of these distributions have a mean value of 1 (by construction), but clearly the distribution of the adjusted sales price to list price ratio is much more tightly concentrated than the distribution of sales price to hedonic value ratios. Evidently there is significant information about the value of the home that affects the seller's decision of what price to list their home at that is not contained in the x variables used to construct the hedonic price predictions. The model we present in section 3 accounts for this extra *private information* about the home that we are unable to observe. However even when this extra information is taken into account, there is still a fair amount of variation/uncertainty in what the ultimate sales price will be, even factoring in the information revealed by the initial list price: the sales price can vary from as low of only 53% of the adjusted list price to 32% higher than the adjusted list price.

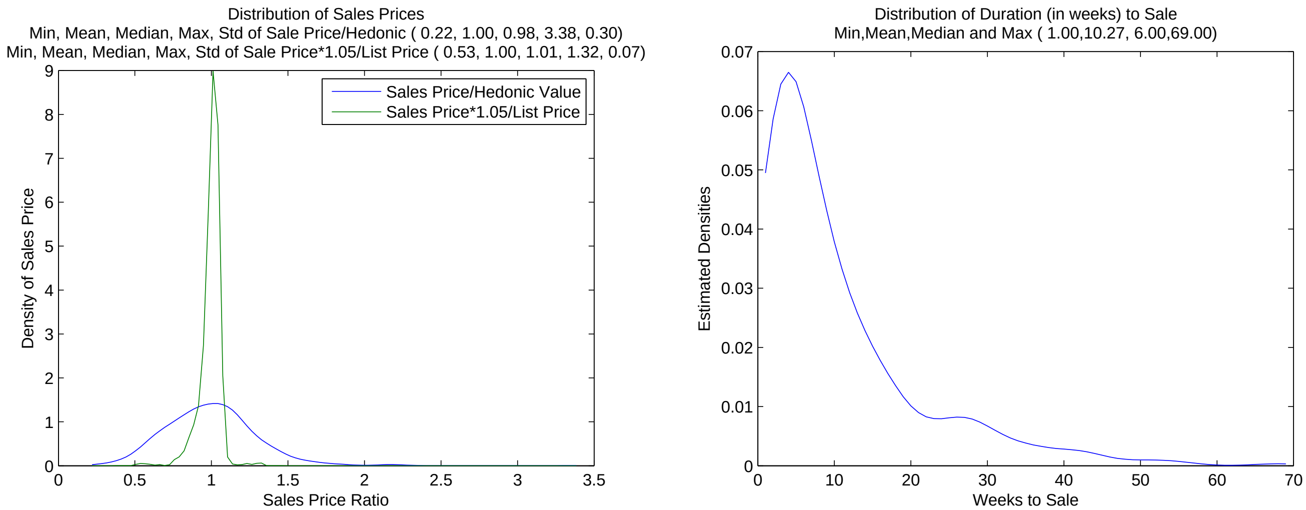

Figure 2.3 Distribution of Sale Prices and Duration to Sale

The right hand panel of figure 2.3 plots the distribution of times to sale. This is a clearly right skewed but unimodal distribution with a mean time to sale of 10.27 weeks and a median time to sale of 6 weeks. As we noted above, over 90% of the properties in our data set were sold within 30 weeks of the date the property was initially listed. Scatterplots relating time to sale to the ratio of the list price to the hedonic value (not shown) do not reveal any clear negative relationship between the degree of "overpricing" (as indicated by high values of this ratio) and longer times to sale.

We conclude our review of the English housing data by showing figure 2.4, which plots the distributions of the first offer received and the best (highest) offer received as a ratio of the current list price for properties with different durations on the market. The left hand panel of figure 2.4 shows the distributions of first offers. We see that in the first week a home is listed, the mean first offer received is 96% of the list price (which is also the initial list price in this case). However first offers range from a low of only 79% of the list price to a high of 104% of the list price. We see that even accounting for declines in the list price with duration on the market, that first offers made on properties tend to decline the longer the property is on the market. There is a notable leftware shift in the distribution of first offers for offers received on homes that have been on the market for 20 weeks, where the mean first offer is only 91% of the list price in effect for properties that are still unsold after 20 weeks.

The right hand panel of figure 2.4 shows the distribution of the best offers received in a match. In the first few weeks the best offers show only modest improvement over the first offers received (e.g. the

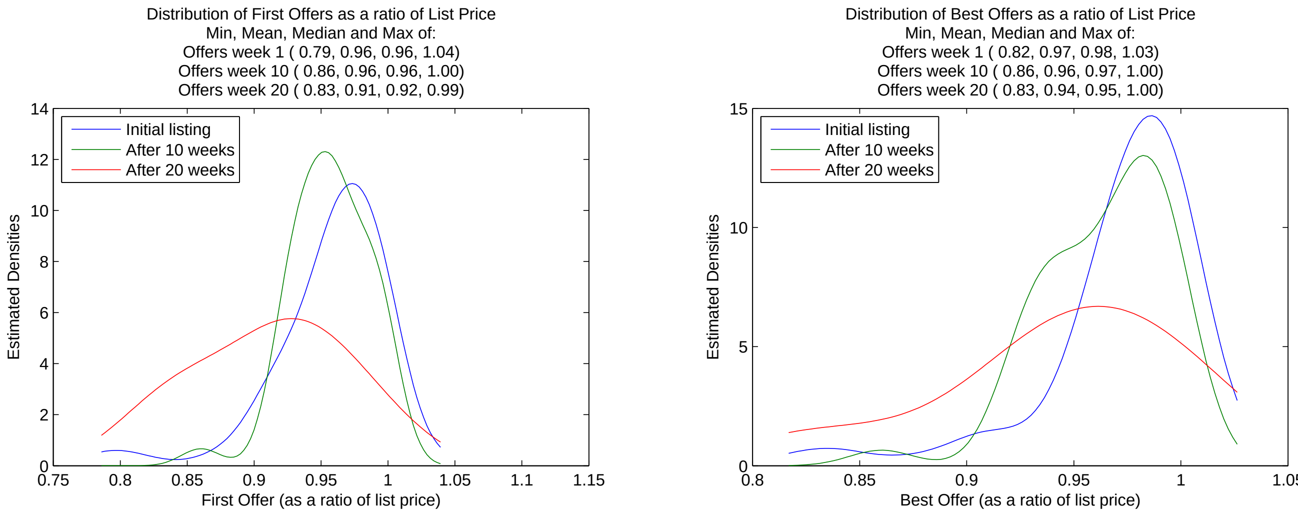

Figure 2.4 Distribution First Offer and Best Offer as a Ratio of List Price

best offer is 97% of the list price, whereas the first offer is 96% of the list price). However we see more significant improvement in offers received for homes that were still unsold after 20 weeks: the best offer received is 94% of the current list price, which is 3 percentage points higher than the ratio of the first offer to the list price.

3 The Seller's Problem

This section presents our formulation of a discrete-time, finite-horizon dynamic programming problem of the seller's optimal strategy for selling a house. The model we propose incorporates several features of the house selling process in England illustrated in the previous section.

Since our data set only includes properties that were listed and sold via a real estate agent, we take the decision to sell a house (via a real estate agency) as a given, and consider the seller's decisions of which price to list the house at initially, how to revise this price over time, whether or not to accept offers that are received, and whether to withdraw the house if insufficiently attractive offers are realized. To make these decisions the seller forms expectations about the probability a potential buyer will arrive and make an initial offer, the probability she will make additional offers if any of her offers are rejected, and the level of each of these offers. These expectations are revised over time based on the realized event history.

We do not explicitly model the behavior of buyers and the bargaining game that leads to the sale of a house. Rather, we capture the salient features of the bargaining environment by specifying a simplified model of buyers' bidding behavior. In particular, we assume that if a potential buyer arrives, she makes up to 3 consecutive offers (where 3 is the maximum number of offers observed in the data), which are drawn from bids distributions that depend, among other things, on the list price and the amount of time the house has been on the market.[11] The seller can either accept or reject each offer, but after any rejection there is a positive probability the buyer "walks" (i.e., she decides not to make a further offer and move on and search for other properties instead). As explained above, the procedure where a potential buyer makes offers that the seller can simply either accept or reject mimics the negotiating protocol in the data.

A decision period is a week, and we assume a finite horizon of 2 years. If a house is not sold after 2 years, we assume that it is withdrawn from sale and the seller obtains an exogenously specified "continuation value" representing the use value of owning (or renting) their home over a longer horizon beyond the 2 year decision horizon in this model.[12]

The seller's continuation value will generally be different from a quantity we refer to as the seller's *financial value* of their home. This is the seller's expectation of what the ultimate selling price will be for their home. While it is clear that the ultimate selling price is endogenously determined and partly under control of the seller, we can think of the financial value as a realistic appraisal or initial assessment on the part of the seller of the ultimate outcome of the selling process. Since the seller's optimal strategy will depend on the financial value of the house, if the financial value is to represent a rational, internally consistent belief on the part of the seller, it will have to satisfy a fixed-point condition that guarantees that it is a "self-fulfilling prophecy". Although we do not explicitly enforce this fixed-point constraint in our solution of the dynamic programming problem, we verify below (via stochastic simulations) that it does hold for the estimated version of our model.[13]

Let F_0 denote the seller's perception about the financial value of their home at the time of listing. We

[11] We describe this component of our model in detail in the next section.

[12] The continuation value may include the option value of relisting the home at a future date, perhaps during a period where conditions in the housing market are more favorable to the seller. However, we do not model the decision that leads either to "entry" (i.e. the initial decision to sell) or to "re-entry" (in case the property is withdrawn and then re-listed) of a house on the market.

