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“Tax Competition and the Efficiency of 'Benefit-Related' Business Taxes"
by
Elisabeth Gugl, George R. Zodrow
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Abstract

The conventional wisdom in public finance is that local governments should finance public services, including those provided to businesses, with user charges that function as benefit taxes, and should in particular avoid inefficient source-based taxes on highly mobile capital. However, if user charges are not available or infeasible, several public finance experts have recently suggested that taxes on local production, such as an origin-based VAT, are a desirable alternative in that they serve as relatively efficient "benefit-related" taxes. We examine this contention formally in a model in which business public services must be financed with either a source-based tax on mobile capital, such as a property tax, or a tax on production, such as an origin-based VAT. In general, both a capital tax and a production tax are inefficient. However, consistent with the "benefit-related" view, the production tax is efficient if the production function belongs to the knife-edge case between log sub- and log supermodularity with respect to capital and public services (e.g., a Cobb-Douglas production function), while the capital tax results in underprovision of public services in this case. Similarly, if the production function is log submodular with respect to capital and public services (e.g., a CES production function with substitution elasticity greater than one), a production tax is again less inefficient than a capital tax, although both taxes result in underprovision of the public service. Finally, if the production function is log supermodular (e.g., a CES production function with substitution elasticity smaller than one), a production tax results in overprovision of the public service, while the effects of a capital tax – and thus the relative efficiency properties of the two taxes – are theoretically ambiguous.

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Authors’ addresses: Gugl: Department of Economics, University of Victoria, P.O. Box 1700 STN CSC, Victoria, BC, V8W 2Y2, fax: (250) 721 6214, phone: (250) 721 8538, email: egugl@uvic.ca; Zodrow: Rice University, Department of Economics, 6100 Main Street, Houston TX 77005, email: zodrow@rice.edu.
1 Introduction

The conventional wisdom in public finance is that both efficiency and equity considerations imply that state and local governments should finance public services, including those provided to businesses, with user charges and fees that function as benefit taxes. Subnational governments, because they are effectively small open economies, should especially avoid inefficient source-based taxes on highly mobile capital that are not directly related to the benefits of business public services. However, if user charges are not available or are infeasible for technical, political or other reasons, state and local governments must use alternative tax instruments to finance public services. In the United States, state and local taxes often take the form of property taxes or corporate income taxes on business capital, which may distort a wide variety of decisions, including those regarding capital accumulation and allocation and the level of public services provided, as stressed in the tax competition literature.\(^1\) In addition, at the state and local level in the US and at the provincial level in Canada, retail sales taxes often apply tax to business inputs, even though in principle they are supposed to be limited to final consumer goods. In light of this situation, several prominent public finance experts have argued recently that, in the absence of explicit benefit taxes or user charges, taxes on local production, such as an origin-based value added tax (VAT), are an attractive option. Such taxes serve as a proxy for user charges – that is, they are relatively efficient "benefit-related" taxes. For example, Bird [1] argues that

The economic – as opposed to the political economy – case for local business taxation is simply as a form of generalized benefit tax. Where possible, specific business enterprises should be paid for by appropriate user charges. Where it is not feasible to recoup the marginal cost of cost-reducing public sector outlays through user charges, some form of broad-based general levy on business activity may well be warranted. (Bird [1], p. 225)

\(^1\)See Zodrow and Mieszkowski [34], Wilson [30], Wilson [31], Zodrow [33], Wildasin and Wilson [32]. Although most of these articles focus on the effects of tax competition on local public services provided to households, Zodrow and Mieszkowski also discuss the case in which local jurisdictions provide a public service that is an input into firm production functions.
Despite such arguments, the use of business taxes at the state and local level in the U.S. is generally declining. Indeed, many states that use a formula-apportioned state corporate income tax have moved away from the production-based components of the tax by switching to formulas that put a relatively small or zero weight on productive factors used by businesses within the taxing jurisdiction (payroll and property); they instead have attached larger weights (sometimes unitary) on a destination-based measure of gross sales. Although these reductions in business taxes may simply reflect a realization that business taxes were far higher than the value of public services received by business, as suggested by Testa and Mattoon [28], they also may reflect the perception that taxes on production within a state do not correspond to the benefit taxes envisioned by Bird.

There is a vast literature analyzing the efficiency of destination-based vs. origin-based value-added taxes (VATs), and a similarly vast literature analyzing the efficiency of capital taxes. However, in this paper, we provide what we believe is the first attempt to systematically analyze the assertion that a production-based tax can be viewed as an approximate benefit tax, and is thus preferable to the often-used alternative of a tax on capital.

In a closed economy, there is no difference between a uniform tax on all consumption and a similar tax on production. In contrast, as economies become increasingly more open, the distinction between a tax on local production and a tax on local consumption becomes important in terms of efficiency (Mintz and Tulkens [19], Kanbur and Keen [11], Lockwood [13], Hauffer and Pfüger [10]). In tax competition models in which firms are perfectly competitive, a tax on local consumption such as a destination-based VAT is efficient when countries are too small to affect world prices (Lockwood [13]). Hauffer and Pfüger [10] investigate the difference between a destination-based and an origin-based VAT in several settings of international duopoly and find that only the former is efficient when competition between countries is imperfect. However, public finance experts have also supported an origin-based VAT against a destination-based VAT. In addition to Bird’s argument cited above, McLure [18] notes that with increasing e-commerce, the typically favoured destination-based VAT is difficult to enforce. Despite the vast literature on different forms of VATs, an origin-based VAT has not been analyzed in models of tax competition where public services are provided to firms.

