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# "Energy Sector Innovation and Growth"

by

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# Energy Sector Innovation and Growth*

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**Abstract**

We study the optimal transition from fossil fuels to renewable energy in a neoclassical growth economy with endogenous technological progress in energy production from fossil fuels and renewable energy sources. Innovations keep fossil energy cost under control even as increased exploitation raises mining costs. Nevertheless, the economy eventually transitions to renewable energy. Learning-by-doing in renewable energy production implies that it is optimal to transition to renewable energy before the cost of fossil fuels reaches parity with renewable energy costs. Since energy costs escalate as the transition approaches, growth of consumption and output decline sharply around the transition. The energy shadow price remains more than double current values for over 75 years around the switch time, resulting in a continued drag on output and consumption growth. The model highlights the important role that energy can play in influencing economic growth.

**Keywords**: Energy innovation, energy transition, energy cost, economic growth

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# 1 Introduction

Since the days of the industrial revolution, economic growth has been powered largely by fossil fuel. In recent decades, large-scale energy production through renewable sources has become technologically feasible, albeit expensive and as yet uncompetitive without subsidies. Yet, since fossil fuel is a finite resource that will become more scarce, non-fossil energy must eventually become the economy's main energy source. Admittedly, technological progress can slow the escalation in fossil fuel costs by expanding the quantity of economically viable resource, as has been dramatically illustrated by the recent expansion of oil and natural gas production from shale, and oil production from Canadian oil sands. Technological progress in the form of improved energy efficiency can also reduce the amount of fuel needed to provide a given level of energy services. Nevertheless, the expanding demand for energy resulting from economic and population growth implies that fossil fuel costs ultimately will rise.[1]

The finite supply of fossil fuel raises concern that the need to transition to more expensive alternatives will impose substantial costs. It is often argued that these costs are sub-optimal and should, if possible, be reduced via appropriate policies.[2] In a simple growth model that allows for technological progress in energy production, however, we show that an "energy crisis" around the time the economy optimally abandons fossil fuels can be *efficient*.

Formally, we study the optimal transition path from fossil fuel to renewable energy sources in a neoclassical growth economy with endogenous technical progress in energy production. As in Hartley and Medlock (2005), we assume that energy is needed to produce the economy's single consumption good. Energy can come from two sources: fossil fuel and a renewable source (the backstop technology). The renewable technology combines capital with an exogenous energy source (for example, sunlight or wind) to produce energy services. Throughout the paper, we assume for simplicity that the energy services of fossil fuels and the renewable

[1]DOE/NETL (2007) summarizes several forecasts for the likely year of peak conventional oil production.

[2]See, for example, Farrell and Brandt (2006). The possible macroeconomic costs of an energy transition are distinct from any environmental or congestion externalities associated with using different energy sources. We do not discuss the latter in this paper. In addition, since our model is long run in nature and does not involve uncertainty, we do not model temporary energy crises due to unanticipated supply or demand shocks and binding short-run production capacity constraints.

technology are perfect substitutes.[3]

A central feature of our analysis involves the explicit modeling of technological progress in *both* the fossil and the renewable energy production technologies. In particular, while mining costs increase with cumulative resource development, advancements in mining technologies can keep the cost of supplying fossil fuel energy services under control for some time.[4]

We model technological progress in renewables by assuming that accumulated knowledge lowers unit production cost until a technological limit is attained. We assume a two-factor learning model, whereby direct R&D expenditure can accelerate the accumulation of knowledge about the renewable technology. Following estimates in a paper by Klaassen et. al. (2005), we assume that direct expenditure on R&D is roughly twice as productive as is learning-by-doing in lowering the amount of capital required to harvest the energy.

We calibrate the model using data from the *Energy Information Administration* (EIA), the *Survey of Energy Resources*, and the *GTAP 7 Data Base* produced by the *Center for Global Trade Analysis* in the Department of Agricultural Economics, Purdue University. The last data source provides a consistent set of international accounts that also take account of energy flows. We then compute the optimal path of investment in both the fossil fuel and the renewable energy sectors. We focus on the consequences of energy costs for macroeconomic growth, including around the time of transition from fossil fuels to renewable energy sources.

The calibrated model gives rise to several different regimes, which are depicted graphically in Figure 2 below. Initially, growth occurs through the use of relatively cheap fossil fuel as investment in the fossil fuel technology keeps energy costs from rising substantially.[5] However, fossil energy investments, which must be made at an increasing rate to keep costs

[3]Fossil and renewable energy sources therefore are not employed at the same time in our model. Extending the model to allow simultaneous use of different energy sources is an important issue for further investigation.

[4]The mining technology variable can also be thought of as a reduced form means of capturing the effect of energy efficiency improvements. By reducing the resource input needed to provide a given level of energy services, efficiency improvements also slow down the rise in costs from resource depletion.

[5]Short-run spikes in energy prices result more from binding production capacity constraints than longer-run depletion. We would need to introduce supply and demand shocks and excess production capacity to model such short-run crises. Energy prices also fluctuate more in reality than in our model because we have assumed gradual technological progress and associated expansions in recoverable resources whereas in practice we see periodic breakthroughs amidst steady improvements.

under control as resources are depleted, eventually cease. Shortly thereafter, fossil fuels become uncompetitive and renewable energy powers the economy. Interestingly, the transition from fossil fuels to renewable energy occurs when the cost of fossil energy is less than the initial cost of renewable energy. The reason is that the learning-by-doing element of renewable energy production lowers the shadow price (or full cost) of renewable energy, making it worthwhile to transition before the explicit cost of fossil energy reaches the initial cost of renewable energy.

Once the economy shifts to renewable energy, investment in the renewable technology makes it more effective over time. Immediately after the transition, a spurt of renewable R&D investment produces a steep decline in renewable energy costs for ten to fifteen years. Renewable costs then decline more gradually for a long time, initially mostly as a result of both learning-by-doing and later through both learning-by-doing and growing investment in R&D. The latter then drops to zero as the technological limit is approached.

Finally, the world economy enters a regime where the technological frontier in renewable energy has been reached and the cost of renewable energy is constant. In this regime, the model becomes a simple endogenous growth model with investment only in physical capital. In addition to being realistic, the technological limit on renewable efficiency facilitates numerical solution of our model by allowing the terminal regime to be solved analytically.

Numerical simulation of the model reveals that per capita output grows at an average rate of 4.22% per annum (p.a.) in the fossil regime, 3.11% p.a. in the renewable regime with investment in R&D, and 4.07% p.a. in the long run with renewable energy at its minimum cost. Growth in per capita consumption averages 3.68% p.a. in the fossil energy regime and 3.33% p.a. in the renewable regime with R&D. In the long run regime, per capita capital, consumption, investment in capital, and energy use all grow at the same rate of 4.07% p.a. The gaps between output and consumption growth in the earlier regimes result from investment in energy production and the changing cost of energy over time. In the fossil regime, consumption grows slower than capital (and output) because the rising cost of energy and rising investment in mining both take increasing resources away from consumption. In the renewable regime with R&D, the declining cost of energy allows consumption to grow faster than output.

Toward the end of the fossil regime, the shadow price of energy rises steeply and substantially. In addition, investments in mining technology become rather large, while investments in renewable technology also tend to be relatively large immediately following the transition to renewable sources. As a result, around the transition point, consumption and investment in non-energy capital decline as shares of output. In this sense, our model predicts an "energy crisis" around the switch point. Although consumption remains at close to its current share of around 60% of output for the first fifty years of the fossil fuel regime, it plunges to well below 40% of output at the switch date. The shadow relative price of energy peaks at the switch time and is more than double current levels for over 75 years around the switch time. After the transition, the high cost of renewable energy prevents the consumption share from rising back above 55% of output for another 150 or so years. We emphasize that these are features of the optimal allocation in our model.

We also emphasize the importance of modeling progress in fossil, as well as in renewable energy production. Currently, there is an active discussion regarding subsidizing renewable energy sources. Advances such as shale oil and gas, oil sands production in Canada, and deep water exploration increase the supply of fossil fuel and imply that the "parity cost target" for renewables is a moving one. This important implication is often missing in the policy discussions. Our quantitative findings suggest that these advancements allow fossil fuels to remain competitive for a longer time than is commonly assumed. Ultimately, the model implies that about 80% of the technically recoverable fossil fuel resources in place will be exploited, with the transition to renewable energy occurring at the end of this century.

## 2 Related literature

Our approach is related to a number of papers in the literature. Parente (1994) studies a model in which firms adopt new technologies as they gain firm-specific expertise through learning-by-doing. He identifies conditions under which equilibria in his model exhibit constant growth of per capita output. As in most of the literature on economic growth, Parente abstracts from issues related to energy.

Chakravorty, Roumasset, and Tse (1997) develop a model with substitution between energy sources, improvements in extraction, and a declining cost of renewable energy. They find that if historical rates of cost reductions in renewables continue, a transition to renewable energy will occur before over 90% of the world's coal is used. Our model is complementary to theirs. By modeling investment in energy technologies, we generate an endogenous transition to renewable energy. The explicit presence of investment in physical capital also allows us to explore the endogenous trade-off between the cost of energy and economic growth, which is the focus of our work. Unlike Chakravorty et. al., we do not study the implications of energy use for carbon dioxide emissions and we do not conduct policy experiments.

Tsura and Zemelb (2003) analyze the role of learning through R&D in the optimal transition from a nonrenewable energy resource to a backstop substitute. They find that, if the initial knowledge level is sufficiently low, R&D should start as early as possible and at the highest affordable rate. Their analysis uses a partial equilibrium model. Our analysis differs in many respects, including using a general equilibrium model, modeling technological progress in fossil fuel, and relying on calibrated experience curves when modeling technological progress in renewables.

More recently, Golosov, Hassler, Krusell, and Tsyvinski (2011) built a macroeconomic model that incorporates energy use and the resulting environmental consequences. They derive a formula describing the optimal tax due to the externality from emissions and provide numerical values for the size of the tax in a calibrated version of their model. However, they abstract from endogenous technological progress in either fossil fuels or renewables. As a result, transitions between different energy regimes are exogenous in their model.

Van der Ploeg and Withagen (2011) use a growth model to investigate the possibility of a green paradox, that is a tendency for promotion of renewable energy to accelerate the exploitation of fossil fuel by lowering resource rents and thus the opportunity cost of extraction. Van der Ploeg and Withagen (2012) study optimal climate policy in a Ramsey growth model with exhaustible oil reserves and an infinitely elastic supply of renewables. They consider climate change issues and characterize the different energy regimes, as well as the optimal carbon tax along the economy's growth path. Our model differs from their studies as we model technological progress in renewables using industry experience curves. In addition, we model technological progress in fossil fuels, and we calibrate our model using world-economy data.

A paper that is closest to ours is Acemoglu et al. (2009). They study a growth model that takes into consideration the environmental impact of operating "dirty" technologies. They examine the effects of policies that tax innovation and production in the dirty sectors. Their paper focuses on long run growth and sustainability and abstracts from the endogenous evolution of R&D expenditures. They find that subsidizing research in the "clean" sectors can speed up environmentally friendly innovation without resorting to taxes or quantitative controls on carbon dioxide emissions with their negative impact on economic growth. Consequently, optimal behavior in their model implies an immediate increase in clean energy R&D, followed by a complete switch toward the exclusive use of clean inputs in production. Our work differs from Acemoglu et al. (2009) by explicitly connecting R&D, energy, and growth. This follows from our focus on the effects of the energy technology transition on growth rather than environmental issues.[6] The resulting transition between energy sources in the two models is very different.