[13] While it is possible to enforce the rationality constraint as a fixed-point condition on our model, from our standpoint it is useful to allow for formulations that relax the rationality constraint. This gives us the additional flexibility to consider models where sellers do not have fully rational, self-consistent beliefs about the financial value of their homes. Indeed, allowing for inconsistent or "unrealistic" beliefs may be an alternative way to explain why some home sellers set unrealistically high listing prices for their homes that would be distinct from the loss aversion approach discussed in the introduction. However, as we show below, we do not need to appeal to any type of irrationality or assume that sellers have unrealistic beliefs in order to provide an accurate explanation of the English housing data.

assume that F_0 is given by the equation

$$F_0 = \exp\{X\beta + \eta_0\} \tag{1}$$

where X are the observed characteristics of the home (the basis for the traditional hedonic regression prediction of the ultimate sales price discussed in Section 2), and η_0 reflects the impact of other variables that are observed by the seller but not by the econometricians that can affect the seller's perception of their home's financial value. These variables could include the seller's private assessment of aggregate shocks that affect the entire housing market, regional or neighborhood level shocks, as well as idiosyncratic house-specific factors. We assume that after consultation with appraisers and the real estate agent, the seller has a firm assessment of the financial value of their home that does not vary over the course of their selling horizon. Hence, η_0 can be interpreted as reflecting the seller's *private information* about the financial value of their home that is not already captured by the observable characteristics X.

Recall the left panel of figure 2.3 that shows that the adjusted list price is a far more accurate predictor of the ultimate selling price of the home than the hedonic value, $\exp\{X\beta\}$. In our estimation of the model, we assume that $\exp\{\eta_0\}$ is a lognormally distributed random variable that is independent of X, and we estimate β via a log-linear regression of the final transaction price on the X characteristics assuming that the random variable $\exp\{\eta_0\}$ satisfies the restriction $E\{\exp(\eta_0)\} = 1$. This restriction represents the *rationality constraint* we refer to above, which we verify is satisfied by our estimated model.

Due to the fact that the seller's optimal selling decisions depend critically on the seller's financial value F_0, which in turn depends on a very high dimensional vector of observed housing characteristics X as well as unobserved components η_0, straightforward attempts to solve the seller's problem while accounting for all of these variables immediately presents us with a significant "curse of dimensionality". In principle, we could treat the estimated hedonic value $\exp\{X_i\hat{\beta}\}$ as a "fixed effect" relevant to property i and solve $N = 780$ individual dynamic programming (DP) problems, one for each of the 780 properties in our sample. However, the problem is more complicated due to the existence of the unobserved "random effect" η_0. This is a one dimensional unobserved random variable and in principle we would need to solve each of the 780 DP problems over a grid of possible values of η_0, and thereby approximate the optimal selling strategy explicitly as a function of all possible values of the unobserved random effect η_0, which would be then "integrated out" in the estimation of the model.

However, by imposing a *linear homogeneity* assumption, we can solve a single DP problem for the seller's optimal selling strategy where the values and states are defined as *ratios relative to the seller's* *financial value.* In particular, define the seller's current list price P_t to be the ratio of the actual list price divided by the seller's financial value F_0. Then $P_t = 1.0$ is equivalent to a list price that equals the financial value, and $P_t > 1.0$ corresponds to a list price that exceeds the financial value and so forth. The implicit assumption underlying the linear homogeneity assumption is that, at least within the limited and fairly homogeneous segment of the housing market in our data set, there are no relevant further "price subsegments" that have significantly different arrival rates and buyer behavior depending on whether the houses in these segments are more expensive "high end" homes or not. The homogeneity assumption reflects a reasonable assumption that arrival rates and buyer bidding behavior are driven mostly by whether a given home is perceived to be a "good deal" as reflected by the ratio of the list price to the financial value. However, as we discuss below, the actual bid submitted by a buyer will depend on the buyer's private valuation for the home (also expressed as a ratio of the financial value F_0).

Let $S_t(P_t, d_t)$ denote the expected discounted (optimal) value of selling the home at the start of week t, where the current ratio of the list price to the financial value is P_t, and where the duration since the last match is d_t, with $d_t = 0$ indicating a situation where no matches have occurred yet. Here, a match is defined as a buyer who makes an offer on the home. We will get into detail about the timing of decisions and the flow of information shortly, but already we can see that this formulation of the seller's problem has three state variables: 1) the current total time on the market t, 2) the duration since the last match d_t, and 3) the current list price to financial value ratio P_t. The value function $S_t(P_t, d_t)$ provides the value of the home as a ratio of the financial value, so to obtain the actual value and actual list price we simply multiply these values by F_0. Thus $F_0 S_t(P_t, d_t)$ is the present discounted value of the optimal selling strategy, and $F_0 P_t$ is the current list price, both measured in UK pounds (£). Via this "trick" we can account for substantial heterogeneity in actual list prices and seller valuations by solving just a single DP problem "in ratio form." However, an important implication of this assumption is that timing of list price reductions and the percentage size of these reductions implied by the seller's optimal selling strategy are homogeneous of degree 0 in the list price and the financial value.

Our model of the optimal selling decision does not require the seller to sell their home within the 2 year horizon: we assume that the seller has the option to withdraw their home from the market at any time over the selling horizon. Since we do not model the default option of not selling one's house, we do not attempt to go into any detail and derive the form of the value to the seller of withdrawing their home from the market and pursuing their next best option (e.g., continuing to live in the house, or renting the home).

Instead we simply invoke a flexible specification of the "continuation value" $W_t(P_t)$ of withdrawing a home from the market and pursuing the next best opportunity.[14]

The seller has 3 main decisions: 1) whether or not to withdraw the property, 2) if the seller opts not to withdraw the property, there is a decision about which list price to set at the beginning of each week the home is on the market, and 3) if a prospective buyer arrives within the week and makes an offer, the seller must determine whether or not to accept the offer, and if the seller rejects the offer and the buyer makes a second offer, whether to accept the second offer, and so on up to (possibly) a third and final offer. We assume that the first two decisions are made at the start of each week and that the seller is unable to withdraw their home or change their list price during the remainder of the week. Within the week, if one or more offers arrive, the seller decides whether or not to accept them.

The Bellman equation for the seller's problem is given in equation (2) below.

$$S_t(P_t,d_t) = \max\left[W_t(P_t), \max_P\left[u_t(P,P_t,d_t) + \delta ES_{t+1}(P,P_t,d_t)\right]\right] \tag{2}$$

The Bellman equation says that at each week t, the optimal selling strategy involves choosing the larger of 1) the continuation value of (permanently) withdrawing the home from the market, or 2) continuing to sell, choosing an optimal listing price P. The function $ES_{t+1}(P,P_t,d_t)$ is the conditional expectation of the week $t+1$ value function S_{t+1} conditional on the current state variables (P_t,d_t) and the newly chosen list price P. Pursuant to the "forward-looking" perspective that we discussed in the introduction, in the version of the model we actually estimate, this expectation depends only on P and not on the previous week's list price P_t. That is, the current list price P is a sufficient statistic affecting the arrival rate of buyers and the magnitude of bids submitted. However, one could imagine a world with information lags where arrival rates and offers could depend on previous list prices, including the last week list price P_t. While it is not hard to allow for such lags without greatly complicating the solution of the model (at least provided we only allow a single week lag), we have found that it was not necessary to account for information lags to enable the model to provide a good approximation to the behavior we observe in the English housing data.

The function $u_t(P,P_t,d_t)$ captures two things: 1) the fixed "menu cost" to the seller of changing their

[14] Alternatively, we could allow for different types of sellers who have different continuation values and specify $W_t(P_t,\tau)$, where the parameter τ could denote the seller's "type." Fortunately, however, although our model can allow for other types of unobserved heterogeneity beyond the privately observed component of the financial value η_0, we did not need to appeal to any type of unobserved heterogeneity in seller types in order for the model to provide a good approximation to the behavior we observe in the English housing data.

list price, and 2) the "holding cost" to the seller of having their home on the market.

$$u_t(P,P_t,d_t) = \begin{cases} -h_t(d_t) - K & \text{if } P \neq P_t \\ -h_t(d_t) & \text{if } P = P_t \end{cases} \tag{3}$$

The function $h_t(d_t)$ is the net disutility (in money equivalent units) of having to keep the house in a tidy condition and to be ready to vacate it on short notice so the real estate agent can show it to prospective buyers. K is the fixed menu cost associated with changing the list price. This fixed cost can include the cost of posting new advertisements in a newspaper and/or websites, and printing up new flyers with the new listing price, and other bureaucratic costs involving in making this change (i.e. consulting with the realtor to determine the best new price to charge). We would expect that K should be a small number since none of the costs listed above would be expected to be large in absolute terms.

We now write a formula for $ES_{t+1}(P,P_t,d_t)$ that represents the value of the within week events when a match occurs. To keep the notation simpler, we will omit P_t from this conditional expectation, since as we noted above, we did not need to include P_t to capture any information lags that might affect arrival of buyers or the bids they might make. In order to describe the equation for ES_{t+1}, we need to introduce some additional information to describe the seller's beliefs about the arrival of offers from buyers, the distribution of the size of the offers, and the probability that the buyer will walk away (i.e. not make a new offer and search for other houses) if the seller rejects the buyer's offer. Given the negotiation protocol described above, within a given week there are at most 3 possible stages of offers by a potential buyer and accept/reject decisions by the seller. To simplify notation, we write ES_{t+1} for the case where at most one buyer arrives and makes an offer on the home in any week.[15]

Let $\lambda_t(P,d_t)$ denote the conditional probability that an offer will arrive within a week given that the seller set the list price to be P at the start of the week and the duration since the last offer is d_t. Let O_j be the highest offer received at stage $j = 1,2,3$ of the "bargaining process." Let $f_j(O_j|O_{j-1},P,d_t)$ denote the seller's beliefs about the offer the buyer would make at stage j given that the buyer did not walk in response to the seller's rejection of the buyer's offer in stage $j-1$. If the seller accepts offer O_j, let $N_t(O_j)$ denote the net sales proceeds (net of real estate commissions, taxes, and other transactions costs) received by the seller. The seller must decide whether to accept the net proceeds $N_t(O_j)$, thereby selling the home and terminating the selling process, or reject the offer and hope that the buyer will submit a more attractive

[15] Note however that our framework also accommodates the possibility of "auctions", i.e. situations where multiple buyers are bidding simultaneously for a home.

offer, or that some better offer will arrive from another potential buyer in some future week.