To be sure, there are many papers analyzing the efficiency of capital taxation when firms receive a public service and jurisdictions compete with each
other for mobile capital. (See, e.g., Zodrow and Mieszkowski [34], Oates and Schwab [21], [22], Noiset [20], Sinn [25], Keen and Marchand [12], Bayindir-Upmann [2], Richter [24], Matsumoto [15], [14], and Dhillon, Wooders and Zissimos [5].) However, these papers focus on the inefficiency of capital taxation, and do not consider the tax on local production envisioned by Bird, or compare such a production tax to a capital tax. In the cases analyzed by Oates and Schwab [22] and Sinn [25], a production tax is found to be undesirable because the models are constructed so that a tax on mobile capital is efficient. For example, Oates and Schwab [22] assume that jurisdictions distribute public services to firms precisely in proportion to the amount of capital that they employ. This rather stringent assumption is of course sufficient to make a capital tax an efficient benefit tax in their model (while taxes on production would be inefficient). Similarly, Sinn [25] constructs a model in which the usage of a public service and capital are perfect complements, which again implies that a tax on capital is an efficient benefit tax.\footnote{More specifically, in Sinn [25] each unit of capital must make a trip on a publicly provided highway before it can be used in the production of a consumption good.}

In this paper, we adopt some alternative and we believe more realistic assumptions about how public services enter firm production functions, and analyze the relative efficiency properties of a broad-based tax on production relative to a capital tax. Specifically, our paper follows the original Zodrow and Mieszkowski (1986) – hereafter, ZM – formulation in assuming that public services enter as a separate input into the firm’s production function that is not linked to capital, so that the consumption good in the model is produced with three factors: labor (the immobile factor), mobile capital, and the public service.\footnote{See Matsumoto and Feehan [17] for a discussion of Oates and Schwab’s [22] rationing system for the unpaid factor.} The public input is exogenous to firms, but endogenous for local governments. We follow Oates and Schwab [22] – and most of the state and local public finance literature in the Tiebout [29] tradition, as discussed, for example, by Hamilton [9] – in assuming that the public input is a publicly provided private good (that is, it is subject to congestion to the full extent of a private good\footnote{Alternative assumptions are, of course possible. See e.g. Richter [24], Matsumoto [15] or Sinn [25] who assume a partly congested public good, or Matsumoto [14] and Feehan and Matsumoto [6] who assume a pure public input.} and that the production technology exhibits constant returns to scale (CRS) in all factors, both public and private.\footnote{Matsumoto [15] also assumes CRS in all factors.}
a technology implies that privatization of the supply of the public input and/or charging user fees is efficient. While this is not true for all types of public investments, Gramlich [6, p. 1193] suggests that many state and local public investment projects fall into this category. We assume throughout the paper that the production function is strictly concave in capital and the public service.

Our analysis reveals that log modularity properties of the production function play a major role in determining the relative efficiency properties of production-based taxes and capital taxes, an issue that has thus far not been examined in models of interjurisdictional tax competition. It is well known that log (super/sub) modularity plays an important role in the matching literature when there are search frictions; positive (negative) assortative matching requires that all matching sets be convex, which is determined by the log modularity properties of the production function (Peters and Siow [23]; Smith [27]). We find that these properties are also crucial in determining the efficiency of tax-financed provision of local business public services.

Our results provide some support for the idea that a production-based tax may be viewed as a "benefit-related" tax, although not surprisingly it does not in general substitute perfectly for an explicit user charge. In particular, in the special case of a production function that is log modular in capital and the public service (e.g., a Cobb-Douglas production function), a production tax is effectively a benefit tax, and is thus analogous to a user charge for public services that ensures an efficient level of public service provision. In the same vein, if the production function is log submodular in capital and the public service (e.g., a CES production function with an elasticity of substitution greater than one), a production tax will be inefficient and lead to underprovision of the public service, but it will result in less underprovision and thus be less inefficient than a capital tax. However, although we can show that the production tax leads to overprovision of public services in the case of a log supermodular production function (e.g., a CES production function with an elasticity of substitution less than one), the ambiguity of the effect of a capital tax on public services in this case implies that a comparison of the relative efficiency properties of the two taxes is impossible without further restrictions on the production technology.
2 The Model

A federation or union consists of \( N \) jurisdictions, each with the same number of residents who are immobile across jurisdictions. All residents have identical preferences and endowments. Individuals work where they live, provide a fixed amount of labor, and obtain utility from consumption of an aggregate composite good. The labor supply of each jurisdiction, \( L \), is therefore fixed.\(^6\) People own an equal share of the union’s capital stock \( K \) which is fixed in total supply. Since capital is perfectly mobile across jurisdictions, the after-tax rate of return to capital, \( r \), is the same in every jurisdiction.

Each jurisdiction produces a single consumption good, \( X \), with a technology characterized by constant returns to scale (CRS) in two private inputs and one public input. Labor and capital are the private inputs in the production of \( X \). In addition, the local government provides a fully congestible business public service \( B \) that is used directly in the production of the consumption good \( X \). The consumption good is assumed to be tradable and is taken as the numeraire. The government can costlessly transform the consumption good into the public service, so its unit cost is also equal to 1.\(^7\) We assume that the number of firms is fixed in each jurisdiction (or equivalently there is a single representative firm) and focus therefore on the aggregate production function in each jurisdiction given by\(^8\)

\[
X = F(L, K, B)
\]

where

\[
F(0, 0, 0) = 0
\]

\[
F_L > 0, \quad F_K > 0, \quad F_B > 0,
\]

\[
F_{KB} > 0, \quad F_{KL} > 0, \quad F_{BL} > 0, \quad F_{ii} < 0.
\]

\(^6\) The fixed factor can also be thought of as a combination of labor and land, as assumed in Zodrow and Mieszkowski (1986).