More generally, most of the literature ignores the key idea that advances in fossil fuel extraction and end-use efficiency technologies are of first-order importance in addressing the energy transition question. The next section presents the main model used in the remainder of the paper.

[6]While we do not explicitly discuss environmental externalities associated with energy use, allocations in our model can be interpreted as laissez-faire or "business as usual" scenarios in models where such environmental externalities are included.

# 3 The Model

## 3.1 Production Technology

We assume that per capita output[7] $y$ can be written as a linear function of a per capita stock of capital, $k$:[8]

$$y = Ak \tag{1}$$

Capital depreciates at the rate $\delta$, while investment in new capital is denoted by $i$:

$$\dot{k} = i - \delta k \tag{2}$$

Energy is also needed to produce output. Denote the per capita energy [9] derived from fossil fuel resources that is used to produce goods by $R \geq 0$ . We assume that per capita renewable energy supply $B \geq 0$ is a perfect substitute for the energy produced from fossil fuel burning. This assumption is admittedly extreme and it is mainly adopted for simplicity.[10] Thus, we assume that at each moment of time:

$$R + B = y \tag{3}$$

Letting $c$ denote per capita consumption of the sole consumption good, we assume that the lifetime utility function is given by:

$$U = \max \int_0^\infty e^{-\beta\tau} \frac{c(\tau)^{1-\gamma}}{1-\gamma} \, d\tau \tag{4}$$

[7]Although we model economic activity in continuous time, indexed by $t$, we usually simplify notation by omitting $t$ as an explicit argument.

[8]As is well known, this could be regarded as a reduced form of a model where investment, for example in human capital, allows the "productive services" supplied by inputs to expand even if the physical inputs remain fixed. Hence, the marginal product of capital does not decline as $k$ accumulates.

[9]Although "energy" is more properly thought of as an input into the energy sector, and "energy services" an output, we use the terms interchangeably.

[10]Using a continuity argument, we can show that our results remain true if the degree of substitutability is large, but not perfect. See Hassler, Krusell, and Olovsson (2011) for a discussion of desirable short and long-run substitution elasticities in this context.

where $e^{-\beta\tau}$ is the discount factor.

## 3.2 Fossil Fuel Supply

Higher rates of both population growth and per capita economic growth will increase the rate of depletion of fossil fuels. Let $Q$ denote the (exogenous) population and assume that it grows at the constant rate $\pi$. The total fossil fuel used will then be $QR$. Depletion then implies that the marginal costs of resource extraction increase with the total quantity of resources mined to date, $S$, which is the integral of $QR$:

$$\dot{S} = QR \tag{5}$$

As a result of technical change in mining and fossil energy use, the marginal cost of extracting the resources needed to supply a unit of fossil energy services, $g(S, N)$, depends not only on $S$ but also on the state of technical knowledge $N$. Investment in mining technology,[11] or the efficiency with which fossil fuel is used to provide useful energy services,[12] leads to an accumulation of $N$:

$$\dot{N} = n \tag{6}$$

which we assume does not depreciate over time.

At any given moment, one can array producers of fossil fuels by increasing costs of producing output from current proved reserves. The result is a "hockey stick" shaped curve, with a long "handle" of slowly rising costs as output increases, followed by a sharply rising "blade" as current short-run productive capacity limits are reached. As a simple parametric approximation to this situation, we assume that $g(S, N)$ is given by the following function, which is illustrated in Figure 1:

$$g(S, N) = \alpha_0 + \frac{\alpha_1}{\bar{S} - S - \alpha_2/(\alpha_3 + N)} = \alpha_0 + \frac{\alpha_1(\alpha_3 + N)}{(\bar{S} - S)(\alpha_3 + N) - \alpha_2} \tag{7}$$

[11] Examples include horizontal drilling and hydraulic fracturing, deep sea/arctic drilling, and 4D-seismic.

[12] Since we have defined the energy supplies in efficiency units, improvements in energy efficiency also reduce the per-unit cost $g(S, N)$ of supplying an additional unit of $R$.

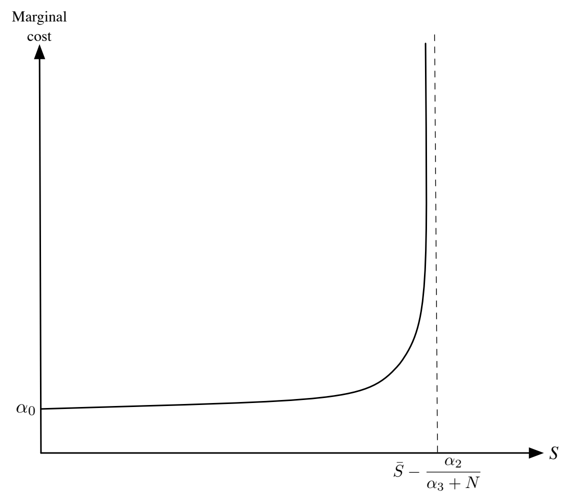


Figure 1: Marginal cost of energy from fossil fuels

For a fixed level of technology $N$, the marginal costs of extraction are increasing and convex in the amount of resources extracted already. The maximum fossil fuel resource that can be extracted at any given time is $\bar{S} - \alpha_2/(\alpha_3 + N)$, and the marginal cost of extraction rises rapidly as this temporary capacity limit is reached. Investment in new technologies expands the temporary capacity limit and the flat portion of the marginal cost curve to the right, extending the competitiveness of fossil fuel.[13] This process inevitably reaches a natural limit as fossil fuel resources are bounded by $\bar{S}$, the absolute maximum technically recoverable fossil fuel resources. This upper limit is only available asymptotically as the stock of investment in new fossil fuel technology $N \to \infty$. Even then, arbitrarily large costs would be incurred in recovering all the technically available resources $\bar{S}$. The terms $\alpha_0, \alpha_1, \alpha_2$ and $\alpha_3$ in (7) are parameters. The partial derivatives of the fossil fuel cost function $g(S, N)$ are presented and discussed in the Appendix.

[13]It may be useful to comment on the contrast between technological change in the fossil fuel industry and the renewable energy sector, which we later assume experiences cost reductions through learning-by-doing. Depletion can be viewed as a form of "inverse learning-by-doing," since cumulative past production raises current costs. Investment in $N$ tends to offset this process in the case of fossil fuels, whereas explicit investment in renewable energy R&D reinforces the cost-reducing effects of learning-by-doing. One could also allow for learning-by-doing in the accumulation of $N$ (so, for example, more widespread application of horizontal drilling and hydraulic fracturing to produce oil and natural gas from shales can result in rapidly declining costs in the short-run as firms learn how to apply the new techniques). This would only exaggerate the point we are trying to make, however, since it would extend the time that fossil fuels remain competitive.

For energy to be productive on net, we need the value of output produced from energy input to exceed the costs of producing that energy input. In particular, whenever fossil fuel is used to provide energy input, we must have $1 > g(S, N)$. Function (7) implies that this constraint eventually must be violated as exhaustion of fossil fuel resources increases $g(S, N)$.

## 3.3 Renewable Energy Technologies

We use $p$ to denote the marginal cost (measured in terms of goods) of the energy services produced using the renewable technology. For the renewable technology to be productive on net, we require $p < 1$. In effect, the renewable technology needs to combine some output (effectively, capital) with an exogenous energy source (for example, sunlight, wind, waves or stored water) to produce more useful output than has been used as an input.

We allow for technological progress by assuming that $p$ declines as new knowledge is gained. Even so, there is a limit, $\Gamma_2$, determined by physical constraints, below which $p$ cannot fall. Explicitly, using $H$ to denote the stock of knowledge about renewable energy production, and $\Gamma_1$ the initial value of $p$ (when $H = 0$), we assume:[14]

$$p = \begin{cases} (\Gamma_1 + H)^{-\alpha} & \text{if } H \leq {\Gamma_2}^{-1/\alpha} - \Gamma_1, \\ \Gamma_2 & \text{otherwise} \end{cases} \tag{8}$$

for constant parameters $\Gamma_1$, $\Gamma_2$ and $\alpha$. We assume that ${\Gamma_1}^{-\alpha} > g(0, 0)$, so that renewable energy is initially uncompetitive with fossil fuels.

Following the learning-by-doing literature, we assume a two-factor learning model, whereby direct R&D expenditure $j$ can accelerate the accumulation of knowledge about the renewable

[14]This functional form is motivated by the literature on experience (or learning) curves. See, for example, International Energy Agency (2000). For a related learning-by-doing modeling of the endogenous technological change in alternative energy sources see Chakravorty, Leach, and Moreaux (2011).

technology arising from its use:[15]

$$\dot{H} = \begin{cases} B^{\psi} j^{1-\psi} & \text{if } H \leq \Gamma_2^{-1/\alpha} - \Gamma_1, \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

In particular, once $H$ reaches its upper limit, further investment in the technology would be worthless and we should have $j = 0$. The parameter $\psi$ determines how investment in research enhances the accumulation of knowledge from experience. Klaassen et. al. (2005) derive robust estimates suggesting that direct R&D is roughly twice as productive for reducing costs as is learning-by-doing.[16] Hence, we assume that $\psi = 0.33$.

## 3.4 The Optimization Problem

Goods are consumed, invested in $k$, $N$, or $H$, or used for producing fossil fuel or renewable energy input. This leads to a resource constraint (in per capita terms):

$$c + i + j + n + g(S, N)R + pB = y \tag{10}$$

The objective function (4) is maximized subject to the differential constraints (5), (6), (2) and (9) with initial conditions $S(0) = N(0) = 0$, $k(0) = k_0 > 0$ and $H(0) = 0$, the resource constraint (10), the definitions of output (1), energy input (3) and the evolution of the cost of renewable energy supply (8). The control variables are $c$, $i$, $j$, $R$, $n$ and $B$, while the state variables are $k$, $H$, $S$ and $N$. Denote the corresponding co-state variables by $q$, $\eta$, $\sigma$ and $\nu$. Let $\lambda$ be the Lagrange multiplier on the resource constraint and $\epsilon$ be the multiplier on the energy constraint (the shadow price of energy). We also need to allow for the possibility that either type of energy source is not used and investment in cost reduction

[15] The Cobb-Douglas form of (9) implies that research alone is not sufficient to reduce the costs of renewable energy production. Experience in deploying the technologies also is essential

[16] Building on an earlier paper by Kouvaritakis et al. (2000), Klaassen et. al. (2005) estimated a two-factor learning curve model that allowed both capacity expansion (learning-by-doing) and direct public R&D to produce cost reducing innovations for wind turbine farms in Denmark, Germany and the UK. They interpret their results as enhancing the validity of the two-factor learning curve formulation.

for the energy technology is zero. To that end, let $\mu$ the multiplier on the constraint $j \geq 0$, $\omega$ the multiplier on the constraint $n \geq 0$, $\xi$ the multiplier on the constraint $R \geq 0$ and $\zeta$ the multiplier on the constraint $B \geq 0$. Finally, let $\chi$ be the multiplier on the constraint $H \leq {\Gamma_2}^{-1/\alpha} - \Gamma_1$ on the accumulation of knowledge about the renewable technology.