If a seller rejects the offer O_j, there is a probability $\omega_j(O_j, P, d_t)$ that the buyer will "walk" and not make a new offer as a function of the last rejected offer, O_j, and the current state (P, d_t). With this notation we are ready to write the equation for the within week problem which determines ES_{t+1} and completes the Bellman equation. We have

$$ES_{t+1}(P,d_t) = [1-\lambda_t(P,d_t)]S_{t+1}(P,d_t+1) + \lambda_t(P,d_t)\int_{O_1} \max\left[N_t(O_1), ES^1_{t+1}(O_1,P,d_t)\right] f_1(O_1|P,d_t)dO_1. \tag{4}$$

The function $ES^1_{t+1}(O_1,P,d_t)$ is the expectation of the subsequent stages of the within-week "bargaining process" conditional on having received an initial offer of O_1 and conditional on the beginning of the week state variables, (P, d_t). We can write a recursion for these within-week expected value functions similar to the overall backward induction equation for Bellman's equation as a "within-period Bellman equations"

$$\begin{aligned} ES^1_{t+1}(O_1,P,d_t) = \quad & \omega_1(O_1,P,d_t)S_{t+1}(P,1) + \\ & [1-\omega_1(O_1,P,d_t)]\int_{O_2} \max\left[N_t(O_2), ES^2_{t+1}(O_2,P,d_t)\right] f_2(O_2|O_1,P,d_t)dO_2. \end{aligned} \tag{5}$$

and

$$\begin{aligned} ES^2_{t+1}(O_2,P,d_t) = \quad & \omega_2(O_2,P,d_t)S_{t+1}(P,1) + \\ & [1-\omega_2(O_2,P,d_t)]\int_{O_3} \max\left[N_t(O_3), S_{t+1}(P,1)\right] f_3(O_3|O_2,P,d_t)dO_3. \end{aligned} \tag{6}$$

What equation (6) tells us is that after receiving 2 offers and rejecting the second offer O_2, the seller expects that with probability $\omega_2(O_2,P,d_t)$ the buyer will walk, so that the bargaining ends and the seller's expected value is simply the expectation of next periods' value $S_{t+1}(P,1)$. However, with probability $1-\omega_2(O_2,P,d_t)$, the buyer will submit a third and final offer O_3 which is a draw from the conditional density $f_3(O_3|O_2,P,d_t)$. Once the seller observes O_3, she can either take the offer and receive the net proceeds $N_t(O_3)$, or reject the offer, in which case the potential buyer leaves for sure and the seller's expected value is the next week value function, $S_{t+1}(P,1)$. Note that the second argument, the duration since last offer, becomes 1 at week $t+1$ reflecting that an offer arrived at week t.

4 A Simplified Model of Bidding by Prospective Buyers

As explained above, in this paper, we do not explicitly model the behavior of buyers and the bargaining game that leads to the sale of a house. Instead, we specify a simplified model of the bidding behavior of prospective buyers that incorporates important features of the data.

One important fact about observed bidding behavior is that *there is a positive probability that a prospective buyer will submit an offer equal to the current list price.* In the English housing data, over 15 percent of all accepted offers are equal to the list price and over 10 percent of all *first* offers are equal to the list price. Thus, any estimation of the offer distributions needs to account for mass points in the distribution, particularly at the list price. Further, we also observe offers in *excess* of the seller's list price. For example, over 2% of all first offers are above the list price, and nearly 4% of all accepted offers are higher than the list price prevailing when the offer was made.

The "semi-reduced form model" of buyers' bidding behavior we specify derives the distribution of offers from two underlying "semi-structural" objects: 1) a specification of buyers' bid functions, $b(v,l,F)$, and 2) a specification of the distribution of buyer valuations, $H(v|F,l)$, where v is the buyer's private valuation of the home, F is the financial value of the home, and l is the current list price.[16] In order to maintain the homogeneity restriction, we assume that l and F only enter b and h in a ratio form, i.e. as $P = l/F$. Thus, in the subsequent notation we will write these objects as $b(v,P)$ and $H(v|P)$.

The simplest specification for bid functions that we could think of that yields an offer distribution with a mass point at the current list price of the house is the following class of piecewise linear bid functions:

$$b(v,P) = \begin{cases} r_1(P)v & \text{if } v \in [\underline{v}, v_1) \\ P & \text{if } v \in [v_1, v_1 + k(P)) \\ r_2(P)v & \text{if } v \in [v_1 + k(P), \overline{v}] \end{cases} \tag{7}$$

where $\underline{v}$ and $\overline{v}$ are the lower and upper bounds, respectively, on the support of the distribution of buyer valuations (to be discussed shortly). To ensure continuity of $b(v,P)$ as a function of v, r_1 and r_2 must satisfy the following restrictions

$$\begin{aligned} P &= r_1(P)v_1 \\ P &= r_2(P)(v_1 + k(P)) \end{aligned} \tag{8}$$

This implies that

$$v_1 = \frac{P}{r_1(P)}$$

[16] Here, we put "structural" in quotes because a fully structural model of buyer behavior would derive the buyers' bid functions from yet deeper structure, like, for example, the solution to their search and bargaining problem. See, e.g., Albrecht, Anderson, Smith and Vroman (2007) for a theoretical model of the housing market with matching and bargaining.

$$r_2(P) \quad = \quad \frac{P}{P/r_1(P)+k(P)} \tag{9}$$

Thus, the bid functions are fully determined by the two functions $r_1(P)$ and $k(P)$. The first function determines how aggressive the bidder will be in terms of what fraction of the buyer's true valuation the buyer is willing to bid, *for the first bid* (we will consider specifications for 2nd and 3rd bid functions below). The closer $r_1(P)$ is to 1 the more "aggressive" the buyer is in her bidding (i.e., the closer the bid function is to truthful bidding). We assume that the buyer interprets the list price l as a signal from the seller about what the seller's reservation value is and as a signal of how reasonable the seller is. If the list price ratio P is substantially bigger than 1, the buyer will interpret this as a sign of an "unreasonable" list price by the seller, and so the buyer will respond by shading her bid to a higher degree. Conversely, a seller that "underprices" their home by setting a list price less than the financial value will result in more aggressive bidding by buyers, i.e. $r_1(P)$ will be closer to 1 when $P<1$. Thus, we posit that $r_1'(P)<0$, so that a seller who considers overpricing their home will expect that buyers will shade their first bids to a greater degree.

The bid functions have a flat segment equal to the list price for valuations in the interval $[v_1, v_1+k(P)]$. As we noted above, this flat section is empirically motivated by the fact that we observe a mass point in bid distributions at the list price. By adjusting the length of this flat segment $k(P)$ we can affect the size of the mass point in the bid distribution and thereby attempt to match observed bid distributions.

We posit that $k'(P)<0$ for reasons similar to the assumption that $r_1'(P)\leq 0$: a seller who overprices her home by setting a list price bigger than 1 will result in a shorter range of valuations over which buyers would be willing to submit a first offer equal to the list price. Conversely, if a seller underprices her home by setting a list price less than 1, there should be a wider interval of valuations over which the buyer is willing to submit a first offer equal to the list price. Observe that since the probability of a first offer equal to the list price is the probability that valuations fall into the interval $[v_1, v_1+k(P)]$, it is not strictly necessary for $k'(P)\leq 0$ in order for the probability of making an offer equal to the list price to be a declining function of l, which is another feature we observe in the English housing data. However initially we will assume that $k'(P)\leq 0$, but we can obviously consider relaxations of this condition later.

The left hand panel of Figure 4.2 plots examples of bid functions for four different values of P. These bid functions were generated from the following specifications for the function $r_1(P)$ and $k(P)$:

$$r_1(P) \quad = \quad .98(1-\gamma(P))+.85\gamma(P)$$

$$k(P) \quad = \quad .12(1-\gamma(P))+.07\gamma(P) \tag{10}$$

where

$$\gamma(P) = \frac{P-\underline{v}}{\overline{v}-\underline{v}}. \tag{11}$$

We see that the bid function for the highest list price, i.e. for a list price of $P = 1.62$ given by the blue dotted line in the left hand panel of figure 4.2, involves the most shading and lies uniformly below the bid functions at other list prices. It follows that the list price of $P = 1.62$ is *dominated* in terms of revenue to the seller by lower list prices. However, at more moderate list prices, the bid functions generally cross each other and so there is no unambiguous ranking based on strict dominance of the bid functions. For example if we compare the bid function for a list price of $P = 1$ with the bid function with a list price of $P = 1.09$ (the former is the orange dotted line and the latter is the solid red line in the left hand panel of figure 4.2), we see that the bid function for the lower list price $P = 1$ is higher for buyers with lower valuations and also for buyers with sufficiently high valuations, but the bid function with $P = 1.09$ (corresponding to a 9% markup over the financial value of the home), is higher for an intermediate range of buyer valuations. Thus the question of which of the two list prices result in higher expected revenues depends on the distribution of buyer valuations: if this distribution has sufficient mass in the intermediate range of buyer valuations where the bid function for the higher list price $P = 1.09$ exceeds the bid function for the lower list price $P = 1$, then the

expected bid from setting the higher list price will exceed the expected bid from setting a lower list price. Of course this statement is *conditional* on a buyer arriving and making a bid: we need to factor in the impact of list price on the arrival rate to compute the overall expected revenue corresponding to different list prices.