\(^7\) We follow most of the literature in assuming constant marginal costs for the public service (Oates and Schwab [21], [22], Sinn [25], Bayindir-Upmann [2], Keen and Marchand [12], Richter [24], and Matsumoto [15]). Two alternative approaches, which Matsumoto [15] points out are equivalent, would be to assume either an imperfectly congestible public input and a constant marginal cost of producing that public input, or a perfectly congestible public input (i.e., our publicly provided private service) and decreasing marginal costs of producing the public service.

\(^8\) See e.g. Zodrow and Mieszkowski [34], Bayindir-Upmann [2], Keen and Marchand [12], Dhillon et al. [5] for the same assumption.
for $i = L, K, B$. For a given $L > 0$, the second-order-derivative matrix of $F(K, B)$ is assumed to be negative definite for $(K, B) > 0$.

We consider two tax scenarios. Under the production tax scenario, all jurisdictions must use a tax on capital ($\tau$) to finance provision of the business public service. Thus each jurisdiction is subject to a balanced budget constraint,

$$\tau K = B.$$  
(5)

Under the alternative capital tax scenario, all jurisdictions must use a production tax ($t$) to finance their business public services

$$tX = B.$$  
(6)

In both cases, we assume that the local taxing jurisdiction is a small open economy, with local governments choosing the various tax instruments available to maximize the income of their immobile residents, taking as given the policies of all competing jurisdictions, the union-wide return on capital, and the price of the tradable composite consumption good. Under the capital tax scenario, the total income of residents $I_{CT}$ equals the sum of their capital income from union-wide investment and firm value added (labor income earned within the taxing jurisdiction).

$$I_{CT} = rK/N + [F(L, K, B) - (r + \tau) K].$$  
(7)

The amount of capital in a jurisdiction is determined by firms’ profit maximizing behavior, taking the level of public services and various taxes in the jurisdiction as given. Under the production tax scenario, total income of residents $I_{PT}$ equals the sum of their capital income from union-wide investment and total net local production minus capital production costs

$$I_{PT} = rK/N + [(1 - t) F(L, K, B) - rK].$$  
(8)

Note that our analysis is short run in the sense that it is possible that perfectly competitive firms will make positive economic profits as they do not have to pay for $B$ directly, e.g., in the form of user charges.

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9 As in the majority of the literature, we assume complementarity between capital and the public service (but not fixed proportions), as we assume a strictly positive cross-derivative between capital and the public service in the production function (e.g., Zodrow and Mieszkowski [34], Bayindir-Upmann [2], Keen and Marchand [12], Feehan and Matsumoto [6], Matsumoto [15], [16], [14], and Dhillon et al. [5]).

10 Negative definiteness implies strict concavity of $F(K, B)$. 

8
2.1 Efficiency

As noted in the introduction, given that \( B \) is a publicly provided private good, it is optimal to charge user fees. With user fees and fixed \( L \), firms maximize profit by

\[
\max_{B,K} F(L, K, B) - B - rK.
\]

The firms’ profit maximizing conditions are

\[
F_K (L, K, B) = r \tag{9}
\]

\[
F_B (L, K, B) = 1. \tag{10}
\]

These conditions are necessary and sufficient to determine a unique optimum since \( F(L, K, B) \) is assumed to be strictly concave. Note that assuming increasing returns to scale in \((K, B)\) would rule out the existence of an interior solution even in the case of user charges, and assuming CRS in \((K, B)\) would not lead to a unique interior solution. Thus, in order to get a solution in which the efficient amount of the public service is uniquely determined by \( F_B = 1 \), we must assume that the production function is at least locally strictly concave in \((K, B)\) for all \((K, B) >> (K, B)\) for some \( \infty >> (K, B) \geq 0 \). See Dhillon et al. [5].

2.2 Capital Tax

Suppose that local governments are constrained – for reasons of political feasibility or otherwise – from utilizing user charges, and instead impose a tax on capital at rate \( \tau \). In this case, total resident income is

\[
I_{CT} = rK/N + [F(L, K, \tau K) - (r + \tau)K]. \tag{11}
\]

In calculating its optimal level of taxation, the local government must predict how the demand for capital in the jurisdiction changes as the tax rate \( \tau \) is increased, assuming that other jurisdictions do not respond. Differentiating (5) and (9) with respect to \( K \) and \( \tau \) yields the response of capital to an increase in the capital tax

\[
\phi \equiv -\frac{dK}{d\tau} = -\frac{1 - F_{KB}K}{F_{KK} + F_{KB}\tau}. \tag{12}
\]

To determine the sign of \( \phi \), note that
Property 1 A proportional increase in $B$ and $K$ will cause $F_K$ and $F_B$, respectively to decrease, i.e. $BF_{BK} + KF_{KK} < 0, KF_{BK} + BF_{BB} < 0$.\footnote{Property 1 would also hold if $F(L,K,B)$ is homothetic in $(K, B)$ instead of assuming CRS in all factors.}

This follows from our assumption that the Hessian of $F(K, B)$ is negative definite and from CRS in production of the consumption good.