Define the current value Hamiltonian and thus Lagrangian by

$$\begin{aligned} \mathcal{H} &= \frac{c^{1-\gamma}}{1-\gamma} + \lambda \left[Ak - c - i - j - n - g(S,N)R - (\Gamma_1 + H)^{-\alpha} B\right] + \epsilon(R + B - Ak) \\ &+ q(i - \delta k) + \eta B^{\psi} j^{1-\psi} + \sigma Q R + \nu n + \mu j + \omega n + \xi R + \zeta B + \chi[\Gamma_2^{-1/\alpha} - \Gamma_1 - H] \end{aligned} \tag{11}$$

The first order conditions for a maximum with respect to the control variables are:

$$\frac{\partial \mathcal{H}}{\partial c} = c^{-\gamma} - \lambda = 0 \tag{12}$$

$$\frac{\partial \mathcal{H}}{\partial i} = -\lambda + q = 0 \tag{13}$$

$$\frac{\partial \mathcal{H}}{\partial j} = -\lambda + (1-\psi)\eta B^{\psi} j^{-\psi} + \mu = 0; \mu j = 0, \mu \geq 0, j \geq 0 \tag{14}$$

$$\frac{\partial \mathcal{H}}{\partial n} = -\lambda + \nu + \omega = 0, \omega n = 0, \omega \geq 0, n \geq 0 \tag{15}$$

$$\frac{\partial \mathcal{H}}{\partial R} = -\lambda g(S,N) + \epsilon + \sigma Q + \xi = 0, \xi R = 0, \xi \geq 0, R \geq 0 \tag{16}$$

$$\frac{\partial \mathcal{H}}{\partial B} = -\lambda(\Gamma_1 + H)^{-\alpha} + \epsilon + \eta\psi B^{\psi-1} j^{1-\psi} + \zeta = 0, \zeta B = 0, \zeta \geq 0, B \geq 0 \tag{17}$$

The differential equations for the co-state variables are:

$$\dot{q} = \beta q - \frac{\partial \mathcal{H}}{\partial k} = (\beta + \delta) q - \lambda A + \epsilon A \tag{18}$$

$$\begin{aligned} &\dot{\eta} = \beta\eta - \frac{\partial \mathcal{H}}{\partial H} = \beta\eta - \lambda\alpha(\Gamma_1 + H)^{-\alpha-1} B + \chi; \\ &\chi[(\Gamma_2^{-1/\alpha} - \Gamma_1 - H] = 0, \chi \geq 0, H \leq \Gamma_2^{-1/\alpha} - \Gamma_1 \end{aligned} \tag{19}$$

$$\dot{\sigma} = \beta\sigma - \frac{\partial \mathcal{H}}{\partial S} = \beta\sigma + \lambda \frac{\partial g}{\partial S} R \tag{20}$$

$$\dot{\nu} = \beta\nu - \frac{\partial\mathcal{H}}{\partial N} = \beta\nu + \lambda\frac{\partial g}{\partial N}R \tag{21}$$

We also recover the resource constraint (10) and the differential equations for the state variables, (2), (9), (5) and (6).

## 3.5 The evolution of the economy

Section 2 of the Appendix provides a detailed analysis of the evolution of the economy through various regimes of energy use and energy technology investment working backwards through time. In this section, we provide an overview of the different regimes.

We assume that parameter values are set so that initially all energy services are provided by lower cost fossil fuels. As fossil fuels are depleted by both population and per capita economic growth, however, the shadow price of energy services ($\epsilon$) will rise. Although investments in $N$ moderate the increase in fossil fuel costs, eventually the value of $\epsilon$ from (16) will rise to equal the value of $\epsilon$ in the renewable regime obtained from (17). At that time, which we will denote $T_1$, the economy switches to use only renewable energy and all use of, and investment in, fossil fuel technologies ceases.

The co-state variable $\sigma$ corresponding to the state variable $S$ satisfies $\sigma = \partial V/\partial S$, where $V$ denotes the maximized value of the objective subject to the constraints. In particular, $\sigma = 0$ at $T_1$ since $S$ has no effect once fossil fuels cease to be used. Also, since an increase in $S$ raises the cost of fossil fuel while fossil fuels are used, $\sigma$ will be negative for $t < T_1$.[17] Hence, (16) implies that the shadow price of energy converges to $\epsilon = \lambda g(S, N)$ as $t \to T_1$.

Once the economy starts to use renewable energy, both the accumulation of experience and explicit R&D investment will raise $H$. Eventually, however, the economy will attain the technological frontier for renewable energy efficiency at another time $T_2$. Explicit investment $j$ in $H$ will then cease. Since changes in $H$ have no further effect on maximized utility beyond $T_2$, the co-state variable $\eta$ corresponding to $H$ must satisfy $\eta = \partial V/\partial H = 0$ at $T_2$.

[17] Formally, if $\sigma(\tau) > 0$ for $\tau < T_1$, since $\partial g/\partial S > 0$, (20) would imply $\dot{\sigma} > 0$ and $\sigma > 0$ for all $t \geq \tau$ contradicting $\sigma(T_1) = 0$. Heal (1976) introduced the idea of an increasing marginal cost of extraction to show that the optimal price of an exhaustible resource begins above marginal cost, and falls toward it over time. This claim is rigorously proved in Oren and Powell (1985). See also Solow and Wan (1976).

For $t \in [T_1, T_2)$, $\eta > 0$ since an increase in $H$ will lower the shadow price of energy services and raise $V$.[18] In particular, we must have $\eta > 0$ at $T_1$ with $\eta \downarrow 0$ as $t \to T_2$.

It is conceivable that investment $j$ in $H$ could become worthwhile prior to $T_1$ when renewable energy begins to be used. Observe from (14), however, that so long as $B > 0$ we must also have $j^{\psi}\lambda \geq (1-\psi)\eta B^{\psi} > 0$. Also, when $j > 0$, it must satisfy

$$j = [(1-\psi)(\eta/\lambda)]^{1/\psi} B \tag{22}$$

and we conclude that $B > 0$. Thus, we cannot have $j > 0$ and $B = 0$. Hence, $j$ also becomes positive for the first time at $T_1$ and we must also have $H = 0$ at $T_1$. Then (17) and continuity of the shadow price of energy at $T_1$ will require

$$\epsilon = \lambda g(S, N) = \lambda \Gamma_1^{-\alpha} - \eta\psi B^{\psi-1} j^{1-\psi} \tag{23}$$

Since the total energy input requirement $R + B = Ak$, $B$ must immediately jump from 0 to $Ak > 0$ as $R$ declines from $Ak$ to 0 at $T_1$. We then must also have $j > 0$ at $T_1$. Equation (23) then implies that the transition from fossil fuels to renewable energy will occur when the mining cost of fossil energy, $g(S, N)$, is strictly less than the initial cost of renewable energy $\Gamma_1^{-\alpha}$. Thus, the benefits of learning by doing make it worthwhile to transition to renewable energy before the cost of fossil fuels reaches parity with the cost of renewable energy.

While we have shown that we cannot have $j = 0$ while $B > 0$, we will have a regime where investment in fossil fuel technology $n = 0$ while fossil fuels continue to be used ($R > 0$). Specifically, since changes in $N$, like changes in $S$, have no effect once the economy abandons fossil fuels at $T_1$, the co-state variable $\nu$ corresponding to $N$ satisfies $\nu = \partial V/\partial N = 0$ at $T_1$. On the other hand, (13) implies $\lambda = q > 0$, so from (15), $\omega = \lambda - \nu > 0$ and hence $n = 0$ at $T_1$. For $t < T_1$, increases in $N$ will reduce fossil fuel mining costs and raise the maximized value of the objective subject to the constraints, so $\nu = \partial V/\partial N > 0$.[19] As we

[18]Formally, if $\eta(\tau) \leq 0$ at $\tau \geq T_1$, (19) would imply $\dot{\eta} < 0$ and $\eta < 0$ for all $t \geq \tau$ contradicting $\eta(T_2) = 0$.

[19]Formally, if $\nu(\tau) < 0$ for $\tau < T_1$, since $\partial g/\partial N < 0$, (21) would imply $\dot{\nu} < 0$ and $\nu < 0$ for all $t > \tau$ contradicting $\nu(T_1) = 0$.

move backwards in time from $T_1$, $\nu$ will be increasing while $\lambda$ is decreasing. Hence, we will arrive at a time $T_0$ when $\nu = \lambda$, and for $t < T_0$ we will have $n > 0$ in addition to $R > 0$. From (15), we will also continue to have $\nu = \lambda$ for $t < T_0$.

In summary, we have shown that the economy will pass through the regimes illustrated in the time line in Figure 2. Section 2 of the Appendix provides a detailed technical characterization of these regimes.

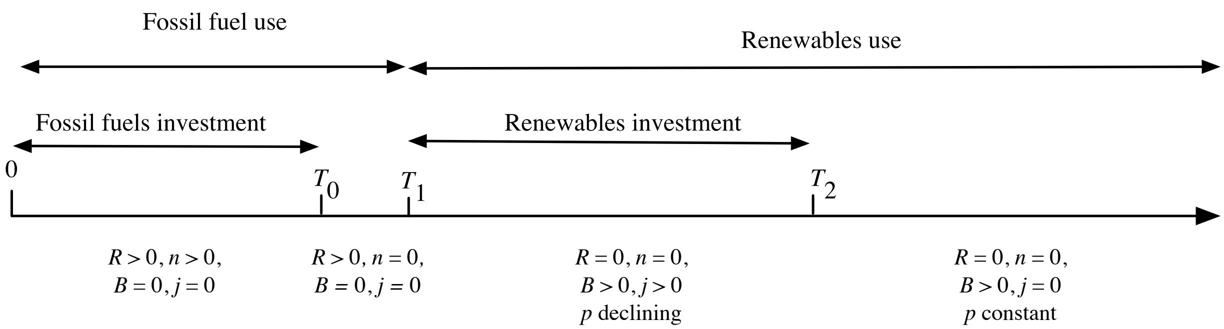


Figure 2: Regimes of energy use and investment

# 4 Calibration

In order to judge whether the effects of energy costs on growth are quantitatively significant, we need to calibrate the parameters and solve the model numerically. The numerical solution procedure is discussed in section 3 of the Appendix. In this section we discuss the data used to calibrate the parameters.

For convenience, we take the current population $Q_0 = 1$ and effectively measure future population as multiples of the current level. We will assume that the world population growth rate is 1%.[20] In line with standard assumptions made to calibrate growth models, we assume a continuous time discount rate $\beta = 0.05$. From previous analyses, we would expect the coefficient of relative risk aversion $\gamma$ to lie between 1 and 10, but there is no strong consensus on what the value should be. For most of our calculations, we assumed

[20] This is consistent with a simple extrapolation of recent world growth rates reported by the Food And Agriculture Organization of the United Nations, http://faostat.fao.org/site/550/default.aspx

that $\gamma = 4$, although we examined some results for a few larger and smaller values of $\gamma$.[21] Note that, as one would suspect from the result that the long run growth rate of the economy is, from (35), given by $-\bar{A}/\gamma$, changes in $\gamma$ primarily alter base level economic growth rates, but do not much affect deviations from those growth rates as a result of energy costs.