The right hand panel of figure 4.2 shows how the bid functions change in successive bidding stages. Bid functions for later bidding stages dominate the bid functions for earlier bidding stages, resulting in a monotonically increasing sequence of bids that is consistent with what we almost always observe in the English housing data. However, there are intervals of valuations where the bids lie on the flat segment of the bidding function, so this model can generate a sequence of bids where a previous bid (equal to the list price) is simply resubmitted by the bidder. This is also something we observe in the English housing data.

We complete the description of the semi-reduced form model of buyers' behavior by describing assumptions about the distribution of buyers' valuations for the home, $H(v|P)$. We assume that $H(v|P)$ is in the Beta family of distributions and thus it is fully specified by two parameters (a,b), as well as its

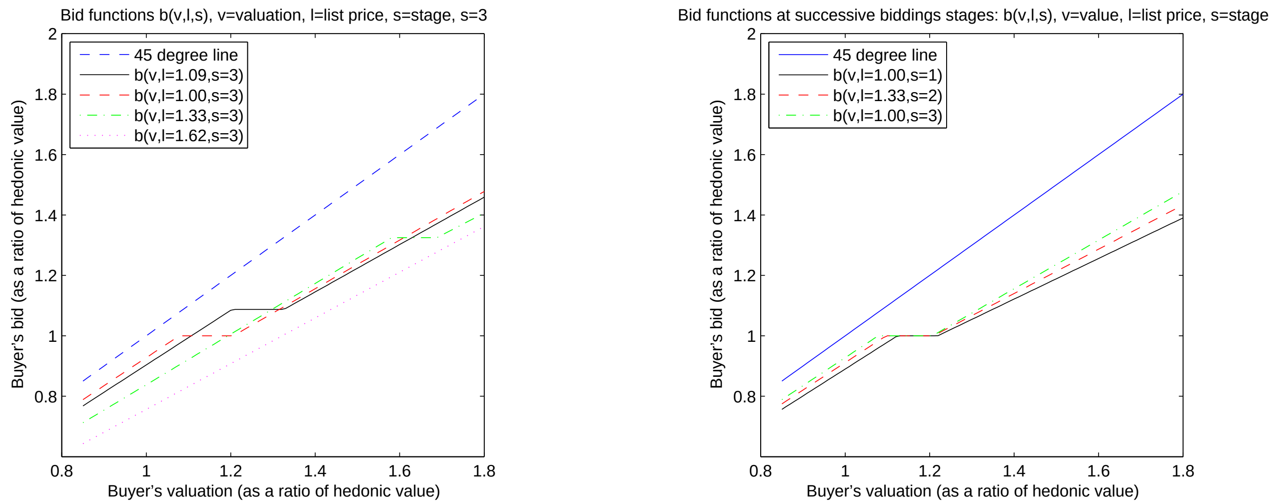

Figure 4.2 Piecewise linear bid functions for different list prices and bidding stages

support, $[\underline{v}, \overline{v}]$. We do not place any restriction on the distribution of valuations. In particular, it might be the case that buyers who have relatively higher than average valuations for a given home may choose to make offers: this would argue for a "positively biased" specification where $E\{v|P\} > P$. The direction of the bias might also depend on the list price: overpriced homes that have been on the market for a long time might be more likely to attract "vultures" (i.e., buyers with lower than average valuations who are hoping to get a good deal if the seller "caves").[17]

Let $B(v|a,b)$ be a beta distribution on the $[0,1]$ interval with parameters (a,b). We can derive the distribution of bids from this distribution by first rescaling this distribution to the $[\underline{v}, \overline{v}]$ interval to get the distribution of valuations $H(v)$ given by

$$H(v) = Pr\{\tilde{v} \leq v\} = B((v - \underline{v})/(\overline{v} - \underline{v})|a,b)\,. \tag{12}$$

The left hand panel of figure 4.3 plots an example of a beta distribution of valuations on the interval $[\underline{v}, \overline{v}] = [.5, 3]$ for different values of the (a,b) parameters. These parameters give us the flexibility to affect both the mode and the tail behavior of the distributions independently of each other. For fixed *a*, increases in *b* decrease the expected value $E\{v\}$ and move the mode towards zero *and* thin out the upper tail, whereas for fixed *b*, increases in *a* increase the mode, the mean, and thickens the upper tail of $H(v)$ although larger

[17] We could imagine many other types of stories or scenarios which may be incorporated into the analysis by allowing for a more general model of valuations of the form $f_t(v|P,d)$ where the distribution of valuations of buyers who make an offer on a home with a price ratio of P also depends on the duration since the last offer d and the length of time that house has been listed, t. We leave these generalizations to future work.

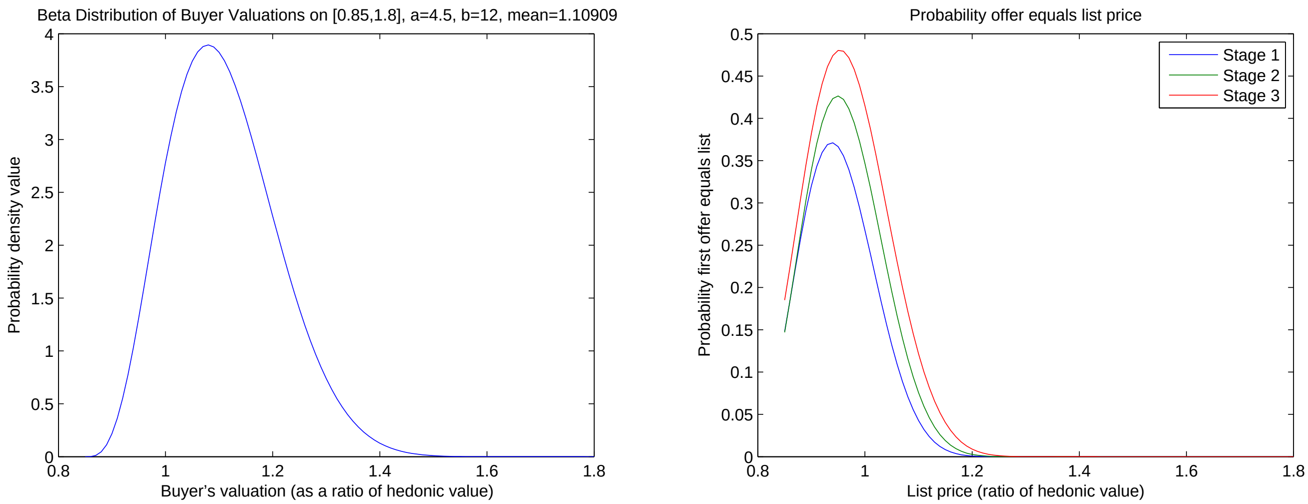

Figure 4.3 Beta distribution of buyer valuations and implied probabilities of bidding the list price

changes are required in a to produce comparably dramatic shifts in $H(v)$ compared with changes in b, at least for $a > 1$.

The right hand panel of Figure 4.3 plots the implied probability that an offer equals the list price, as a function of P at successive stages of the within week bargaining process for buyers whose distribution of valuations is a beta distribution on the support $[.85, 1.8]$ with parameters $(a,b) = (4.5, 12)$. We see that these implied probabilities are roughly in line with the data for the limited range of list prices that we observe in the English housing data (i.e. a mean first offer that is roughly equal to the financial value, i.e. $E\{b(v,P)\} \simeq 1$, where the mean value of P is approximately equal to 1.05. This implies that $r_1(P) \simeq .95$ when $P \simeq .95$. Actually, for the specification of $r_1(P)$ given above, we have $r_1(1.05) = .9248$.

The implied distribution of offers, $f(O|a,b,P)$, is given by

$$\begin{aligned} f(O|a,b,P) &= Pr\{b(\tilde{v},P) \leq O\} \\ &= Pr\left\{\tilde{v} \leq b^{-1}(O,P)\right\} \\ &= B\left(b^{-1}(O,P) - \underline{v})/(\overline{v} - \underline{v})|a,b\right). \end{aligned} \tag{13}$$

Due to the presence of the flat segment, the usual notion of an inverse of the bid function does not exist. However, if we interpret the inverse of the bid function at the value P as the interval $[v_1, v_1 + k(P)]$, we obtain a distribution of offers that has a mass point at the list price, consistent with what we observe in the English housing data.