Lemma 1 The denominator of $\phi$, given by $F_{KK} + F_{KB} \tau$, is negative.

The proof of Lemma 1, given in the appendix, depends only on property 1 and the assumption of budget balance. Lemma 1 coincides with ZM’s stability assumption (\cite{34}, equation 17) and Dhillon et al. \cite{5} show that this assumption is a necessary condition for the existence of a capital tax equilibrium. Given this condition, the perceived change in capital in response to an increase in $\tau$ depends on the sign of $(1 - F_{KB}K)$. This expression measures from the firms’ perspective the net impact of an increase in the capital tax rate, reflecting the cost an increase in the marginal cost of capital, given by 1, and the benefit of the associated increase in $B$ on the marginal productivity of capital, given by $F_{KB}K$. If the marginal cost and marginal benefit from the firm’s perspective are not equal, firms will lower their demand for capital if $1 > F_{KB}K$, and increase it if $1 < F_{KB}K$.

With local jurisdictions choosing $\tau$ to maximize the income of the residents (11), the first order condition is

$$K (F_B - 1) - F_B \tau \phi = 0$$

(13)

and the optimal level of capital taxation is determined by

$$\tau = K \frac{F_B - 1}{F_B \phi}.$$  

(14)

For any interior solution with $\tau > 0$, overprovision or underprovision of the public service is determined by the sign of $\phi$. If $\phi > 0$, then $F_B > 1$ and the equilibrium is characterized by underprovision of business public services, as in the case of public services provided to residents analyzed by ZM. In contrast, if $\phi < 0$, then $F_B < 1$ and overprovision results. Note that from condition (13) $\phi = 0$ is also a possibility, which implies $F_B = 1$ and efficient provision of business public services.
Consider next a property that characterizes some production functions:

**Property 2** The marginal productivity of the public service is greater than the marginal impact of a unit of the public service on the marginal productivity of the existing capital stock, or equivalently $F_B$ is an increasing and strictly concave function in $K$

$$F_B > KF_{KB}.$$  \hspace{1cm} (15)

As shown in the appendix, Property 2 is crucial in allowing us to prove:

**Proposition 1** Given properties 1 and 2, an interior solution with capital tax $\tau > 0$ has the following properties: (1) $F_B > 1$, that is, the business public service is underprovided, and (2) $\phi > 0$, that is, each jurisdiction expects to drive out capital if it increases $\tau$. However, if property 2 is not satisfied, the capital tax equilibrium can lead to over-, efficient, or under-provision of the business public service.

This result can be related to the existing literature on the efficient provision of business public services. Matsumoto [14] notes (p.471): “If the number of firms is constant in each jurisdiction (normalized to one), [...] $[F_B]$ may be below one because the sign of $[F_B - KF_{KB}]$ is indeterminate under linear homogeneity with respect to all inputs. This argument corresponds to the Noiset [20] result of potential overprovision in the [ZM] model where the variability of the number of firms is not explicitly considered.” Dhillon et al. [5] construct a model with two production inputs (capital and public services only), and assume Lemma 1 as do ZM in their paper. They develop alternative conditions that guarantee existence of a unique interior solution to the optimal capital tax, and then show that over-, efficient, or underprovision of the public service can occur depending on whether the production function is only locally or globally strictly concave in $(K, B)$. In the next section we introduce the concept of log (sub/super) modular production functions and show that both log submodularity and log modularity of $F(L, K, B)$ in $(K, B)$ together with strict concavity in $(K, B)$ satisfy property 2.\(^{12}\)

\(^{12}\)Matsumoto [14] also notes that property 2 holds in the case in which $F(L, K, B)$ is linearly homogeneous in $(L, K)$. In this case

$$F = F_L L + F_K K$$
2.3 Production Tax

Consider next the production tax scenario, in which case the government budget constraint is

$$tF(L, K, B) = B. \quad (16)$$

Since firms receive the consumer price minus the tax for each unit of the consumption good sold, the income of residents is now given by (8) and the profit maximizing condition for capital demand is

$$(1 - t) F_K(L, K, B) = r, \quad (17)$$
or

$$F_K(L, K, B) = \frac{r}{(1 - t)}. \quad (17)$$

In order to find the optimal production tax, we first need to evaluate the impact of capital and the production tax on the amount of the public service, $B$. Implicit differentiation of (16) yields

$$\frac{\partial B}{\partial K} = \frac{tF_K}{(1 - tF_B)},$$
and

$$\frac{\partial B}{\partial t} = \frac{F}{(1 - tF_B)}.$$ 

Using (17) and the balanced budget constraint, the predicted effect of an increase in $t$ on the capital stock is

$$\frac{dK}{dt} = \frac{\left( \frac{F_K}{1 - t} - F_K B \frac{\partial B}{\partial t} \right)}{\left( F_{KK} + F_{KB} \frac{\partial B}{\partial K} \right)}$$

$$= \frac{F_K (1 - F_B t) - F_K B F (1 - t)}{(1 - t) (F_{KK} (1 - F_B t) + F_{KB} F_K t)},$$

where, as shown in the appendix, the denominator is unambiguously negative.\(^\text{13}\) Again, the perceived change in capital as $t$ increases depends on taking the derivative with respect to $B$

$$F_B = F_{BL} L + F_{BK} K$$

and given $F_{ij} > 0$, implies $F_B > KF_{KB}$.