To calibrate values for the initial production, capital and energy quantities we used data from the *Energy Information Administration* (EIA),[22] the *Survey of Energy Resources 2007* produced by the *World Energy Council*, [23] and *The GTAP 7 Data Base* produced by the *Center for Global Trade Analysis* in the Department of Agricultural Economics, Purdue University.[24] The last mentioned data source is useful for our purposes because it provides a consistent set of international accounts that also take account of energy flows.

One of the first issues we need to address is that national accounts include government spending in GDP, which does not appear in the model.[25] We therefore subtracted government spending from the GDP measures before calibrating the remaining variables.[26] After excluding government, the investment share of private sector expenditure is 0.2273. Effectively defining units so that aggregate output is 1, we therefore identify 0.2273 as the sum $i + n$ at $t = 0$. We would expect most of this to be investment in capital used to produce output rather than improvements in fossil fuel mining or energy efficiency.

Converting the GTAP data base estimates of the total capital stock to units of GDP, we obtain the initial $k(0) = 3.6071$. Then if we choose units so that output equals 1, the parameter $A$ would equal the ratio of output to capital, that is, $A \approx 0.2772$. We also use the GTAP depreciation rate on capital of 4%.

[21] More precisely, as there is no uncertainty in our model, this parameter relates to intertemporal elasticity of substitution.

[22] International data is available at http://www.eia.doe.gov/emeu/international/contents.html

[23] This is available at http://www.worldenergy.org/publications/survey_of_energy_resources_2007/default.asp The data are estimates as of the end of 2005.

[24] Information on this can be found at https://www.gtap.agecon.purdue.edu/databases/v7/default.asp The GTAP 7 data base pertains to data for 2004.

[25] In the GTAP data base, aggregate world exports equal aggregate world imports so world GDP equals consumption plus investment plus government expenditure.

[26] Government spending would not affect the equilibrium if it was financed by lump sum taxes and the utility obtained from it was additively separable from the utility obtained from private consumption. In any case, since government spending is concentrated on services rather than production processes using energy as a significant input, we believe that including government spending would not significantly alter the results.

From the resource constraint, the difference between total output and the sum of the investments, namely 0.7727 would equal consumption plus the current costs $gR$ of supplying fossil fuel energy. We separated these two components using sectoral data from the GTAP data base. Specifically, we classified "energy expenditure" as combined spending on the primary fuels coal, oil and natural gas[27] and the energy commodity transformation sectors of refining, chemicals, electricity generation and natural gas distribution.[28] This produced a value for $gR = 0.1107$. Subtracting the initial value for $gR$ from 0.7727 we obtain the initial value of $c(0) = 0.6620$. The *model* solution for $c(0)$ would follow from the first order condition $\lambda(0) = c(0)^{-\gamma}$. To obtain the calibrated value for $c(0)$ we then need to free up an additional parameter. We will return to this issue below.

After we set the initial values of $S$ and $N$ to zero, the initial value for $gR$ would imply

$$\frac{0.1107}{R} = \alpha_0 + \frac{\alpha_1}{\bar{S} - \alpha_2/\alpha_3} \tag{24}$$

We can obtain a value for total fossil fuel production, $R$, from the EIA web site. It gives world wide production of oil in 2004 of 175.948 quads (where one quad equals $10^{15}$ BTU), of natural gas 100.141 quads and of coal 116.6 quads. Summing these gives a total of 392.689 quads. We then choose energy units so that the initial value of $R = 1$.

To obtain an estimate of total fossil fuel resources $\bar{S}$ in the same units, we begin with the proved and estimated additional resources in place from the World Energy Council. The millions of tonnes of coal, millions of barrels of oil, extra heavy oil, natural bitumen and oil shale and trillions of cubic feet of natural gas given in that publication were converted to quads using conversion factors available at the EIA. The result is 115.2 quintillion BTU, or almost 300 times the annual worldwide production of fossil fuels in 2004. These resources

[27] Recall that we abstract from initial renewable and nuclear energy production.

[28] A component of the investment expenditure in the energy transformation sector would, in practice, be directed toward increasing energy efficiency and conceptually should be counted as part of $n$. Like mining investments, it would reduce the cost of providing a given level of energy services from a given reserve of primary fuels. However, we do not have sufficient information to split investment in energy transformation into a component that raises energy efficiency and a component that simply expands existing transformation capacity. The assumption we have made is that the vast majority of the expenditure is not aimed at increasing energy efficiency.

are nevertheless relatively small compared to estimates of the volume of methane hydrates that may be available. Although experiments have been conducted to test methods of exploiting methane hydrates, a commercially viable process is yet to be demonstrated. Partly as a result, resource estimates vary widely. According to the National Energy Technology Laboratory (NETL),[29] the United States Geological Survey (USGS) has estimated potential resources of about 200,000 trillion cubic feet in the United States alone. According to Timothy Collett of the USGS,[30] current estimates of the worldwide resource in place are about 700,000 trillion cubic feet of methane. Using the latter figure, this would be equivalent to 719.6 quintillion BTU. Adding this to the previous total of oil, natural gas and coal resources yields a value for $\bar{S} = 834.8$ quintillion BTU or around 2126.0527 in terms of the energy units defined so that $R = 1$.

We still need to specify values for the $\alpha_i$ parameters in the $g$ function. Equation (24) with $R \equiv 1$ and $\bar{S} = 2126.0527$ will give us one equation in four unknowns. Noting that $\bar{S} - \alpha_2/\alpha_3$ is the level of fossil fuel extraction $S$ at $t = 0$ at which marginal costs of extraction $g(S, 0)$ become unbounded, we associate $\bar{S} - \alpha_2/\alpha_3$ with current proved and connected reserves of fossil fuel.[31] A recent report from Cambridge Energy Research Associates (Jackson, 2009), for example, gives weighted average decline rates for oil production from existing fields of around 4.5% per year. They also note that this figure is dominated by a small number of "giant" fields and that, "the average decline rate for fields that were actually in the decline phase was 7.5%, but this number falls to 6.1% when the numbers are production weighted." Hence, we shall use 6% as a decline rate for oil fields. If we use United States production and reserve figures as a guide, we find that natural gas decline rates are closer to 8 per year but coal mine decline rates are closer to 6% per year. In accordance with these figures, we assume the ratio of fossil fuel production to proved and connected reserves equals the share weighted average of these figures, namely $(175.948 * 0.06 + 100.141 * 0.08 + 116.6 * 0.06)/392.689 = 0.0651$. Thus, in terms of the energy units defined so that $R = 1$, the initial value of $\bar{S} - \alpha_2/\alpha_3$

[29] http://www.netl.doe.gov/technologies/oil-gas/FutureSupply/MethaneHydrates/about-hydrates/estimates.htm

[30] http://www.netl.doe.gov/kmd/cds/disk10/collett.pdf

[31] Current official reserves are not the relevant measure since many of these are not connected and thus are unavailable for production without further investment, denoted $n$ in the model.

would equal 1/0.0651=15.361. Using the previously calculated value for $\bar{S}$, this leads to $\alpha_2/\alpha_3 = 2110.538$.

We can obtain two more equations by specifying the partial derivatives of $g$ at $t = 0$. Using GTAP data on capital shares by sector, we estimate that around 3.66% of annual investment occurs in the oil, natural gas, coal, electricity, and gas distribution sectors.[32] We noted above that in the GTAP data, total investment $i + n = 0.2273$, implying that $n(0) \approx 0.0083$ in private sector output units. We use this as an additional target value. Thus, we choose values for $\alpha_3$ and the partial derivative $g_S(0)$ of $g$ with respect to $S$ at $t = 0$, and hence $\alpha_2 = \alpha_3 * 2110.538, \alpha_1 = g_S(0)/0.0651^2$ and $\alpha_0 = g(0) - \alpha_1 * 0.0651$, to target $c(0)$ and $n(0)$.

Turning next to the learning curve (8), the literature provides a range of estimates for $\alpha$. An online calculator provided by NASA[33] gives a range of learning percentages between 5 and 20% depending on the industry. A learning percentage of $x$, which corresponds to a value of $\alpha = -ln(1 - x)/ln(2)$, has the interpretation that a doubling of the experience measure will lead to a cost reduction of $x$%. Thus, $x = 0.2$ is equivalent to $\alpha = 0.322$ while $x = .05$ corresponds to $\alpha = 0.074$. In a study of wind turbines, Coulomb and Neuhoff (2006) found values of $\alpha$ of 0.158 and 0.197. Grübler and Messner (1998) found a value for $\alpha = .36$ using data on solar panels. Bentham et. al. (2008) report several studies finding a learning percentage of around 20% ($\alpha = 0.322$) for solar panels. We conclude that for renewable energy technologies $\alpha$ could range from a low of 0.15 to a high of 0.32, so we chose a middle value of $\alpha = 0.25$.

The other parameter affecting the incentive to invest in renewable energy sources is the initial value $\Gamma_1^{-\alpha}$ of the cost of using renewable energy as the primary energy source. Using a document available from the Energy Information Administration (EIA)[34] the cost of new

[32] Since we have defined $R$ to be energy services input, investments in energy efficiency in addition to mining increase the effective supply of fossil fuels. Hence, we include investments in the energy transformation sectors. While some of these would not increase energy efficiency, some investments in the transportation and manufacturing sectors that have not been included would be aimed at raising energy efficiency.

[33] Available at http://cost.jsc.nasa.gov/learn.html

[34] *Assumptions to the Annual Energy Outlook, 2009*, "Electricity Market Module," Table 8.2, available at http://www.eia.doe.gov/oiaf/aeo/assumption/pdf/electricity.pdf #page=3

onshore wind capacity is about double the cost of combined cycle gas turbines (CCGT), while offshore wind is around four times as expensive, solar thermal more than five times as expensive and solar photovoltaic more than six times as expensive. However, these costs do not take account of the lower average capacity factor of intermittent sources such as wind or solar. The same document gives a fixed O&M cost of onshore wind that is around two and a half times the corresponding fixed O&M for CCGT, although the latter also has fuel costs. The corresponding ratio is around 7 for offshore wind, while fixed O&M for solar photovoltaic are similar to the fixed O&M for CCGT. As a rough approximation, we will assume $\Gamma_1^{-\alpha}$ is around 4 times the initial value of $g$. Following the EIA, we also assume that the renewable technologies can ultimately experience a five-fold reduction in costs, so $\Gamma_2 = \Gamma_1^{-\alpha}/5$. This would result in an energy cost that is below the current cost of fossil fuel technologies.

Finally, we need to specify a value for $\psi$, the relative effectiveness of direct investment in research versus learning by doing in accumulating knowledge about new energy technologies. Klaassen et. al. (2005) estimated a model that allowed for both learning-by-doing and direct R&D. Although they assume the capital cost is multiplicative in total R&D and cumulative capacity, while we assume the *change in knowledge* $\dot{H}$ is multiplicative in new R&D and current output, we can take their parameter estimates as a guide. They find direct R&D is roughly twice as productive for reducing costs as is learning-by-doing.[35] Consequently, we assume that $\psi = 0.33$.