In summary, we can write the distribution of offers implied by our semi-reduced form specification of

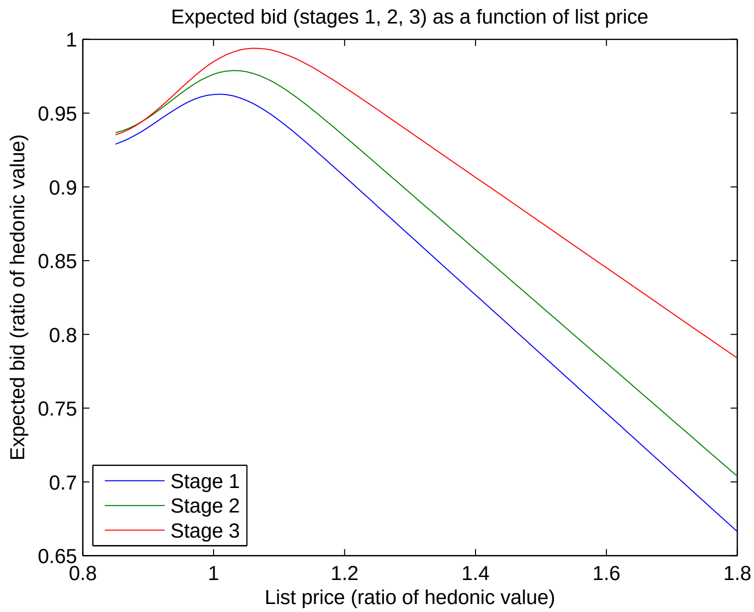

Figure 4.4 Expected bids as a function of the list price and bidding stage

bidding behavior explicitly in terms of the functions $r_1(P)$ and $k(P)$ as

$$f(O|a,b,P) = \begin{cases} B((O/r_1(P) - \underline{v})/(\overline{v} - \underline{v})|, a, b) & \text{if } O \in [\underline{v}, P) \\ B((k(P) + P/r_1(P) + k(P) - \underline{v})/(\overline{v} - \underline{v})|a,b) - B((P/r_1(P) - \underline{v})/(\overline{v} - \underline{v})|a,b) & \text{if } O = P \\ B((O(P/r_1(P) + k(P)) - \underline{v})/(\overline{v} - \underline{v})|a,b) & \text{if } O \in (P, \overline{v}] \end{cases} \tag{14}$$

Using this distribution function, we can compute the *expected bid function* $E\{\tilde{b}|P\}$ as

$$\begin{aligned} E\{\tilde{b}|P\} &= \int O f(dO|a,b,P) \\ &= \int_{\underline{v}}^{\overline{v}} b(v,P) H(dv). \end{aligned} \tag{15}$$

Note that the expectation depends both on the list price and on the financial value because offers are interpreted as ratios of list price to the financial value of the home.

Figure 4.4 plots the expected bid functions for several different specifications of the distribution of valuations. We see that the expected bid functions are unimodal and are maximized at list prices that are higher than 1, providing an incentive for the seller to “overprice” when the seller sets a list price. Of course this is not the full story, since the seller must also account for the effect of the list price on arrival rates of buyers. The dynamic programming problem takes both factors into account, as well as other dynamic considerations and the fixed menu costs involved in changing the list price.

5 Empirical Results

This section presents econometric estimates of our model of the home selling problem via a simulated minimum distance (SMD) approach. In general terms, the objective of the estimation method is to find estimates of the unknown parameters of our semi-reduced form model of bidding behavior and of the other structural parameters of the model that enable the predicted optimal selling strategy from our dynamic programming model to best fit the actual selling behavior that we observe in the data.

As we noted in Section 3, we have adopted a "full solution" approach to estimation — that is, we estimate the seller's belief parameters by repeatedly numerically resolving for the optimal selling strategy for different trial values of the parameters in an inner dynamic programming subroutine while an outer optimization algorithm searches for parameters that minimize a quadratic form in a vector of actual versus simulated moments of interest from the real and simulated housing data. We found that the full solution approach resulted in much more sensible outcomes, because this approach enforces the requirement that the implied optimal selling strategy should be close to the selling behavior we observe.

The SMD estimator, sometimes also referred to as a "simulated method of moments estimator", estimates the vector of unknown model parameters θ by minimizing a distance function constructed as a quadratic form between an $N \times 1$ vector of moments about housing transactions that we actually observe in the English housing data, call this m, and a conformable $N \times 1$ vector of simulated moments, call this $m_S(\theta)$, formed by creating an artificial data set with the same set of 780 homes with the same set of observable characteristics X and same hedonic values $\exp\{X\beta\}$ (where the β coefficients are computed from a first stage regression using the data, independent of the housing model), but simulated S times and the individual moments from each *IID* simulation are averaged to form the vector of simulated moments $m_S(\theta)$. Then the SMD criterion is

$$\hat{\theta} = argmin[m - m_S(\theta)]' W [m - m_S(\theta)] \tag{16}$$

where W is an $N \times N$ positive definite weighting matrix. The specification of the model we consider has 30 unknown parameters that we estimate using $N = 137$ moments and the optimal weighting matrix equal to the inverse of the variance-covariance matrix of these 137 moments. We chose the 137 moments to reflect a wide array of features in the English housing data, and the actual moments used will be listed in detail below.

Before we present our estimates and discuss the overall fit of the model, it is useful to describe the parametric specification that we estimated. Recalling the discussion in section 4, we can write the piecewise linear bid functions as functions of the parameter vector θ as follows

$$
\begin{aligned}
r_{1s}(P) &= \underline{r}_{1s}(\theta)(1-\gamma(P)) + \overline{r}_{1s}(\theta)\gamma(P) \\
k_s(P) &= \underline{k}_s(\theta)(1-\gamma(P)) + \overline{k}_s(\theta)\gamma(P)
\end{aligned}
\tag{17}
$$

where

$$
\gamma(P) = \frac{P - \underline{v}}{\overline{v} - \underline{v}}, \tag{18}
$$

and s denotes the s^{th} stage of the bargaining subgame, $s = 1,2,3$. Thus, $r_{1s}(P)$ is the bid ratio (the ratio of the buyer's valuation v that the bidder is willing to bid) in the first linear segment of the bid function in stage s of the bargaining subgame. Similarly, $k_s(P)$ is the length of the flat segment of the bid function at the list price. This determines the probability that the buyer will submit a bid equal to the list price. The final segment of the bid function is $r_{2s}(P)$. We assume that this is given by

$$
r_{2s}(P) = \overline{r}_{2s}(\theta) r_{1s}(P), \tag{19}
$$

so that we need only three additional coefficients $(\overline{r}_{21}, \overline{r}_{22}, \overline{r}_{23})$ to specify the upper linear segment of the bid functions corresponding to bids in excess of the list price.

Thus, there are a total of 15 coefficients required to specify the piecewise linear bid functions: the 6 coefficients $(\underline{r}_{1s}(\theta), \overline{r}_{1s}(\theta))$, $s = 1,2,3$ determining the first linear segment of the bid functions below the list price, the 6 coefficients $(\underline{k}_s(\theta), \overline{k}_s(\theta))$, $s = 1,2,3$ determining the length of the flat segments corresponding to bids equal to the list price, and the 3 remaining ratio terms $(\overline{r}_{2s}(\theta))$, $s = 1,2,3$ determining the slope of the positively sloped component of the bid function for bids above the list price. Due to concerns about identification, we only estimated the first 7 coefficients (denoted $(\theta_1, \ldots, \theta_7)$), and fixed the remaining 8 coefficients.[18]

The next set of parameters pertain to the arrival probabilities and the probabilities that a buyer will walk if a previous offer was rejected. The arrival probabilities are given by

$$
\lambda_t(P, d_t) = \frac{\exp(\Lambda)}{1 + \exp(\Lambda)} \tag{20}
$$

[18] In particular, we initially iterated on all 15 coefficients until there was no longer any significant improvement in the value of the criterion function, and then fixed the last eight coefficients at those values prior to obtaining standard errors for the remaining model parameters. These values are $\underline{k}_{12} = 0.156$, $\underline{k}_{13} = 0.165$, $\overline{k}_{11} = 0.073$, $\overline{k}_{12} = 0.089$, $\overline{k}_{13} = 0.095$, $\underline{r}_{21} = 0.762$, $\underline{r}_{22} = 0.795$, and $\underline{r}_{23} = 0.845$.

where

$$\begin{aligned}\Lambda &= \theta_8 + \theta_9 P + \theta_{10} I\{2 \le t \le 5\} + \theta_{11} I\{6 \le t \le 10\} + \\ &\quad \theta_{12} I\{d_t = 1\} + \theta_{13} I\{d_t = 2\} + \theta_{14} I\{d_t = 3\} + \theta_{15} I\{d_t \ge 4\} + \\ &\quad \theta_{16} P * I\{d_t = 1\} + \theta_{17} P * I\{d_t = 3\} + \theta_{18} P * I\{d_t \ge 4\} \qquad (21)\end{aligned}$$

Similarly, the probability of walking is also specified as a binomial logit model involving 6 coefficients $(\theta_{19}, \ldots, \theta_{24})$ where, for example, the stage 1 probability of walking (i.e. the probability the buyer leaves after the seller rejects the buyer's first offer) is given by

$$\omega_1(O_1, P, d_t) = \frac{\exp(\theta_{19} + \theta_{20}(O_1/P))}{1 + \exp(\theta_{19} + \theta_{20}(O_1/P))}. \qquad (22)$$

The expressions for $\omega_2(O_2, P, d_t)$ and $\omega(O_3, P, d_t)$ are the same as above, but involve the coefficients $(\theta_{21}, \theta_{22})$ and $(\theta_{23}, \theta_{24})$, respectively.

Parameter θ_{25} is K, the fixed menu cost of changing the list price, and θ_{26} is σ, the standard deviation of η_0 in equation (1). The next two parameters, $(\theta_{27}, \theta_{28})$, are the parameters of the Beta distribution of buyer valuations, (a, b).[19] Finally, θ_{29} is the weekly "holding" cost to the seller of having her home on the market, h_t, and θ_{30} is the seller's subjective discount factor δ.

As per our previous discussion about the difficulty of identifying the continuation value given that none of the 780 sellers in our sample withdrew their homes from the market (i.e., all were eventually successful in selling their homes), we simply assumed that $W_t(P_t) = .2$ (i.e., the continuation value is 20% of the seller's estimate of the financial value of the home). The only other parameters in our model are the fixed and variable costs associated with selling the home, mainly due to real estate fees and other closing costs. The real estate commissions charged by the British real estate agency we are studying are admirably low by U.S. standards, the commission rate is only 1.8% of the sale price of the home. We assume that the entire commission is paid by the seller but the buyer pays for all other fixed selling expenses associated with the final closing, including the seller's legal fees and taxes. Thus, we used the following specification for the net sale proceeds from selling the home as a function of the accepted offer O

$$N_t(O) = .982 * O. \qquad (23)$$

[19] Due to concerns about identification we did not attempt to estimate the support of the distribution and set $[\underline{v}, \overline{v}] = [1.0, 1.8]$. Recall these values are ratios of the financial value of the home, so $\overline{v} = 1.8$ indicates a buyer whose private valuation of the home is 1.8 times its financial value.