\(^{13}\)As in the case of $\phi$ in the derivation in section 3.1, this result does not require property 2.
whether the real cost of capital, given by \( \frac{r}{1-t} \) increases by more than, the same as, or less than the marginal productivity of capital due to the increase in \( B \) financed with an increase in \( t \).

The first order condition for the optimal production tax rate, found by maximizing (8) with respect to \( t \), is

\[
F_K t (F_{KB} F - F_B F_K) - F_{KK} F (F_B - 1) = 0. \tag{18}
\]

or

\[
t = \frac{K (-F_{KK}) (1 - F_B)}{F_B F_K (\varepsilon_{KB} - \varepsilon_K)}. \tag{19}
\]

where \( \varepsilon_{KB} = F_{KB} K / F_B \) is the capital elasticity of the marginal productivity of the public service, and \( \varepsilon_K = F_K K / F \) is the capital elasticity of production of the consumption good. With a positive production tax rate, underprovision (overprovision) of business public services occurs if \( \varepsilon_{KB} < (>) \varepsilon_K \). The relative magnitudes of these two elasticities are determined by whether the production function can be characterized as log submodular or log supermodular in \((K, B)\). As shown in Smith [27], the production function \( F(L, K, B) \) is log submodular in \((K, B)\) if and only if

\[
F(L, K, B) F(L, K', B') < F(L, K', B) F(L, K, B')
\]

for all \( K' > K \) and \( B' > B \), while log supermodularity holds if the inequality is reversed. Intuitively, log supermodularity implies that an increase in the capital stock of a given size leads to a proportionately larger increase in output if the level of public services also increases. Analogously, log submodularity implies that an increase in the capital stock of a given size leads to a proportionally smaller increase in output if the level of public services also increases. In the appendix we show that for a twice differentiable function, log submodularity implies \( F_{KB} F - F_B F_K < 0 \) or \( \varepsilon_{KB} < \varepsilon_K \), and log supermodularity implies \( F_{KB} F - F_B F_K > 0 \) or \( \varepsilon_{KB} > \varepsilon_K \). In the knife-edge case of log modularity, in which case

\[
F(L, K, B) F(L, K', B') = F(L, K', B) F(L, K, B')
\]

for all \( K' > K \) and \( B' > B \), the corresponding conditions are \( F_{KB} F = F_B F_K \), and \( \varepsilon_{KB} = \varepsilon_K \). These results imply

\[14\]The derivation of the first order condition is found in the appendix.
Proposition 2 A production tax is inefficient whenever the production function is log sub- or log supermodular in capital and the public service. Log submodularity (supermodularity) leads to underprovision (overprovision) of public services to firms. When a production function is log modular, a production tax is efficient.

Note that Proposition 2 does not depend on property 2. However, there is a relationship between log (sub)modularity and property 2. In particular,

Lemma 2 If \( F(L, K, B) \) is log (sub)modular and strictly concave in \((K, B)\), then property 2 holds.

The proof is provided in the appendix.

Suppose the production function is log submodular and hence property 2 holds, so that both the optimal capital tax and the optimal production tax result in underprovision. In this case, we can unambiguously rank the two tax scenarios in terms of relative efficiency, as shown in

Proposition 3 Suppose that the production function is log submodular in \((K, B)\), and that interior solutions to both the optimal capital tax and the optimal production tax problems exist. Under these conditions, the production tax is unambiguously more efficient than the capital tax.

Proof Suppose the level of public services is the same under both tax regimes. That is,

\[
tF = \tau K = B, \tag{20}
\]

in which case total output is the same in all jurisdictions (given symmetry and a fixed national aggregate capital stock) and thus all derivatives are also identical. Multiplying the first order condition for each tax regime by its respective tax yields

\[
\tau (F_{KB}B - F_B\tau) - F_{KK}B (F_B - 1) = 0 \tag{21}
\]
\[
F_Kt (F_{KB}B - F_BF_Kt) - F_{KK}B(F_B - 1) = 0. \tag{22}
\]

Since both regimes lead to underprovision of public services, \(-F_{KK}B(F_B - 1) > 0\), and therefore the first terms in both (21) and (22) must be negative. We need to show that, given the optimal level of the public service under the capital tax equilibrium, the taxing jurisdiction would
want to increase the production tax on a revenue neutral basis, which occurs if the first term in (22) is less negative than the first term in (21).\(^\text{15}\)

\[ F_K t < \tau. \]

Substituting from (20) this occurs if

\[ F_K \frac{\tau K}{F} < \tau \]
\[ F_K \frac{K}{F} < F \]

which is the case since \( F > F_K K \).