# 5 Results

Next we summarize the results from the calibrated version of our model economy. The calculations were done in MatLab.[36] The transition to a renewable energy regime occurs

[35] Of course, the learning-by-doing has the advantage that it directly contributes to output at the same time it is adding to knowledge.

[36] The long time horizon resulted in calculations being close to the limit of numerical accuracy. For example, we needed to set the tolerance levels for the differential equation solvers to $5.0 * 10^{-14}$. We could not use an optimization procedure to calculate starting values and instead conducted a grid search over values for $\alpha_3$ and $g_S(0)$. For each pair of values for $\alpha_3$ and $g_S(0)$, we adjusted $T_2, k(T_2)$ and $S(T_1)$ to match the initial

after $T_1 = 88.41$ years. Following that, renewable energy is used for a little more than 227 years (until $T_2 = 315.8$) before $H$ attains its maximum value and direct R&D expenditure $j$ is no longer worthwhile. World output per capita grows at an average annual rate of 4.22% in the fossil regime, 3.11% per annum (p.a.) in the renewable regime with investment in R&D, and 4.07% p.a. in the long run with renewable energy at its minimum cost.[37]

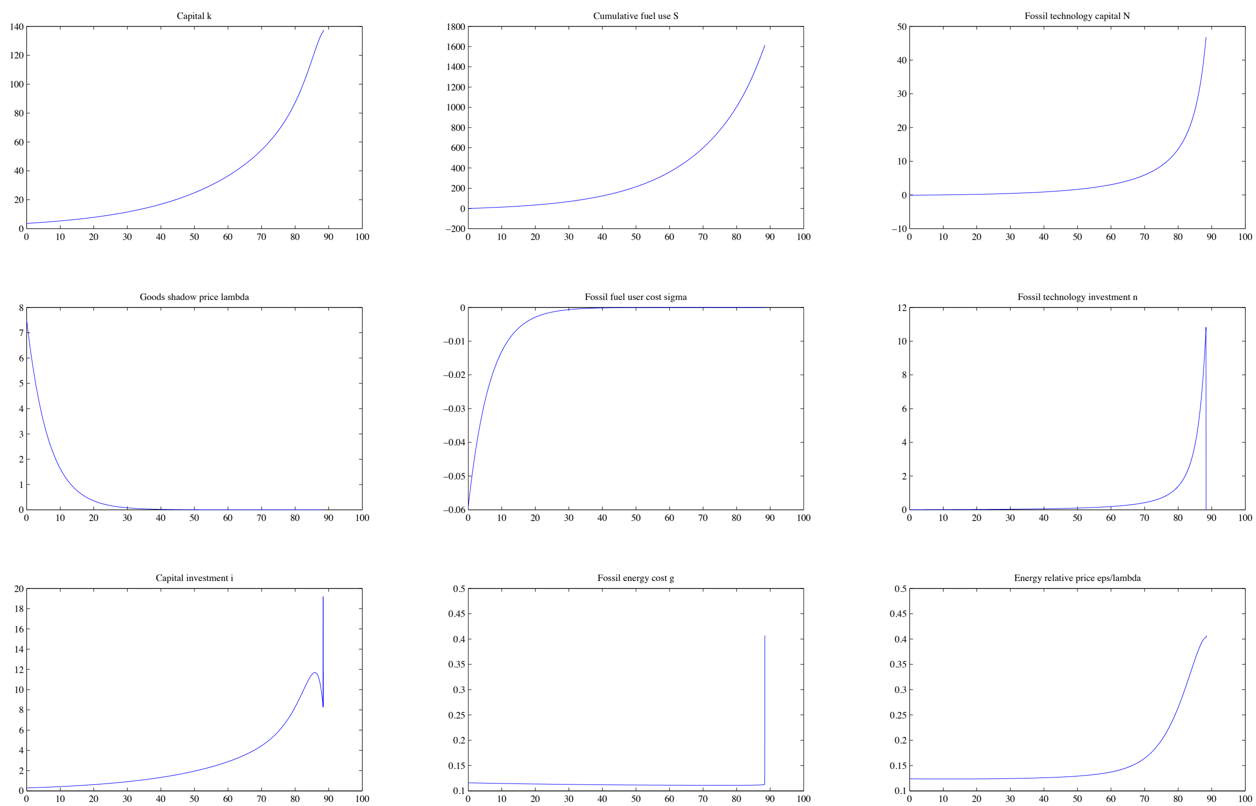


Figure 3: Fossil fuel regime

Figure 3 shows the behavior of the main variables in the economy during the fossil fuel regimes. The period over which $n = 0$ is very short, lasting just 0.0982 of a year. Once investment $n$ ceases, mining cost rises dramatically and the transition to renewables follows

values of $k(0) = 3.6071, N(0) = 0 = S(0), n(0) = 0.0083$ and $c(0) = 0.6620$. The closest we could get resulted from setting $\alpha_3 = 15, g_S(0) = 0.00015, T_2 = 315.8, k(T_2) = 141704.98998437249$ and $S(T_1) = 1613$, which yielded calculated initial values of $k(0) = 3.6644, S(0) = -0.0003179340598, N(0) = -0.07501979386, n(0) = 0.0077708$ and $c(0) = 0.60402$. We also solved a discrete time approximation to the continuous time model and verified that we get essentially the same solution for the same parameter values.

[37]In the long run regime, per capita consumption, investment in capital, and energy use all grow at the same average annual rate of 4.07%.

soon thereafter. Prior to its plunge to zero, however, $n$ rises dramatically as increasing amounts of investment are needed to offset the effects of depletion and maintain $g$ roughly constant. The rise in $n$ in turn constrains $i$, slowing the accumulation of $k$.

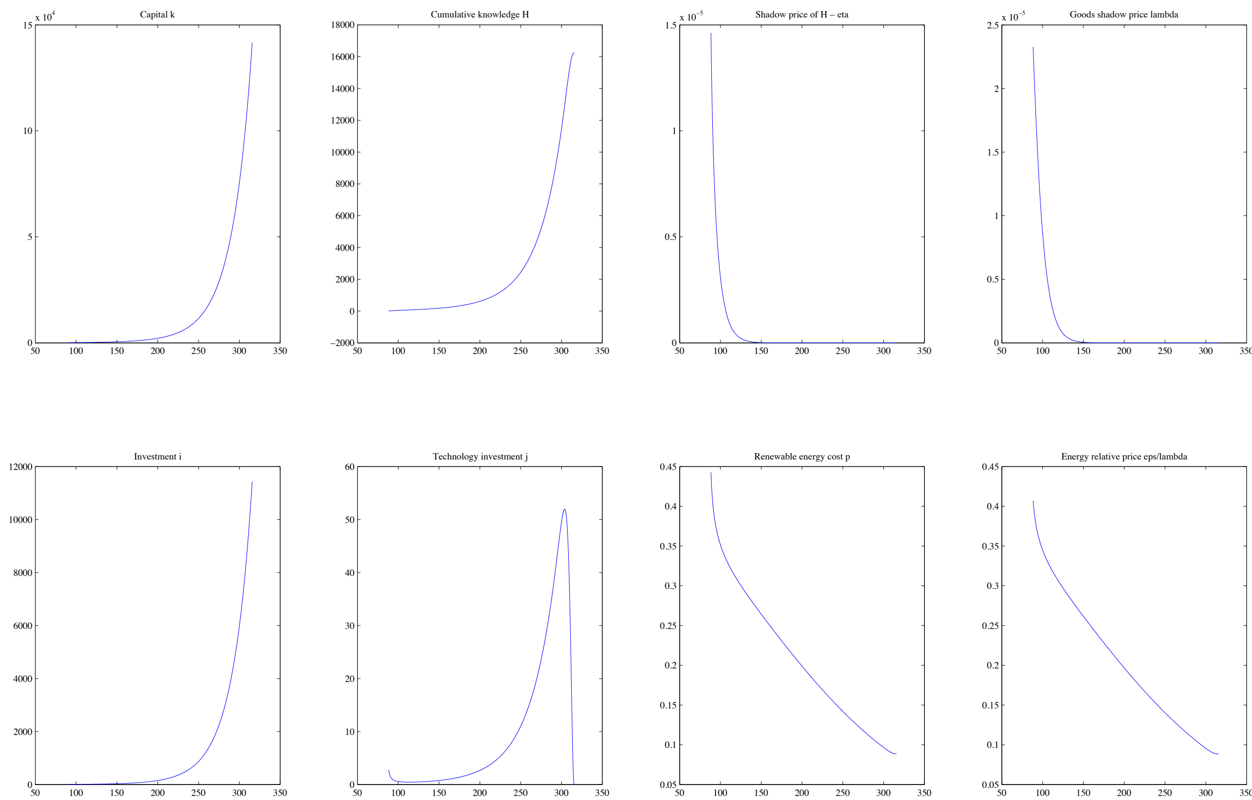


Figure 4: Renewable regime

Figure 4 shows the behavior of the main variables in the renewable energy regime where technological progress continues to reduce renewable energy costs. After a brief initial "burst" of investment in renewable R&D right after the transition, which steeply cuts the cost of renewable energy, direct investment in renewable energy R&D then drops close to zero. It subsequently gradually increases over time before plunging toward zero again as the technological frontier for renewable energy efficiency looms. Evidently, for much of the "middle period" of this regime, learning-by-doing is a major source for the accumulation of technical knowledge.

Having seen the overall behavior of the model, we now look at some particular issues in more detail. The explicit cost of mining stays fairly constant during the fossil fuel regime. This is due to investment in fossil fuel extraction and energy efficiency which allows the $g$ function to decline even as increased exploitation, $S$, otherwise raises mining costs. This process is reflected in more detail in Figure 5, which plots $g(S, N)$ as a function of $S$ for several years. The circled points give the actual costs as determined by the relevant value of $S$ for each year.

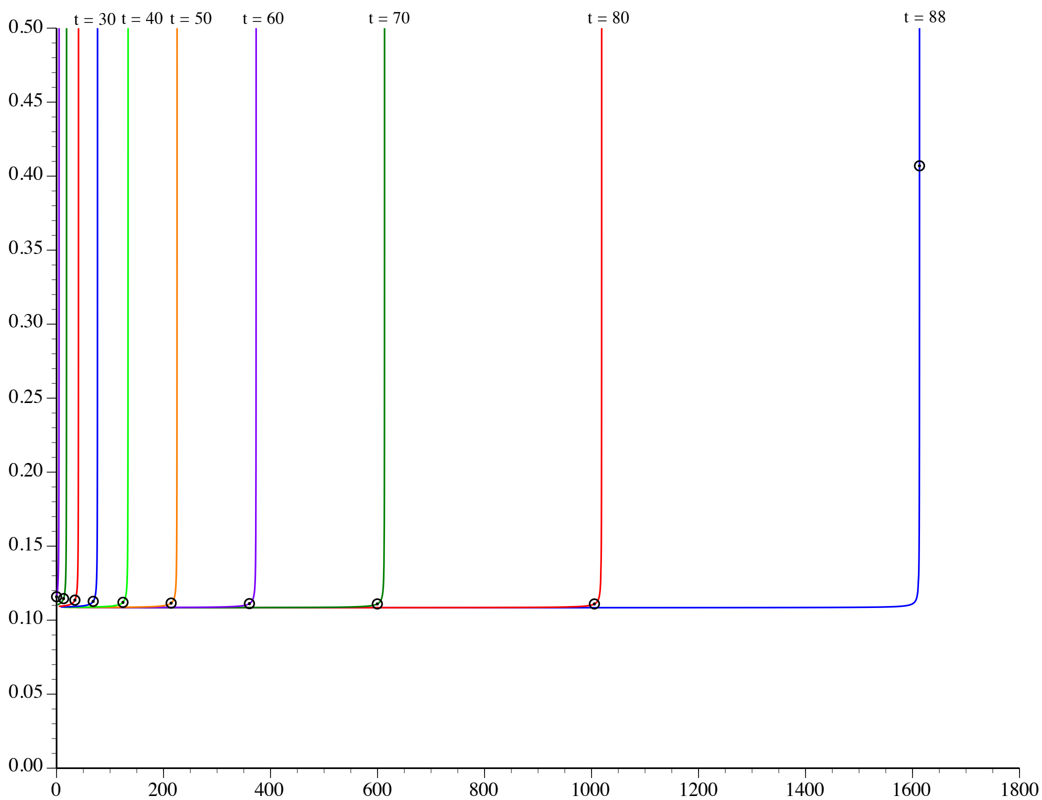


Figure 5: Selected mining cost functions

Figure 5 shows that, apart from the terminal period where $n = 0$, the values of $S$ in each year are very near the "capacity limit" of current proved and connected fossil fuel reserves. This is intuitive. Since there is no uncertainty in the model, it would be wasteful to maintain excess reserves.