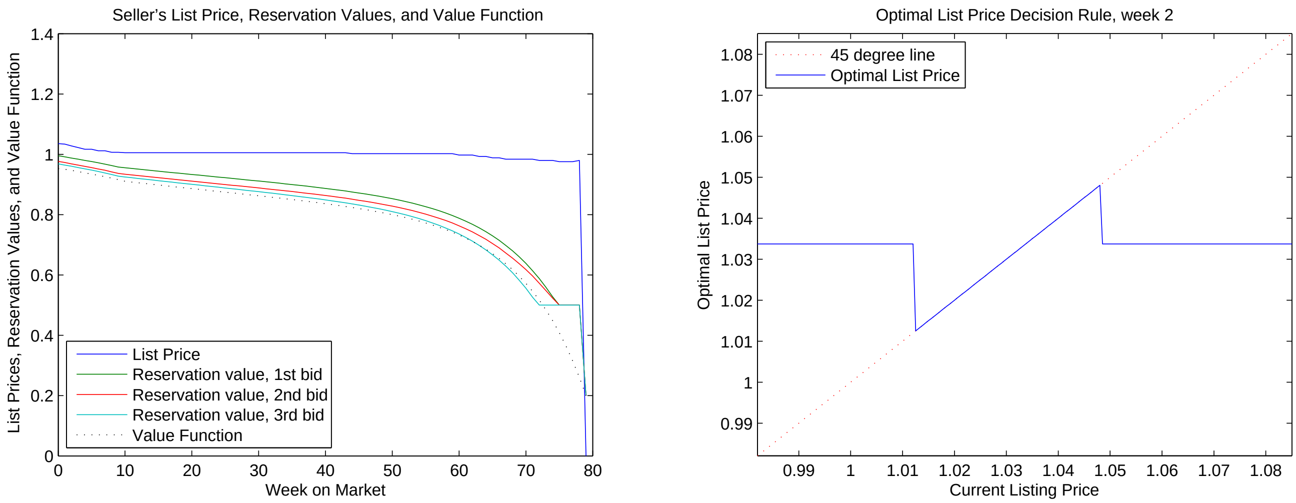

Figure 5.1 Estimated Optimal Selling Strategy

Table 5.1 presents the SMD parameter estimates and standard errors. We illustrate some of our empirical findings in figure 5.1 below. As we noted in the introduction, our main empirical finding is that our model of optimal selling by a rational seller is able to fit the key features we observe in the English housing data, particularly the observed stickiness in list prices. The left hand panel of figure 5.1 plots the optimal list prices, reservation values and the value function corresponding to the estimated parameters from the model. The top blue line is the optimal list price, and notice that it is nearly flat as a function of weeks on the market.

There are no significant drops in the list price over the entire 80 week selling horizon other than in the final period where the list price is plotted as being cut to zero, though this actually corresponds to the seller's decision to withdraw the home from the market if she has not sold it 79 weeks after initially listing the house for sale. The optimal initial list price is $P^* = 1.0356$ (recall that the list price is represented as a ratio of the actual list price of the home in £ to the seller's unobserved financial value of the home), corresponding to a 3.6% "markup" over the price the seller realistically expects to receive from selling her home. The model predicts that there are series of small reductions in the *notional, unconditional optimal list price* over the first several weeks. These reductions lower this notional optimal list price from its initial $P^* = 1.0356$ in the first week to to $P^* = 1.0056$ by week 10, and ultimately to $P^* = 0.9797$ in the last week before the home is withdrawn from the market.

Recall that apart from the the initial list price $P^* = 1.0356$, these subsequent *notional* unconditional optimal list prices are *not* the *actual list prices* that the seller will choose due to the presence of the small fixed transaction cost involved in changing the list price. This is illustrated in the right hand panel of figure 5.2 that shows the substantial *inaction region* about the notional unconditional optimal list price of $P^* = 1.0337$ at the start of the second week the house is on the market. As long as the *current actual list price* is sufficiently close to this notional optimal list price $P^* = 1.0337$ (i.e. any actual list price in the interval $[1.01, 1.05]$), it will not be optimal for the seller to incur the fixed transaction cost to adjust the list price. Since the initial (actual) list price $P^* = 1.0356$ is in this inaction region, it follows that the seller *will not find it optimal to adjust the actual list price to the notional unconditional optimal value* and thus the actual list price will remain at its initial value $P = 1.0356$ that the seller set when the house first went on the market.

We emphasize that the estimated value of the fixed "menu costs" of changing prices, K, is very small in our model. From table 5.1 we see that the point estimate is $K = .0001$, or 10 thousandths of 1% of the seller's financial value of the home. This would be £10 for a home with a financial value of £$100,000$. While K is not precisely estimated (its estimated standard error is also equal to 0.0001), our model strongly rejects the hypothesis that $K = 0$ because setting $K = 0$ leads the model to predict much more frequent (weekly) changes in list prices than we actually observe in the English housing data.

The other three solid color lines in the left hand panel of figure 5.1 are the seller's reservation values at the three stages in the "bargaining process" of our model. We see that even though list prices are essentially flat as a function of duration since listing, the reservation prices decline more or less continuously over time, and their rate of decrease accelerates after a house has been on the market unsold for over one year. At this point the price the seller is willing to accept drops rapidly, falling to 80% of the seller's estimate of the financial value, even though the seller maintains the list price at slightly above his/her estimate of the financial value of the home and the notional, unconditional optimal list price value is still $P^* = 1.002$ at week 52.

In summary, our model predicts that the seller will optimally choose a 3.6% markup in the initial list price, setting it to $P = 1.0356$ times the seller's estimated financial value of their home. We have shown that even a very small menu cost implies a wide inaction region where it is optimal for the seller to leave her initial choice of list price unchanged. In fact, in simulations of the optimal strategy, it will not be optimal for a seller who has not received any acceptable offers on his/her home to reduce the list price until *the 8th week that the home is on the market.* At that point gain from reducing the list price from

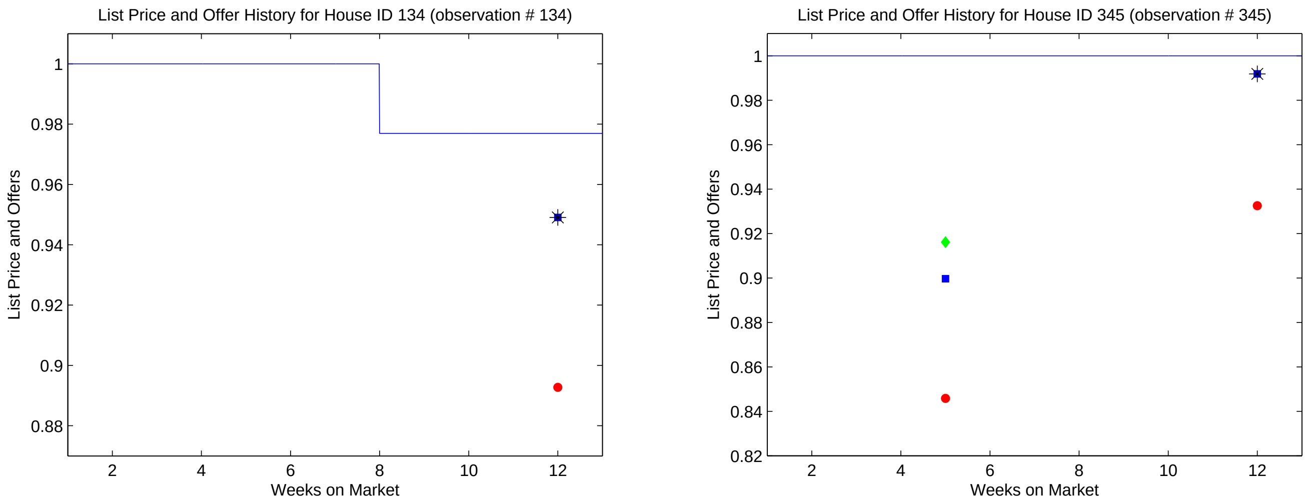

Figure 5.2 Simulated Outcomes of the Optimal Selling Strategy

the initially optimal value of $P = 1.0356$ to the optimal value that prevails in week 8, $P = 1.0068$ is large enough to overcome the menu cost, and so the seller makes a large, discontinuous cut in the list price at this time.

Figure 5.2 illustrates the foregoing discussion by plotting two simulated realizations of the optimal selling strategy. In the left hand panel we see that the seller maintains his/her initial list price for the first 7 weeks, but no offers were received. Then in the 8th week the seller reduced the list price by nearly 3%. In the 12th week a buyer arrives and makes an initial bid that is below 90% of the initial list price, which the seller rejects. This is illustrated by the red dot in the left hand panel of figure 5.2. Then the buyer increased his offer with a bid equal to 95% of the list price, which is above the seller's reservation value so the seller accepts this offer at this point (illustrated by the blue square with a star around it).

The right hand panel of figure 5.2 illustrates the history-dependence in the optimal list price strategy. In this case, there are no changes in the list price over the full 12 week duration from the initial listing until the home was sold. Why were there no list price reductions in this case? We see that for observation 345, an offer was received by the seller in week 5, but the initial offer (red dot) indicated a very "low ball" offer of just over 84% of the list price, which the seller rejects. The buyer then makes a counter offer equal to 90% of the list price which the seller also rejects, and a final offer of 92% of the list price which the seller also rejects, and the buyer walks after making the third rejected offer. Though this bargaining match was unsuccessful, it raised the expectations for simulated seller 345 in comparison to simulated 34 who had no offers until week 12. Due to the greater optimism about subsequent offers resulting from the arrival of an offer in week 8, the simulated seller 345 decided not to reduce her list price and also caused seller 345 to reject an initial offer in week 12 that simulated seller 134 would have rejected.