Note that this proof is valid even if there are multiple candidates for the optimal tax rates. The provision rule under each tax regime indicates how a jurisdiction should choose the optimal tax rate, given \( K \). In the case in which the rule does not provide a unique tax rate, jurisdictions will choose the tax rate from all the candidates provided by the provision rule that leads to the highest residents’ income, given \( K \). Residents’ income as a function of the public service level, which is calculated from the tax rate and \( K \), is well-behaved and strictly concave in the public service level with its maximum at the efficient provision level of the public service. That is, the candidates for the optimal \( \tau \) are chosen from

\[ (F_K B - F_B \tau) - F_{KK} K (F_B - 1) = 0, \]

which, after multiplying both sides by \( K \), yields

\[ (F_K B K - F_B B) - F_{KK} K^2 (F_B - 1) = 0, \]

and gives us the candidates for the optimal \( B \) as a function of given \( K \). The objective function in the case of capital taxation can be written as

\[ I_{CT} (B) = r\overline{K}/N + [F (L, K, B) - B - r K], \]

where the \( B \) argument indicate that the objective function is evaluated for a given value of \( K \), and \( B \) is selected from the candidates for the optimal

\(^{15}\text{Recall that in order to obtain (22) we multiplied with two negative terms.}\)
capital tax. Hence the tax rates that we are evaluating are all associated with the same $K$. Then

\[
\begin{align*}
\frac{\partial I(B)}{\partial B} &= F_B - 1 \\
\frac{\partial^2 I(B)}{\partial B^2} &= F_{BB} < 0.
\end{align*}
\]

Hence the function $I_{CT}(B)$ is strictly concave in $B$, and in Proposition 3 we pick the $\tau$ that gives us the highest value for $I_{CT}(B)$ among all the $\tau's$ satisfying the optimality condition (21).

Similarly, the objective function in case of taxation of production can be written as

\[
I_{PT}(t, K) = r\overline{K}/N + [(1-t) F(L, K, B) - rK],
\]

where we can calculate from the optimality condition for the production tax (22), given $K$, the corresponding $B$ by using $tF = B$. That is, we can rewrite 22 by multiplying the equation by $\frac{F}{t}$ to obtain

\[
F_KB (F_{KB}F - F_BF_K) - F_{KK}F^2 (F_B - 1) = 0.
\]

From this equation, we can find the candidates of the optimal $B$, as a function of $K$ and hence for given $K

\[
I_{PT}(B) = r\overline{K}/N + [F(L, K, B) - B - rK]
\]

\[
\begin{align*}
\frac{\partial I(B)}{\partial B} &= F_B - 1 \\
\frac{\partial^2 I(B)}{\partial B^2} &= F_{BB} < 0.
\end{align*}
\]

Hence the function $I(B)$ is strictly concave in $B$, and in proposition 3 we pick the $B$ that gives us the highest value for $I_{PT}(B)$ among all the $B's$ satisfying the optimality condition (22). However, this $B$ and its corresponding tax rate will not maximize the function $I_{PT}(B)$ in the case of log submodularity of the production function, as we know that both tax regimes lead to underprovision of the business public service since the taxing jurisdiction anticipates an outflow of capital in response to the imposition of the tax.
To compare the levels of underprovision under a capital tax and a production tax when the production function is log submodular, we can assume the jurisdiction has adopted the optimal capital tax and the corresponding level of public service, and then examine whether the jurisdiction would find it desirable to increase the level of public service provision using a production tax. We find that this is the case. What is the interpretation of this result? It is true that by finding that the jurisdiction would increase the production tax at this point, we could be moving toward a suboptimal production tax rate. However, given the assumption that we have an interior solution under both tax regimes, the optimal production tax must be larger than any suboptimal production tax rate, so that switching to a production tax equilibrium increases the level of public service provided and results in less underprovision and thus less inefficiency than under the capital tax equilibrium.

Our results thus far validate Bird’s conjecture that a broad-based tax on local production is more efficient than a tax on capital in the case of log submodular production functions. However, there is no analog to proposition 3 in the case of log supermodular production functions, although there are prominent examples of such functions. In fact, the family of CES production functions includes examples of all three properties, depending on whether the elasticity of substitution between capital and public services/labor is greater than, equal to or smaller than one. In a companion paper, Gugl and Zodrow [8] examine more closely this class of production functions in the context of tax competition and the relative efficiency of capital taxation and production taxation.

Note that the converse of Lemma 2 is not true; even if property 2 holds, no restriction is imposed on the log modularity property of \( F(L, K, B) \). Matsumoto [15] considers the case in which \( F(L, K, B) \) is linearly homogenous in \((L, K)\) and points out that this implies \( KF_{KB} < F_B \), i.e. property 2 holds. While property 1 no longer follows, Matsumoto makes a similar stability assumption so that the numerator of \( \phi \) is negative. In this case, capital taxation always leads to underprovision of public services, while production taxes can still lead to efficiency or overprovision of the public service. Applying the same assumptions in the case of taxing production would keep Proposition 3 valid in the case in which \( F(L, K, B) \) is linearly homogenous in \((L, K)\).
3 Conclusion and Extensions

Can a production tax, such as an origin-based value-added tax, approximate a benefit tax for public services provided to businesses, as suggested by Bird and others? And how does a source-based capital tax such as the property tax compare to a production tax as a proxy for a benefit tax? Using the Zodrow-Mieszkowski [34] model of interjurisdictional tax competition, we find that a production tax more closely approximates a benefit tax than does a capital tax in many instances. In particular, although a production tax is strictly efficient only when the production function is log modular in the public service and capital, it is less inefficient than a capital tax in the case of log submodular production technologies.

Although we focus in our examples on CRS technologies, propositions 1–3 do not depend on this assumption if property 1 is assumed. For example, these three propositions also obtain with increasing returns to scale in all factors and CRS in the private factors, as long as the assumption of strict concavity (implying decreasing returns to scale) in public services and capital and homotheticity in those two factors is made. Our results are also robust to a change in the nature of the public input, as they would not be changed by assuming a pure public input instead of a publicly provided private input. Incorporating imperfectly congestible inputs, however, would require us to modify the model considerably more than accommodating pure public inputs. In this case, we would need to deal with the crowding externality stressed by Matsumoto [15], although the factors driving the results obtained in this paper would still be highly relevant.