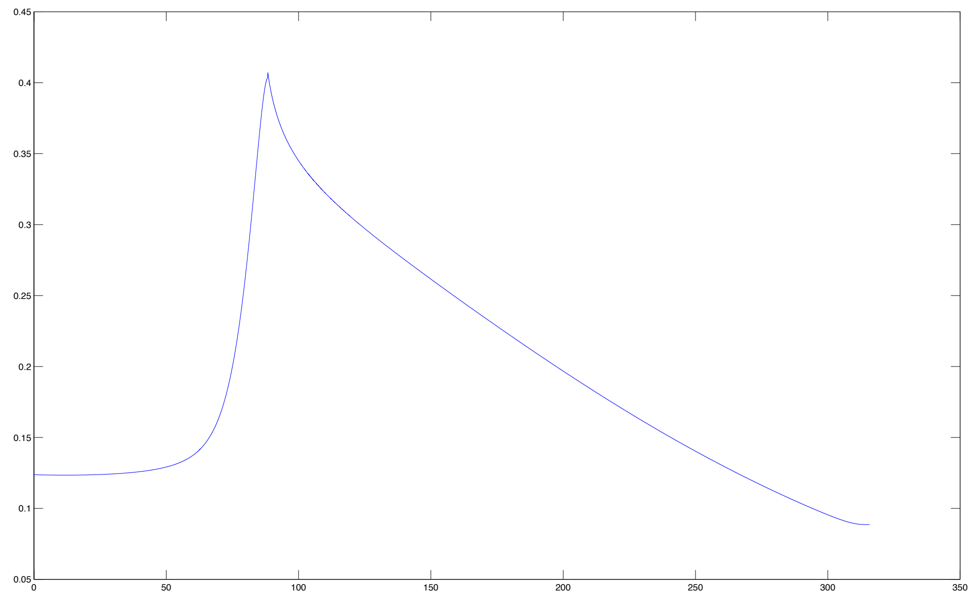


Figure 6: Relative price of energy

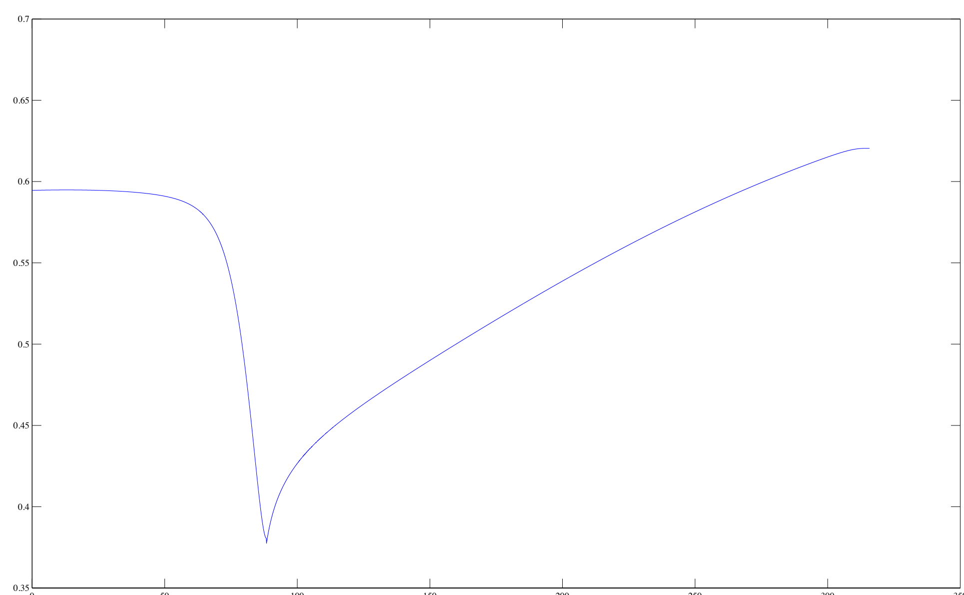


Figure 7: Consumption share of output

Another implication of the results graphed in Figure 5 is that the "cost parity target" for renewables is a moving one. Technological change in the production and use of fossil fuel energy sources allows them to remain competitive for longer. Ultimately, the model implies about 80% of the technically recoverable fossil fuel resources are exploited, with the transition occurring at the end of this century.

Although the explicit cost of mining slightly declines during most of the fossil fuel regime, Figure 6 shows that the shadow relative price of energy ($\epsilon/\lambda$) rises continuously. The gap is the result of the rising user cost, or scarcity rent, for fossil fuels in terms of goods. Specifically, $\sigma/\lambda$ becomes more negative over time up until the point where investment $n$ ceases, at which point it quickly jumps to zero.

Towards the end of the fossil fuel regime, the costs associated with fossil fuel use and increased mining investment are large in real terms. In particular, the shadow relative price of energy rises substantially and Figure 7 shows that the consumption share of output falls substantially around the time of transition to renewables.[38]

Figure 8 shows the annual growth rates of per capita output and consumption. As we would expect given concave utility, consumption growth is somewhat smoother than output growth, but the fluctuations in consumption growth are substantial. Per capita consumption grows by an average 3.68% in the fossil energy regime, which is less than the average output growth. By contrast, in the renewable regime with R&D, although R&D investment takes resources away from consumption and investment in $k$, the declining cost of energy allows consumption to grow at 3.33% compared to average annual growth in output of 3.11%.

Although Figure 8 shows that the per capita output growth rate rises substantially for some time before the switch point, all of the additional output and more is absorbed into producing, and investing in, fossil energy leaving fewer resources for consumption. Right around the switch point, annual per capita output growth actually becomes negative. In summary, our model predicts an "energy crisis" around the switch point.

[38] The alert reader may observe that the initial consumption share in Figure 7 is not equal to the calibrated value. The reason is that the calculated initial value of $k$ is about 10% too high and the calculated value of $c$ about 10% too low.