The other significant point to notice about the optimal selling strategy at this point is that the seller's reservation values *decline* at each successive stage of the "bargaining process." The reason we obtain this prediction in our model is due to the assumptions underlying the bidding automata that constitute our model of buyer behavior. Our seller does use all information to determine the "type" of the buyer based on the buyer's initial bid. Indeed, we presume that the seller also knows the coefficients of the piecewise linear bid function used by the buyer and inverts this function to determine the buyer's bid (unless the buyer bids at the seller's list price, in which case the seller only knows that the buyer's valuation is on the flat segment of the piecewise linear bid function). However, because of the exogenous probability that a buyer will walk if the seller rejects the buyer's previous bid, the model tells us that it is optimal for the seller to lower his/her reservation price when evaluating a new offer by the same buyer. The intuition is that the seller regards the buyer as a "fish nibbling at the bait" and it would be better to sell now at a somewhat lower price than to try to be too greedy and risk the chance that the buyer would walk if the seller rejected the buyer's new offer. If the current buyer leaves, the seller knows that it could be many weeks before the next interested buyer arrives who is willing to make an offer on the home.

Before we turn to a discussion of the overall fit of the model, it is useful to illustrate some of the rich implications of our model for some counterfactual parameter values. Figure 5.3 illustrates the impact on the value function and reservation prices if we change the seller's beliefs about the rate of arrival of buyers to make the arrival rate significantly more sensitive to the list price than our estimation results indicate are the case. In our binary logit specification of the arrival rate, there are eleven coefficients: a constant term θ_8 that governs the overall rate of arrival, a coefficient on the list price θ_9, and nine other dummy variables that are designed to capture differences in the rate of arrival of buyers during the time a home is listed for sale, $(\theta_{10},\ldots,\theta_{18})$. Our parameter estimates result in an estimated constant term of $\hat{\theta}_8 = -1.9526$ and an estimated coefficient of the list price equal to $\hat{\theta}_9 = -0.4536$. In figure 5.3 below we illustrate how the solution changes when we change these coefficients to $\theta_8 = -1.0$ and $\theta_9 = -1.5$. The sum of these two coefficients is -2.5, which is slightly lower than the sum of of the two estimated coefficients, thus implying a somewhat lower rate of arrival of buyers under the counterfactual of setting a list price at $P = 1$.

The changes in the optimal selling strategy resulting from this seemingly small change in the seller's

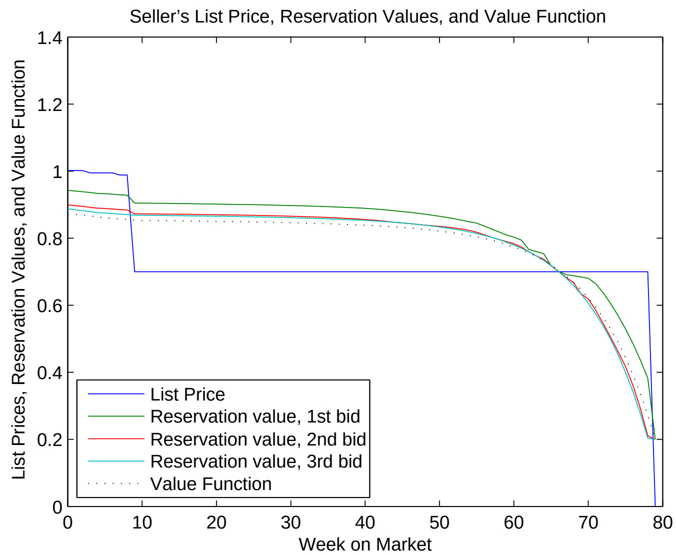

Figure 5.3 Optimal Selling Strategy Under Different Arrival Rate Beliefs

beliefs are striking: while the initial list price is somewhat smaller than the previous (estimated) model illustrated in figure 5.1 (i.e. $P = 1.0017$ versus $P = 1.0356$), the optimal solutions diverge dramatically after the 9th week on the market. In the version of the model where the arrival rate is more sensitive to the list price, the seller reduces the list price to $P = 0.7$ in the 9th week and keeps this value in all subsequent weeks of the selling horizon. We also see an interesting situation with an "inverted" selling strategy, i.e., where the seller's reservation values are *higher* than the list price. This is an example of an *underpricing strategy* that we discussed in the introduction: the seller lowers the list price significantly below the seller's belief about the true financial value of the home in order to "get buyers through the door". Once the buyers actually come to view the home they are willing to pay more than the list price, and this is reflected by the seller's reservation price functions, which are not dramatically lower than the reservation prices illustrated in the left hand panel of figure 5.1. Indeed, simulations of this model show that the seller expects to earn 96% of the financial value from following this underpricing strategy — only slightly lower than what the seller would expect to earn under the original model using the estimated arrival rate parameters.

Turning our attention to the fit of the model, the SMD criterion we used in estimation was based on a total of $N = 137$ individual moments. Table 5.2 compares each simulated and actual moment. By and large, table 5.2 shows that the model captures a broad array of features in the England housing data, not just the stickiness of list prices. Starting with the first moment in table 5.2, we see that the SMD parameter estimates do satisfy the “rationality constraint” that the seller’s financial value is an unbiased expectation of the ultimate selling price. The first row of the table compares the mean of the ratios of the actual sale price for each of the 780 houses sold to the hedonic price $\exp\{X\beta\}$ (in the Actual column) to the mean of the same ratio from 5 *IID* simulations of the model with the same 780 houses and the same hedonic values (in the Simulated column), but with the difference being that the simulated transaction price is generated from our model. We see that the actual moment has a mean of nearly 100%, which is to be expected given that the hedonic value is by construction an unbiased predictor of the actual sales price. The fact that the simulated moment is also equal to 1 indicates that the rationality constraint (i.e., that the financial value is a conditional expectation of the actual sales prices), does hold in our model. To see this recall that the financial value is given by $F = \exp\{X\beta + \eta_0\}$ where η_0 captures unobservable characteristics of the home. Recall that we assumed η_0 to be normal with mean μ and standard deviation σ, but we constrained μ such that for any value of σ, the mean of the lognormally distributed random variable $\exp\{\eta_0\}$ is 1. This implies that if the hedonic price component of the financial value $\exp\{X\beta\}$ is an unbiased predictor of the sales price of the home, then so will the financial value $F = \exp\{X\beta + \eta_0\}$. We regard the fact that the best fitting parameter estimates “automatically” enforce the rationality constraint (without us having to impose it) as further evidence in favor of the hypothesis that the selling behavior that we observe in the data can be well approximated by a model of rational sellers.

The estimated model is also capable of reproducing many of the other key features of the data, including: the fraction of sales that occur with no changes in the listing price (moment 2); the distribution of times to sale (moments 9-48); the distribution of listing prices (moments 84-119); the number and timing of ”matches” between a seller and a potential buyer (moments 8 and 49-69). On the other hand, the model does not do quite as well in terms of matching the fraction of accepted offers equal to, below, and above the list price (moments 4-5), or the distribution of acceptance rates across offers at different percentages of the list price (moments 123-130).

6 Conclusions

In spite of advances in theoretical research on the behavior of buyers and sellers in the housing market (see, e.g., Albrecht et al. (2007, 2012), Arnold (1999), Salant (1991), Taylor (1999), Yavaş (1992), and Yavaş and Yang (1995)), the lack of adequate data has limited the scope of empirical research on housing transactions. Existing data sets typically include property characteristics, time to sale, initial listing price, and sale price. They do not contain information on the buyer's side of the transaction (e.g., the timing and terms of offers made by potential buyers), or on the seller's behavior between the listing and the sale of a property (e.g., the seller's decision to reject an offer or to revise the listing price). This explains why most of the empirical literature on housing transactions has either focused on the determinants of the sale price or on the role of the listing price and its effect on the time to sale (see, e.g., Anglin et al. (2003), Carillo (2011, 2012), Glower et al. (1998), Haurin (1988), Horowitz (1992), Kang and Gardner (1989), Knight et al. (1998), Miller and Sklarz (1987) and Zuehlke (1987)).

In this paper, we have taken advantage of the availability of a rich data set containing detailed information on the sequence of all relevant events since the initial listing of a house through its sale (including all list price changes and all offers received) for a large sample of residential transaction histories in England, to specify and estimate a dynamic model of the "home selling problem" which incorporates several realistic features of this important process. We have shown that the estimated model is capable of reproducing many important features of the data, including the relatively high degree of "stickiness" of listing prices.

One of the main limitations of our analysis is that we focused attention on the dynamic problem of the seller and did not explicitly model the behavior of buyers and the bargaining game that leads to the sale of a house. Incorporating these additional features into a dynamic equilibrium model of housing transactions represents a challenge both from a theoretical point of view and in terms of data availability. We intend to take on this challenge in future work.