Moving from a retail sales tax that is known to tax business inputs (due to problems in its administration) to a pure consumption tax as occurred in British Columbia in July 2010 spurred fierce opposition. While some of that opposition was sparked by the way the HST was introduced by the provincial government, critics of the HST also emphasized its shift away from "taxing business." The opposition was so forceful that continuation of the HST was the subject of a mail-in referendum, in which almost 55% of voters opposed the HST. As a result, British Columbia had to make the painful transition to re-establishing its provincial retail sales tax.

Our analysis suggests that a production tax may be a viable business tax alternative to the provincial retail sales tax, which is typically characterized by the taxation of business inputs similar to that which occurs under our capital tax. By comparison, at least under certain circumstances in our
admittedly highly stylized model, a production tax is less distortionary than the capital tax portion of a retail sales tax. Now that the voters of British Columbia have rejected the HST, an origin-based VAT might lead to less inefficiency than a provincial retail sales tax. We plan to investigate this issue in more general models in future work.

4 Appendix

4.1 Proof of Lemma 1

We show that

\[ F_{KK} + F_{KB} \tau < 0. \]  \hspace{1cm} (23)

Multiply (23) by \( K \) to obtain

\[ F_{KK}K + F_{KB} \tau K. \]

Budget balance implies

\[ \tau K = B. \]

Hence

\[ F_{KK}K + F_{KB} \tau K \leq F_{KK}K + F_{BB}B. \]

By property 1 \( F_{KK}K + F_{KB}B < 0 \), hence \( F_{KK} + F_{KB} \tau < 0 \).

4.2 Proof of Proposition 1

Substituting for \( \phi \) in (14) yields

\[ \tau = \frac{K F_{KK} (1 - F_B)}{F_B - K F_{KB}}. \]

Note \( K F_{KK} < 0 \). Then \( \tau > 0 \) if and only if

- case 1: \( 1 - F_B < 0 \) and \( F_B - K F_{KB} > 0 \),
- case 2: \( 1 - F_B > 0 \) and \( F_B - K F_{KB} < 0 \).

By property 2, \( F_B > K F_{KB} \). This means we are in case 1, and \( F_B > 1 \). Moreover by (14)

\[ F_B = \frac{K}{K - \tau \phi}, \]
which implies that $\phi > 0$, concluding the proof.

Note that in the case in which taxes on residents or firms are imposed at an inefficiently low level, such that

$$\tau K = B,$$

the same logic applies. Thus proposition 1 holds also if jurisdictions rely to some extent on these other taxes.

Without property 2, case 2 is possible. From condition (13) it is also possible that $F_B = 1$, and $\phi = 0$. In this case the optimal $\tau$ is found by solving

$$F_B = 1.$$ 

4.3 Denominator of $\frac{dK}{dt}$ is negative

From (16) we know that $B = tF$. We need to sign

$$(F_{KK} (1 - F_B t) + F_{KB} F_K t)$$

$$= \frac{1}{F} (F_{KK} (F - F_B t F) + F_{KB} F_K t F)$$

$$= \frac{1}{F} (F_{KK} (F - F_B B) + F_{KB} B F_K)$$

$$= \frac{1}{F K} (F_{KK} K (F - F_B B) + F_{KB} B F_K F_K).$$

By property 1, $-F_{KK} K > F_{KB} B$. Strict concavity of $F(K, B)$ implies $F(L, K, B) > F_B B + F_K K$ and hence $F - F_B B > F_K K$. Thus $F_{KK} (1 - F_B t) + F_{KB} F_K t < 0$.

4.4 Proof of Proposition 2

When the government chooses $t$ to maximize the income of residents, the FOC is

$$-F + (1 - t) F_B \frac{F}{1 - F_B t} + \left( F_B \frac{F_K t}{1 - F_B t} \right) \frac{F_K (1 - F_B t) - F_{KB} F (1 - t)}{(F_{KK} (1 - F_B t) + F_{KB} F_K t)} = 0.$$ 

(24)

We now investigate the conditions for an interior solution with $1 > t > 0$. Note that $(1 - F_B t) > 0$, so we can multiply both sides of (24) by this term.$^{16}$

$^{16}$ $F_B t = F_B \frac{t}{F}$. Since $F_B B < F$, $(1 - F_B t) > 0$. 

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After collecting terms

\[ F (F_B - 1) + F_B F_K t \frac{F_K (1 - F_B t) - F_K B F (1 - t)}{(F_K K (1 - F_B t) + F_K B F_K t)} = 0 \]  \quad (25)

Note that an interior solution must have the following properties: If there is underprovision of public services, i.e. \( F_B > 1 \), then the second term must be negative and hence \( \frac{dK}{dt} < 0 \). If there is overprovision, i.e. \( F_B < 1 \), then the second term must be positive and hence \( \frac{dK}{dt} > 0 \). If \( t \) is efficient, \( \frac{dK}{dt} = 0 \). At this point we cannot exclude any of these possibilities. The denominator of \( \frac{dK}{dt} \) is negative, hence we can multiply both sides of (25) by this term to obtain

\[ F (F_B - 1) (F_K K (1 - F_B t) + F_K B F_K t) + F_B F_K t (F_K (1 - F_B t) - F_K B F (1 - t)) = 0 \]

\[ (F_K K F (F_B - 1) + F_B F_K F_K t)(1 - F_B t) + F_K B F_K t F (F_B - 1) - F_K B F_K t F (1 - t) = 0 \]

\[ (tF_B - 1) (F_K t (F_K B F - F_B F_K) - F_K K F (F_B - 1)) = 0 \]