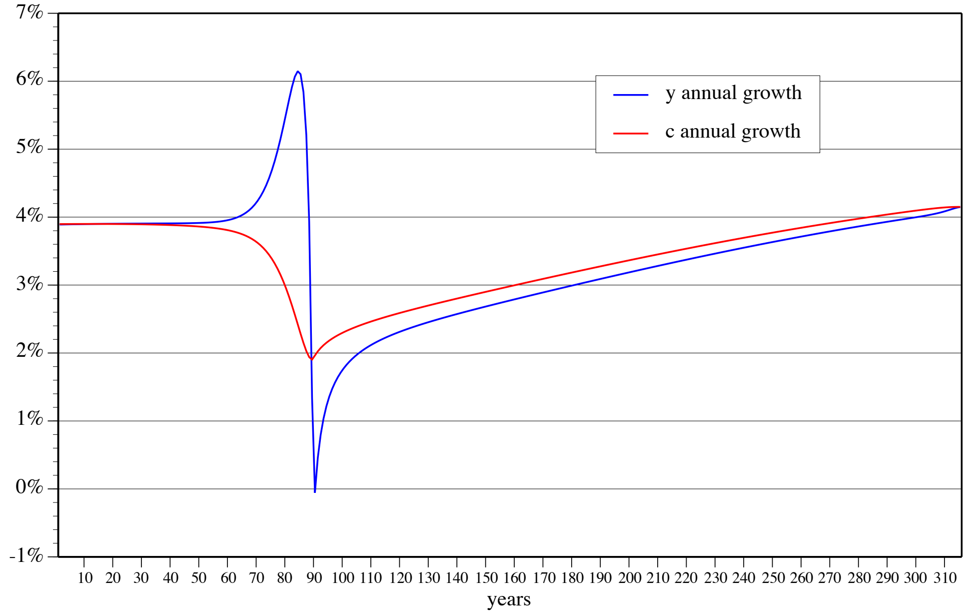


Figure 8: Annual growth rates of per capita output and consumption

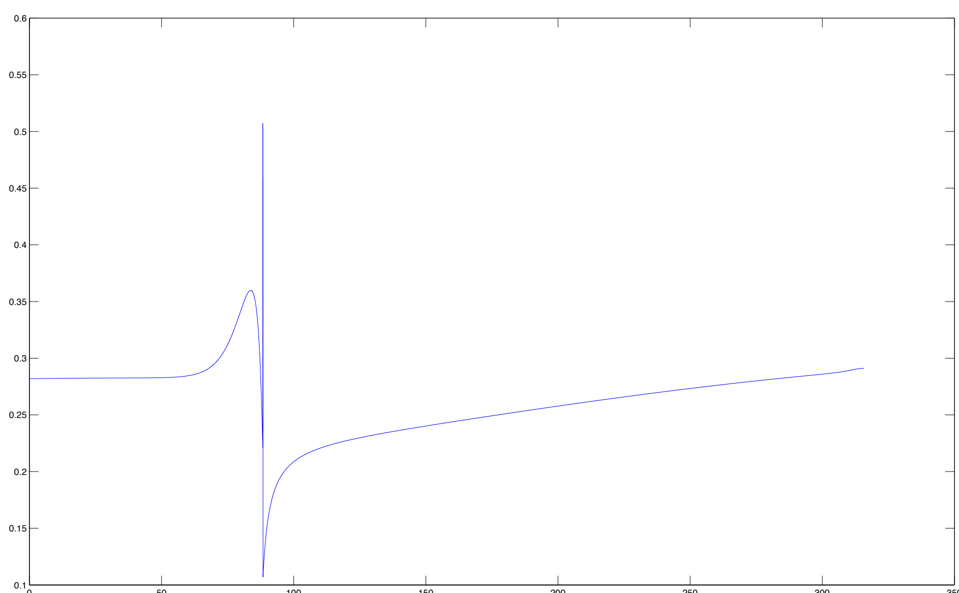


Figure 9: Output share of investment $i$ in $k$

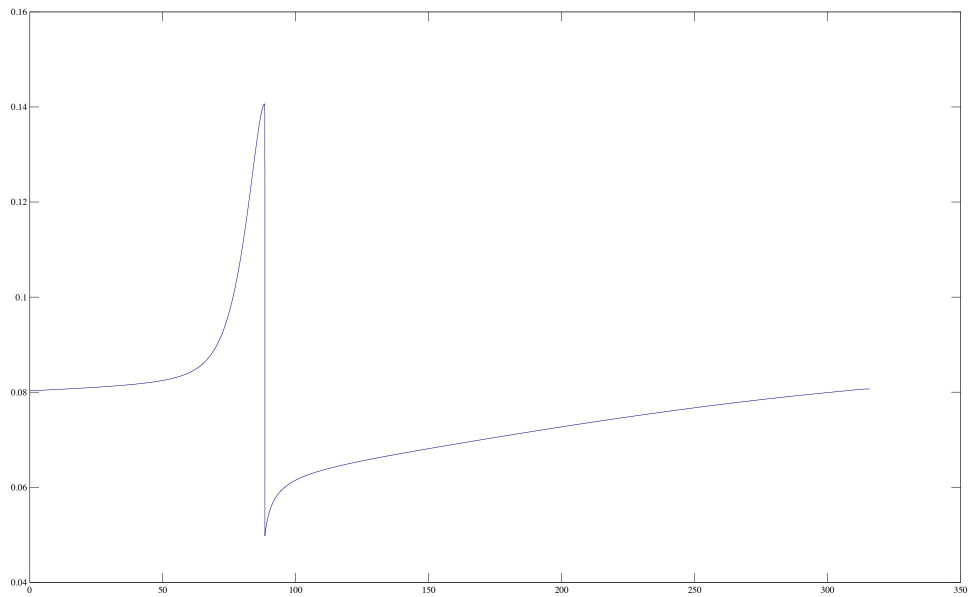


Figure 10: Output share of $i+n$ and $i+j$

The consumption share, and the growth in per capita output and consumption, also take a long time to recover to levels attained in fossil fuel era once the renewable regime begins. The explanation is that the cost of energy remains above the initial cost of fossil fuels for a substantial period of time. This is apparent in Figure 6, which shows that the shadow relative price of energy remains more than double the current level for over 75 years around the switch time (10 years before, 65 years after the switch time).

While the shadow price of energy is continuous, the explicit cost of mining is below the cost of renewable energy when the transition occurs. As we explained qualitatively above, the reason is that the learning by doing element of renewable energy production has a shadow price that lowers the"full cost" of renewable energy, making it worthwhile to transition before the explicit cost of fossil energy reaches the initial cost of renewable energy.

The sharp fluctuations in investment in $n$ and $j$ noticed in Figures 3 and 4 come at the expense of similar sharp fluctuations in investment $i$ in $k$. This is illustrated in Figures 9 and 10, which show different investment shares in output. In particular, Figure 10 shows that the sums of investment in $k$ and energy technologies are much smoother than any of the investments taken alone.

## 6 Conclusion

We studied the optimal transition path from fossil fuel to renewable energy sources in a neoclassical growth economy. We computed the optimal path of investment in new technology in both the fossil fuel and the renewable energy sectors and calibrated the model using data on world energy consumption and cost data from the US.

We found that innovations in technology keep the cost of mining fairly constant even as increased exploitation raises mining costs. Thus, renewable technologies face a moving "parity target." Nevertheless, anticipation of the benefits of learning-by-doing imply that it is optimal to shift from fossil to renewable energy sources *before* fossil fuel costs rise to match the cost of renewables. Ultimately, the model predicts that the transition to renewable energy will occur at the end of this century when about 80% of the available fossil fuels will have been exploited.

For several decades before the switch point, the share of consumption in output and the growth rate in per capita consumption both decline even though per capita output growth increases. The reason for the gap in growth rates is that the rising cost of energy and increased investment in fossil energy technology absorb more than the increase in output. The shadow price of energy peaks at the end of the fossil fuel regime and it remains more than double current levels for over 75 years around the switch time. In addition, a large investment in mining technology is needed to offset the effects of depletion and control energy costs toward the end of the fossil regime. These results should carry over to any model that allows for investment in fossil fuel technologies.

Immediately around the switch point, per capita output growth becomes negative, while the high cost of energy and the need for continuing investment in improving renewable energy technologies continue to constrain the growth in per capita consumption and output for an extremely long time. Thus, our model predicts an "energy crisis" around the switch point and continuing slow growth from some time thereafter. This crisis is part of the efficient arrangement in our economy.

It is worth discussing the robustness of our findings, especially to modifications of our two-factor learning model. Although the specification we have used is standard in the literature, one can question its relevance since if requires both R&D and learning-by-doing for cost reductions to take place. As a result, there is no regime in our model where the use of fossil fuel and progress in renewable technologies coexist. If R&D investment alone could reduce costs of deploying renewable technologies, the cost of renewable technologies at the transition point could be lower. This would hasten the transition and, since the increase in the relative price of energy would be moderated, the energy crisis would be less severe. However, there would still be an energy price maximum during the transition, and the increased investment in fossil fuels (to offset the effects of depletion) would remain. Thus, we believe that our results would not change qualitatively. Our model emphasizes the connection between the magnitude of the energy crisis at the transition point and the substitutability between innovation and learning-by-doing effects in renewable energy production.

Our analysis can be extended in many ways. Our model involves perfect substitutability between alternative energy sources. Using a continuity argument, we can show that our results remain true if the degree of substitutability is high, but not perfect. Introducing energy-source specific capital could allow us to more accurately capture the trade-off between fossil versus renewable energy production and may allow simultaneous use of different energy sources. Studying decentralized allocations will permit us to explicitly account for externalities associated with the investment process and the possibility of under-investment in R&D. Such deviations from efficiency would also allow a possible role for policy. We believe, however, that the higher relative price of energy, and higher levels of investment in energy technologies around the transition point, would remain features of such extended models.

Finally, we reiterate that our analysis abstracts from two important factors relevant to policy: energy independence and the environmental costs from fossil fuel combustion. While environmental factors will almost certainly favor renewables, incorporating benefits from energy independence could favor both renewable and (unconventional) fossil fuel sources. Investigating these considerations is another important topic for future research.

# 7 Appendix

## 7.1 Partial derivatives of $g(S, N)$

The fist partial derivatives are given by

$$\frac{\partial g}{\partial S} = \frac{\alpha_1(\alpha_3 + N)^2}{[(\bar{S} - S)(\alpha_3 + N) - \alpha_2]^2} > 0 \tag{25}$$

and

$$\frac{\partial g}{\partial N} = -\frac{\alpha_1\alpha_2}{[(\bar{S} - S)(\alpha_3 + N) - \alpha_2]^2} < 0 \tag{26}$$

so that increases in $S$ increase marginal cost, while improved technology reduces the costs of providing fossil fuel energy. The second order partial derivatives with respect to $S$ and $N$ are given by

$$\frac{\partial^2 g}{\partial S^2} = \frac{2\alpha_1(\alpha_3 + N)^3}{[(\bar{S} - S)(\alpha_3 + N) - \alpha_2]^3} > 0 \tag{27}$$

and

$$\frac{\partial^2 g}{\partial N^2} = \frac{2\alpha_1\alpha_2(\bar{S} - S)}{[(\bar{S} - S)(\alpha_3 + N) - \alpha_2]^3} > 0 \tag{28}$$

In particular, this function implies that cumulative exploitation $S$ increases fossil fuel energy cost at an increasing rate, while investment in fossil fuel technology decreases costs at a decreasing rate. In fact, we can conclude from (26) that $\partial g/\partial N \to 0$ as $N \to \infty$. The latter fact should imply that eventually it becomes uneconomic to invest further in reducing the costs of fossil fuel energy. Thus, fossil fuel resources will likely be abandoned long before all known deposits are exhausted as rising costs make renewable energy technologies more attractive.

Finally, the cross second partial derivative will be given by

$$\frac{\partial^2 g}{\partial N \partial S} = -\frac{2\alpha_1\alpha_2(\alpha_3 + N)}{[(\bar{S} - S)(\alpha_3 + N) - \alpha_2]^3} < 0 \tag{29}$$

Hence, investment in fossil fuel technology delays the increase in costs of fossil fuel energy accompanying increased exploitation.

## 7.2 Energy Regimes

We begin our detailed analysis with the last regime, describing economic growth once the technological limit in energy production is reached.

### 7.2.1 The Long Run Endogenous Growth Economy

When the cost of the renewable energy source is constant at $p = \Gamma_2$, the stock of knowledge about renewable energy production $H$ is no longer relevant. The model becomes a simple endogenous growth model with investment only in physical capital. We retain the first order conditions (12), (13) and (17), the first co-state equation (18), the resource constraint (10) and the differential equation (2) for the only remaining state variable $k$. However, (17) changes to simply $\epsilon = \lambda\Gamma_2$. From (13) we will obtain $q = \lambda$ and hence $\dot{q} = \dot{\lambda}$, and the co-state equation (18) becomes

$$\dot{\lambda} = [\beta + \delta - (1 - \Gamma_2)A]\,\lambda \equiv \bar{A}\lambda \tag{30}$$

where $\bar{A}$ is a constant. If we are to have perpetual growth, we must have $c \to \infty$ as $t \to \infty$, which from (12) will require $\lambda \to 0$ and hence $\bar{A} < 0$, that is[39]

$$A(1 - \Gamma_2) > \beta + \delta \tag{31}$$

Condition (31) has an intuitive interpretation. With $B = y$ and $p = \Gamma_2$, $A(1 - \Gamma_2)$ equals output per unit of capital *net* of the costs of supplying the renewable energy input. To obtain perpetual growth, this must exceed the cost of holding capital measured by the sum of the depreciation rate (the explicit cost) and the time discount rate (the implicit opportunity

[39]Note that (31) will require $A > (\beta+\delta)/(1-\bar{p}) > \beta+\delta$, which is the usual condition for perpetual growth in a simple linear growth model.

cost). Hereafter, we assume (31) to be valid. The solution to (30) can be written

$$\lambda = \bar{K} e^{\bar{A}t} \tag{32}$$

for some constant $\bar{K}$ yet to be determined. Thus, in this final regime, the resource constraint, the first order condition (12) for $c$ and (32) imply

$$\dot{k} = (\beta - \bar{A})k - \bar{K}^{-1/\gamma} e^{-\bar{A}t/\gamma} \tag{33}$$

which, for another constant $C_0$, has the solution

$$k = C_0 e^{(\beta - \bar{A})t} + \frac{\gamma \bar{K}^{-1/\gamma} e^{-\bar{A}t/\gamma}}{\beta\gamma - \bar{A}(\gamma - 1)} \tag{34}$$

However, the transversality condition at infinity, $\lim_{t\to\infty} e^{-\beta t}\lambda k = 0$, requires $C_0 = 0$ and $\bar{A}(\gamma - 1) < \beta\gamma$.[40] In summary, we conclude that the value of $k$ in the final endogenous growth economy will be given by

$$k = \frac{\gamma \bar{K}^{-1/\gamma} e^{-\bar{A}t/\gamma}}{\beta\gamma - \bar{A}(\gamma - 1)} \tag{35}$$

with $\lambda$ given by (32) and $\bar{K}$ is a constant yet to be determined.