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Table 5.1 Parameter Estimates and Standard Errors

Parameter	Description	Estimate	Standard Error
θ_1	$\underline{r}_{11}$	0.8531	0.0123
θ_2	$\underline{r}_{12}$	0.8953	0.0111
θ_3	$\underline{r}_{13}$	0.9047	0.0105
θ_4	$\overline{r}_{11}$	0.4920	0.0099
θ_5	$\overline{r}_{12}$	0.5259	0.0115
θ_6	$\overline{r}_{13}$	0.5927	0.0164
θ_7	$\underline{k}_{11}$	0.1699	0.0075
θ_8	arrival prob constant	-1.9526	0.0250
θ_9	coefficient of P	-0.4536	0.0113
θ_{10}	coefficient of $I\{1 \leq t \leq 5\}$	0.7581	0.0227
θ_{11}	coefficient of $I\{6 \leq t \leq 10\}$	0.65321	0.1980
θ_{12}	coefficient of $I\{d_t = 1\}$	1.5420	0.5250
θ_{13}	coefficient of $I\{d_t = 2\}$	1.6030	0.2980
θ_{14}	coefficient of $I\{d_t = 3\}$	1.5700	0.0655
θ_{15}	coefficient of $I\{d_t \geq 4\}$	1.8247	1.2900
θ_{16}	coefficient of $P * I\{d_t = 1\}$	-0.6577	0.4850
θ_{17}	coefficient of $P * I\{d_t = 3\}$	-0.6872	0.0667
θ_{18}	coefficient of $P * I\{d_t \geq 4\}$	-0.6373	0.0247
θ_{19}	walk prob constant $(s = 1)$	-6.0095	0.2340
θ_{20}	coefficient of O_1/P	3.5210	0.2540
θ_{21}	walk prob constant $(s = 2)$	-3.9552	0.0564
θ_{22}	coefficient of O_2/P	4.9346	1.7000
θ_{23}	walk prob constant $(s = 3)$	-7.6150	0.4380
θ_{24}	coefficient of O_3/P	8.6788	3.1500
θ_{25}	K (menu cost)	0.0001	0.0001
θ_{26}	σ (standard deviation of η_0)	0.2561	0.0357
θ_{27}	a beta parameter	3.8943	0.0065
θ_{28}	b beta parameter	13.3433	0.0543
θ_{29}	h (holding cost)	0.0104	0.0075
θ_{30}	δ (subjective discount factor)	0.0016	0.0019

Table 5.2 Actual and Simulated Moments

Moment	Simulated	Actual	Moment	Simulated	Actual
Mean sale price to hedonic ratio	1.008	0.997	% unsold after 33 weeks	0.029	0.050
% sales with 0 list price changes	0.787	0.773	% unsold after 34 weeks	0.028	0.044
% sales with 1 list price change	0.163	0.208	% unsold after 35 weeks	0.024	0.040
% accepted offers equal to list	0.219	0.153	% unsold after 36 weeks	0.022	0.037
% accepted offers below list	0.764	0.808	% unsold after 37 weeks	0.021	0.033
% sales with 1 match	0.828	0.767	% unsold after 38 weeks	0.019	0.031
% sales with 2 matches	0.092	0.180	% unsold after 39 weeks	0.018	0.028
Mean number of matches	1.300	1.438	% unsold after 40 weeks	0.014	0.027
Mean duration to sale	9.583	10.274	Mean time to 1st match	8.129	8.917
% unsold after 1 weeks	0.944	0.924	% 1st matches $\leq$ 2 weeks	0.229	0.237
% unsold after 2 weeks	0.824	0.806	% 1st matches $\leq$ 3 weeks	0.359	0.342
% unsold after 3 weeks	0.717	0.710	% 1st matches $\leq$ 4 weeks	0.464	0.435
% unsold after 4 weeks	0.628	0.633	% 1st matches $\leq$ 5 weeks	0.536	0.515
% unsold after 5 weeks	0.565	0.547	% 1st matches $\leq$ 6 weeks	0.617	0.578
% unsold after 6 weeks	0.491	0.490	% 1st matches $\leq$ 7 weeks	0.665	0.622
% unsold after 7 weeks	0.428	0.445	% of 1st matches $\leq$ 8 weeks	0.710	0.665
% unsold after 8 weeks	0.381	0.409	% 1st matches $\leq$ 9 weeks	0.755	0.699
% unsold after 9 weeks	0.337	0.365	% 1st matches $\leq$ 10 weeks	0.783	0.737
% unsold after 10 weeks	0.300	0.318	% 1st matches $\leq$ 15 weeks	0.863	0.841
% unsold after 11 weeks	0.273	0.292	% 1st matches $\leq$ 20 weeks	0.915	0.888
% unsold after 12 weeks	0.251	0.268	% 1st matches $\leq$ 25 weeks	0.942	0.914
% unsold after 13 weeks	0.228	0.250	Mean time to 2nd match	5.007	4.456
% unsold after 14 weeks	0.201	0.227	% 2nd matches $\leq$ 2 weeks	0.882	0.886
% unsold after 15 weeks	0.183	0.197	% 2nd matches $\leq$ 5 weeks	0.940	0.937
% unsold after 16 weeks	0.163	0.191	% 2nd matches $\leq$ 15 weeks	0.996	0.987
% unsold after 17 weeks	0.145	0.169	Mean time to 3rd match	4.532	4.690
% unsold after 18 weeks	0.135	0.156	% 3rd matches $\leq$ 2 weeks	0.942	0.978
% unsold after 20 weeks	0.112	0.138	% 3rd matches $\leq$ 5 weeks	0.981	0.990
% unsold after 21 weeks	0.103	0.133	% 3rd matches $\leq$ 10 weeks	0.992	0.992
% unsold after 22 weeks	0.096	0.128	% sales $\leq$ 0.5	0.003	0.027
% unsold after 23 weeks	0.087	0.122	% sales $\leq$ 0.6	0.029	0.072
% unsold after 24 weeks	0.079	0.117	% sales $\leq$ 0.7	0.095	0.149
% unsold after 25 weeks	0.073	0.106	% sales $\leq$ 0.8	0.224	0.253
% unsold after 26 weeks	0.071	0.097	% sales $\leq$ 0.9	0.392	0.371
% unsold after 27 weeks	0.065	0.085	% sales $\leq$ 1.0	0.553	0.521
% unsold after 28 weeks	0.053	0.076	% sales $\leq$ 1.1	0.695	0.660
% unsold after 29 weeks	0.051	0.069	% sales $\leq$ 1.2	0.790	0.801
% unsold after 30 weeks	0.045	0.059	% sales $\leq$ 1.3	0.850	0.879
% unsold after 31 weeks	0.036	0.055	% sales $\leq$ 1.4	0.914	0.927
% unsold after 32 weeks	0.031	0.053	% sales $\leq$ 1.5	0.950	0.958

Table 5.2 Actual and Simulated Moments (cont'd)

Moment	Simulated	Actual	Moment	Simulated	Actual
% sales ≤ 1.6	0.967	0.972	Offer Acceptance rates:		
% sales ≤ 1.7	0.983	0.982	1st offer/list ≤ 0.9	0.097	0.522
% sales ≤ 1.8	0.991	0.988	1st offer/list in (0.9, 0.95]	0.383	0.484
Mean list/hedonic price week 1	1.034	1.050	1st offer/list in (0.95, 0.96]	0.618	0.517
Mean list/hedonic price week 2	1.037	1.050	1st offer/list in (0.96, 0.97]	1.000	0.488
Mean list/hedonic price week 3	1.040	1.049	1st offer/list in (0.97, 0.98]	1.000	0.792
Mean list/hedonic price week 4	1.032	1.050	1st offer/list in (0.98, 0.99]	1.000	0.897
Mean list/hedonic price week 5	1.034	1.052	1st offer/list in (0.99, 1]	1.000	0.939
Mean list/hedonic price week 6	1.027	1.051	Match prob week 1	0.085	0.108
Mean list/hedonic week 7	1.031	1.044	Match prob week 3	0.171	0.156
Mean list/hedonic week 8	1.027	1.036	Match prob week 6	0.170	0.155
Mean list/hedonic week 9	1.032	1.038	Match prob week 15	0.102	0.153
Mean list/hedonic week 10	1.034	1.040	Match prob week 20	0.115	0.061
Mean list/hedonic week 11	1.035	1.025	% list/hedonic ≤ 0.5 at $t = 0$	0.001	0.015
Mean list/hedonic week 12	1.046	1.030	% list/hedonic ≤ 0.6 at $t = 0$	0.022	0.037
Mean list/hedonic week 13	1.044	1.022	% list/hedonic ≤ 0.7 at $t = 0$	0.079	0.100
Mean list/hedonic week 14	1.031	1.025	% list/hedonic ≤ 0.8 at $t = 0$	0.190	0.197
Mean list/hedonic week 15	1.042	1.040	% list/hedonic ≤ 0.9 at $t = 0$	0.346	0.322
Mean list/hedonic week 16	1.033	1.030	% list/hedonic ≤ 1.0 at $t = 0$	0.514	0.462
Mean list/hedonic week 17	1.028	1.030	% list/hedonic ≤ 1.1 at $t = 0$	0.665	0.582
Mean list/hedonic week 18	1.025	1.021	% list/hedonic ≤ 1.2 at $t = 0$	0.763	0.728
Mean list/hedonic week 19	1.032	1.012	% list/hedonic ≤ 1.3 at $t = 0$	0.832	0.828
Mean list/hedonic week 20	1.037	1.016	% list/hedonic ≤ 1.4 at $t = 0$	0.901	0.897
Mean 1st offer/list	0.954	0.947	% list/hedonic ≤ 1.5 at $t = 0$	0.933	0.935
Mean 2nd offer/list	0.947	0.955	% list/hedonic ≤ 1.6 at $t = 0$	0.962	0.965
Mean 3rd offer/list	0.924	0.964	% list/hedonic ≤ 1.7 at $t = 0$	0.973	0.974
% of 1st offers equal to list	0.191	0.114	% list/hedonic ≤ 1.8 at $t = 0$	0.988	0.983
% of 1st offers below list	0.795	0.858	% list/hedonic ≤ 1.9 at $t = 0$	0.992	0.990
Offer/list, accepted 1st offers	0.972	0.957	% list/hedonic ≤ 2.0 at $t = 0$	0.997	0.991