Multiplying both sides with \( tF_B - 1 < 0 \) yields

\[ F_K t (F_K B F - F_B F_K) - F_K K F (F_B - 1) = 0. \]

### 4.4.1 Log Super- and Log Submodularity

In this section we show that the property of log supermodularity implies \( F_K B F - F_B F_K > 0 \), the property of log submodularity implies \( F_K B F - F_B F_K < 0 \), and the knife edge’s case of log modularity (which falls between log super- and log submodularity) implies \( F_K B F - F_B F_K = 0 \).

\( F (L, K, B) \) is log supermodular if and only if

\[ F (L, K, B) F (L, K', B') > F (L, K', B) F (L, K, B') \]  \quad (26)

for all \( K' > K \) and \( B' > B \). Checking for log supermodularity using differentiation

\[ \frac{\partial F (L, K, B')}{\partial K} \frac{1}{F (L, K, B')} > \frac{\partial F (L, K, B)}{\partial K} \frac{1}{F (L, K, B)} \]  \quad (27)

if and only if \( B' > B \).\(^{17}\) With \( F () \) increasing, a necessary condition for this inequality to hold is \( \frac{\partial F (L, K, B)}{\partial KB} > 0 \). For sufficiency, the output elasticity of \( K \)

\(^{17}\)See e.g. Smith [27].
must be increasing with the increase in $B$ as (27) can be written as

$$\frac{\partial F(L, K, B')}{\partial K} \frac{K}{F(L, K, B')} > \frac{\partial F(L, K, B)}{\partial K} \frac{K}{F(L, K, B')}.$$  

Let $\Phi(L, K, B) = \frac{\partial F(L,K,B)}{\partial K} \frac{1}{F(L,K,B)}$. Then if $F(L, K, B)$ is log supermodular, $\Phi(L, K, B)$ must be increasing in $B$. Thus take derivative wrt $B$ of $\Phi(K, B)$:

$$\frac{\partial \Phi}{\partial B} = \frac{1}{F} F_{KB} - \frac{F_{K} F_{B}}{F_{K}^2} > 0$$

This inequality can be rewritten

$$F_{KB} F - F_{K} F_{B} > 0.$$  

How is Substitution elasticity related to log (sub/super)modularity?

$$\begin{align*}
| \frac{\partial (\frac{B}{K})}{\partial MRTS} \frac{MRTS}{B/K} | &= \frac{F_{BK}}{F_{KB}} \left( \frac{\partial \left( F_{B} \left( \frac{B}{K} K, 1 \right) / F_{K} \left( \frac{B}{K} K, 1 \right) \right)}{\partial (B/K)} \right) \\
&= \frac{F_{BK}}{F_{KB}} \left( \frac{F_{BB} F_{K} - F_{KB} K F_{B}}{F_{K}^2} \right) \\
&= \frac{F_{BK}}{B \left( F_{BB} F_{K} - F_{KB} F_{B} \right)} \\
F(B, K) &= \left( B^{\sigma} + K^{\alpha} \right)^{\frac{1}{\sigma}} \\
MRTS &= \frac{F_{B}}{F_{K}} = \left( \frac{B}{K} \right)^{\sigma - 1} \\
\left( \frac{B}{K} \right) &= MRTS^{\frac{1}{\sigma - 1}} \\
| \frac{\partial (\frac{B}{K})}{\partial MRTS} \frac{MRTS}{B/K} | &= \frac{1}{(1 - \sigma)} MRTS^{-\frac{1}{1 - \sigma} - 1 + 1} \left( \frac{K}{B} \right) \\
| \frac{\partial (\frac{B}{K})}{\partial MRTS} \frac{MRTS}{B/K} | &= \frac{1}{(1 - \sigma)} MRTS^{-\frac{1}{1 - \sigma} + \frac{1}{1 - \sigma}} = \frac{1}{(1 - \sigma)}
\end{align*}$$

Therefore the efficiency condition under a production tax depends on whether $F(L, K, B)$ is log supermodular or log submodular in $(K, B)$. Log submodularity corresponds to underprovision, log supermodularity to overprovision, and the knife-edge case of log modularity corresponds to efficiency.
4.5 Proof of Lemma 2

Log (sub) modularity requires

\[ F_{KB}F - F_KF_B \leq 0. \]

Rearranging and multiplying by \( K \)

\[ \frac{F_{KB}K}{F_B} \leq \frac{F_KK}{F} \]

By strict concavity of \( F \) in \((K, B)\)

\[ F_KK < F \]

Hence

\[ \frac{F_{KB}K}{F_B} < 1 \]

Rearranging yields the desired result

\[ F_B - F_{KB}K > 0. \]

References


[8] Gugl, Elisabeth and George R. Zodrow (2014), The Efficiency of “Benefit-Related” Business Taxation” Paper prepared for presentation at a conference on “Subnational Government Competition,” sponsored by The Office of Tax Policy Research at the University of Michigan, the Center for Business and Economic Research at the University of Tennessee, and the Department of Economics at the University of Georgia and held on April 25–26, 2014 at the University of Tennessee, Knoxville, TN.