#### 7.2.2 Renewables with Technological Progress

Working backwards in time, we consider next the regime where $B = Ak > 0, j > 0$ and $H < \Gamma_2^{-1/\alpha} - \Gamma_1$. As we observed in Section 3.5, the solution for $j$ in this regime is given by

[40] Note that since $\bar{A} < 0$ the inequality will be satisfied if $\gamma > 1$, while if $0 < \gamma < 1$, it will require $\Gamma_2 > 1 - [\beta/(1-\gamma) + \delta]/A$. Thus, for $\gamma < 1$, we need $\beta/(1-\gamma) > A(1-\Gamma_2) - \delta > \beta$.

(22). Hence, $\dot{H}$ will be given by:[41]

$$\dot{H} = [(1-\psi)(\eta/\lambda)]^{(1-\psi)/\psi} B = [(1-\psi)(\eta/\lambda)]^{(1-\psi)/\psi} Ak \tag{36}$$

For $B > 0$, (17) implies $\zeta = 0$ , while $H < \Gamma_2^{-1/\alpha} - \Gamma_1$ and (19) imply $\chi = 0$. The solution (22) for $j$ therefore also implies that the shadow price of energy will be given by:

$$\epsilon = \lambda(\Gamma_1 + H)^{-\alpha} - \psi(1-\psi)^{(1-\psi)/\psi} \lambda^{(\psi-1)/\psi} \eta^{1/\psi} \tag{37}$$

Substituting (37) into (18) and noting that $q = \lambda$ implies $\dot{q} = \dot{\lambda}$, we obtain

$$\dot{\lambda} = \left[\beta + \delta - A(1 - (\Gamma_1 + H)^{-\alpha})\right]\lambda - \psi A(1-\psi)^{(1-\psi)/\psi} \lambda^{(\psi-1)/\psi} \eta^{1/\psi} \tag{38}$$

From (19) with $B = Ak$, we obtain

$$\dot{\eta} = \beta\eta - \lambda\alpha(\Gamma_1 + H)^{-\alpha-1} Ak \tag{39}$$

The resource constraint, the first order condition (12) for $c$ and the solution (22) for $j$ with $B = Ak$ determines $i$ and hence the differential equation for $\dot{k}$:

$$\dot{k} = Ak\left[1 - (\Gamma_1 + H)^{-\alpha} - (1-\psi)^{1/\psi} \eta^{1/\psi} \lambda^{-1/\psi}\right] - \lambda^{-1/\gamma} - \delta k \tag{40}$$

In summary, we conclude that the economy with renewables and technological progress will be characterized by four simultaneous differential equations (36), (38), (39) and (40) for the four state and co-state variables $k$, $H$, $\eta$, and $\lambda$.

[41] Hence, if $\eta/\lambda$ evolves slowly over time $H$ will approximately equal a constant times accumulated past production of renewable energy $B$. Under these circumstances, empirical studies may find that energy production cost and accumulated output *alone* follow a power law relationship, and that experience is more productive in reducing costs than the underlying structural model assumes. As (22) shows, the reason is that direct R&D also grows along with $B$.

### 7.2.3 The Initial Fossil Fuel Economy

Finally, we consider the initial regime where $R > 0$. Then (16) implies $\xi = 0$ and the shadow price of energy will be

$$\epsilon = \lambda g(S, N) - \sigma Q \tag{41}$$

As noted in Section 3.5, $\sigma$ will be negative until the end of the fossil fuel regime at $T_1$ when $\sigma = 0$. It then follows from (41) that the shadow price of energy $\epsilon$ is unambiguously positive.

While investment in fossil fuel technology is productive, that is $n > 0$, (15) implies $\omega = 0$ and hence $\nu = \lambda$. But then $\dot{\nu} = \dot{\lambda}$ and (21) implies

$$\dot{\lambda} = \beta\lambda + \lambda\frac{\partial g}{\partial N}R \tag{42}$$

If we also have $i > 0$, (13) will imply $\lambda = q$ and from (18) and (41), we will also have $\dot{\lambda} = (\beta + \delta + g(S, N)A - A)\lambda - \sigma QA$. Using (42) we then conclude

$$\left[\delta + g(S, N)A - \frac{\partial g}{\partial N}R - A\right]\lambda = \sigma QA \tag{43}$$

Note that since $\sigma < 0$ and $\lambda = c^{-\gamma} > 0$, a necessary condition for (43) to hold is that

$$\delta + g(S, N)A - \frac{\partial g}{\partial N}R < A \tag{44}$$

Since we have assumed, however, that $g(S, N)$ eventually increases above 1 as $S$ grows, and $\partial g/\partial N < 0$, constraint (44) must eventually be violated and we cannot have $R > 0$ and $n > 0$ forever. We will assume, however, that parameters are chosen so that $R > 0$ and $n > 0$ at $t = 0$.

Substituting $R = Ak$ into (43), we obtain an equation relating $N$ and $k$, which is maintained by active investment in the two types of capital. Specifically, differentiating the resulting expression with respect to time, substituting for $\dot{N}, \dot{\lambda}/\lambda = \dot{\nu}/\nu, \dot{S}, \dot{\sigma}$ and $\dot{Q} = \pi Q$ (since the exogenous growth rate of $Q$ is $\pi$), and using (43), we obtain a condition relating

$i$ and $n$:

$$\lambda\left[\frac{\partial g}{\partial N}(n+\delta k+\frac{\sigma QAk}{\lambda}-i)-\frac{\partial^2 g}{\partial S\partial N}QAk^2-\frac{\partial^2 g}{\partial N^2}nk\right]=\sigma\pi Q \tag{45}$$

We obtain a second relationship from the resource constraint. Specifically, using the result that $j=0$ if $B=0$, the first order condition (12) for $c$, the production function (1), and the energy input demand requirement (3) the resource constraint (10) implies:

$$i=Ak[1-g(S,N)]-\lambda^{-1/\gamma}-n \tag{46}$$

Substituting (46) into (45), we then obtain an equation to be solved for energy technology investment $n$ in the fossil fuel regime:

$$\begin{aligned} &n\lambda\left(\frac{\partial^2 g}{\partial N^2}k-2\frac{\partial g}{\partial N}\right)=\\ &\lambda\left[\frac{\partial g}{\partial N}[k(\delta+g(S,N)A-A+\frac{\sigma QA}{\lambda})+\lambda^{-1/\gamma}]-\frac{\partial^2 g}{\partial S\partial N}QAk^2\right]-\sigma\pi Q \end{aligned} \tag{47}$$

Using the signs of the partial derivatives of $g$ given in the appendix, one can show that (47) likely yields $n>0$ as hypothesized.[42] Using the solution for $n$ and the current values of the state and co-state variables, (46) can be solved for $i$.

In summary, we conclude that the initial period of fossil fuel use with both $i>0$ and $n>0$ produces five differential equations for $k$, $S$, $N$, $\sigma$, and $\lambda$:

$$\dot{k}=i-\delta k \tag{48}$$

$$\dot{S}=QAk \tag{49}$$

[42] Since $\partial g/\partial N<0$ and $\partial^2 g/\partial N^2>0$, the coefficient of $n$ on the left hand side of (47) is positive. From the resource constraint (46), $\delta k+Ak(g-1)+\lambda^{-1/\gamma}=\delta k-i-n\leq\delta k-n$. Thus $n>0$ if

$$-\frac{\partial^2 g}{\partial S\partial N}QAk^2+\frac{\partial g}{\partial N}(\delta+\frac{\sigma QA}{\lambda})k-\sigma\pi Q>0$$

Since $\partial^2 g/\partial S\partial N<0$ and $\sigma<0$, the quadratic in $k$ has a positive second derivative and positive intercept, so even if $\delta+\sigma QA/\lambda>0$, so the roots are both positive, we conclude that the expression must be positive for large $k$. For small values of $k$, we are likely to have $\dot{k}=i-\delta k>0$, in which case the right hand side of (47) is guaranteed to be positive.

$$\dot{N} = n \tag{50}$$

$$\dot{\sigma} = \beta\sigma + \lambda\frac{\partial g}{\partial S}Ak \tag{51}$$

$$\dot{\lambda} = \lambda(\beta + \delta + (g(S, N) - 1)A) - \sigma QA \tag{52}$$

together with the exogenous population growth $Q = Q_0e^{\pi t}$.

As we argued in Section 3.5, the region where $R > 0$ and $n > 0$ will end at some $T_0 < T_1$ and between $T_0$ and $T_1$, we will $n = 0$ and $R > 0$. In this region, $N$ is fixed at $\bar{N}$, and the resource constraint together with the first order condition (12) for $c$ will imply

$$i = Ak[1 - g(S, \bar{N})] - \lambda^{-1/\gamma} \tag{53}$$

In addition, we will now have separate differential equations for $\lambda$ and $\nu$. Specifically, $\dot{\nu}$ will now be given by (21) whereas, since we will sill have $\lambda = q$ and $R > 0$, $\dot{\lambda}$ will continue to satisfy (52). The equations for $\dot{k}, \dot{S}$ and $\dot{\sigma}$ will also continue to satisfy (48), (49) and (51).

### 7.3 The Numerical Solution Procedure

Mirroring the theoretical analysis, we found it easier to solve the model backwards through time. As observed in Section 3.5, we know the values of the co-state variables at the various transition points. The known initial values $S(0) = N(0) = 0$, $k(0) = k_0 > 0$ of the state variables at $t = 0$ become targets. We have three free variables to set in order to hit these three target values.

Specifically, if we guess values for the transition time $T_2$ and the value of the capital stock at that time $k(T_2)$, the values of the constant $\bar{K}$ and hence $\lambda(T_2)$ are also determined. We also know that at $T_2$ we must have $\eta(T_2) = 0$ and $p = (\Gamma_1 + H)^{-\alpha} = \Gamma_2$, which will determine the value of $H$ at $T_2$, namely $H = \Gamma_2^{-1/\alpha} - \Gamma_1$. The differential equations (36), (38), (39) and (40) are then solved backward until $T_1$, when $H = 0$. The values of $k$ and $\lambda$ at $T_1$ then provide initial conditions for the differential equations (48) and (52) in the fossil fuel regime. Using (37) and (41), the fact that $\sigma(T_1) = 0$, and the requirement that the shadow price of energy has to be continuous across the region boundaries we conclude that

$$\Gamma_1^{-\alpha} - \frac{\eta}{\lambda} = \frac{\epsilon}{\lambda} = g(S, N) \tag{54}$$

For the values of $\eta(T_1)$ and $\lambda(T_1)$ obtained from the backward solution in the renewable regime, and the exogenously specified $\Gamma_1^{-\alpha}$, (54) would then determine the value of the mining cost $g(S(T_1), N(T_1))$ at $T_1$. Thus, $N(T_1)$ will be determined once we guess the value of $S(T_1)$. Finally, the requirements that $\sigma(T_1) = 0 = \nu(T_1)$ will provide the remaining initial conditions for the five differential equations (21), (48), (49), (51) and (52). The initial fossil fuel regime with $n > 0$ then starts at $T_0$ when $\nu = \lambda$. For all $t \leq T_0$, we then have $k, S, N, \sigma$ and $\lambda$ given by the solutions to the differential equations (48)–(52).

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