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# Renewable Technology Adoption and the Macroeconomy[*]

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### Abstract

We study the adaptation of new technologies by renewable energy-producing firms in a dynamic general equilibrium model where energy is an input in the production of goods. Energy can come from fossil or renewable sources. Both require the use of capital, which is also needed in the production of final goods. Renewable energy firms can invest in improving the productivity of their capital stock. The actual improvement is random and subject to spillovers. Productivity improvements by renewable firms require "scrapping" some of their existing capital. Together with spill-overs, this leads to under-investment in improving the productivity of renewable energy capital. In the presence of environmental externalities, the optimal allocation can be implemented through a Pigouvian tax on fossil fuel, together with a policy which promotes adaptation of new renewable technologies by taxing firms proportional to their under-scrapping. An implication of our analysis is that it is not optimal to make large investments in new technologies where progress is fast and where current capital becomes obsolete before long. We calibrate the model using world-economy data in order to study the implications of various proposed tax/subsidy scenarios for economic growth.

# 1  Introduction

Economic growth generates a tremendous demand for energy. Historically, this need has been met largely through the use of fossil fuel. In recent decades, renewable energy sources (such as solar and wind) have been increasing their representation in many nations' energy supply. As concerns about the consequences of climate change become more prevalent, and fossil fuel becomes more scarce, it is likely that renewable energy will become even more widely used. Unfortunately, renewable energy is still too costly to directly compete with fossil fuel sources. Yet, the transition towards renewable energy supplies is expected to accelerate as investment in this sector increases and the resulting technological progress reduces costs.

A widely held view holds that societies currently under-invest in renewable energy. This argument can take many different forms. Under-investment might refer to resources spent on R&D, or to actual installation and usage of facilities that harvest renewable energy. Similarly, the reasons for under-investment range from externalities associated with climate change, to spillovers associated with innovation. In order to evaluate alternative policies, we need to have an idea about the rate at which declining costs will lead to increased competitiveness for renewables. What determines the productivity improvements in renewable energy production? How does the rate of productivity improvement respond to policy? What are the consequences for the fossil fuel sector and the macroeconomy? Our paper attempts to study these questions in the context of a structural dynamic general equilibrium model.

Our analysis investigates the full transition from a mainly fossil energy fueled to a mainly renewable energy fueled world economy. This transition involves several ingredients. First, although fossil fuel sources are plentiful, they constitute an exhaustible resource. Increasing scarcity rents resulting from accumulated use, lead to an increased demand for a substitute. A second important ingredient involves environmental considerations. We follow Golosov, Hassler, Krusell, and Tsyvinski (2011) to model this externality. As fossil fuel generates externalities due to carbon emissions, the need for a clean substitute becomes more prevalent. Lastly, we model the process of innovation in the renewable energy sector. This process too has two important ingredients. First, productivity improvements by renewable firms require "scrapping" some of their existing capital. Second, spillover effects imply that productivity improvements depend on the average level of scrapping in the industry. Incorporating costs associated with scrapping existing capital is a novel feature of our analysis. What we have in mind is that an innovation often requires that part of a firm's capital stock is replaced. While costly, this investment makes the remaining capital stock more productive. On the other hand, if innovations are frequent, the scrapping costs can overwhelm the benefits from innovation. Thus, it is not efficient to invest heavily in technologies where future technological progress will make the existing capital stock obsolete before long. While we believe that such costs are relevant for other industries, we think they are particularly relevant for the energy sector.

We develop a model where energy is an input in the production of a consumption good. Energy can be produced from either fossil or renewable sources. Both require capital, which is also needed for the production of the final good. At each point in time, renewable energy-

producing firms can improve their productivity by scrapping some of their capital stock. The actual improvement is random and subject to a spillover: it depends on the aggregate scrapping in the renewable sector. The spill-over effect leads to an overall under-investment in the productivity of renewable energy-related capital. The optimal mix of energy supply involves a declining use of fossil fuel. We demonstrate that the optimal allocation can be implemented through a policy which promotes adaptation of new technologies by subsidizing(taxing) firms proportional to their over(under) scrapping, together with a Pigouvian tax on the environmental externality created by fossil fuel use. In equilibrium, the adaptation policy is revenue-neutral, even at the firm level. However, the policy reduces the households' profits from the renewable sector, as a result of inducing additional scraping. An important implication of our analysis is that it is not optimal to make large investments in technologies that will become obsolete before long. We calibrate the model using world-economy data and study the implications of various proposed tax/subsidy scenarios for economic growth.

While our analysis concentrates on the energy sector, the modeling of productivity improvements through the scrapping of less productive capital stock might have applications in other areas. Parente (1994) studies a model in which firms choose to adopt new technologies as they gain firm-specific expertise through learning-by-doing. He identifies conditions under which equilibria in his model exhibit constant growth of per capita output. As in most of the literature on economic growth, Parente abstracts from issues related to energy. Acemoglu et al. (2012) study a growth model that takes into consideration the environmental impact of operating "dirty" technologies. They examine the effects of policies that tax innovation and production in the dirty sectors. They find that subsidizing research in the "clean" sectors can speed up environmentally friendly innovation without the corresponding slowdown in economic growth. Consequently, optimal behavior in their model implies an immediate increase in clean energy R&D, followed by a complete switch toward the exclusive use of clean inputs in production. Our model identifies a different effect which might put a limit to how fast renewable technologies should be adopted. Scraping costs imply that it might not be optimal to invest too much in these technologies before they mature.

More recently, Golosov, Hassler, Krusell, and Tsyvinski (GHKT, 2011) built a macroeconomic model that incorporates the use of energy and the resulting environmental consequences. They derive a formula describing the optimal tax due to the externality from emissions and provide numerical values for the size of the tax in a calibrated version of their model. However, they abstract from the scraping costs associated with endogenous technological progress, which are the focus of our study. Acemoglu, D., U. Akcigit, D. Hanley, and W. Kerr (2012), use the structure in GHKT, 2011 to study questions related to the transition to clean technologies. They employ a "ladder" model to study technological progress in both the clean and the dirty sectors, and they estimate the model using R&D and patent data. They find that, in addition to carbon taxes, quantitatively significant R&D subsidies are a necessary ingredient of optimal policy. the reason is that subsidies encourage technological progress without taxing short run future output too much.[1]

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[1] Other related papers include Hartley, Medlock , Temzelides, and Zhang (2012) and Van der Ploeg F., and C. Withagen (2011), who study the possibility of a *Green Paradox.*

The paper proceeds as follows. The next section introduces the economic environment and discusses efficiency. Section 3 studies equilibrium allocations and optimal policy. Section 4 introduces our calibration and policy scenarios. A brief conclusion follows.

# 2   The Economic Environment

We assume discrete time and infinite horizon, $t = 0, 1, ...$ There is a single consumption good per period and all markets are competitive. The economy is populated by a representative infinite-lived household. The household discounts the future at rate $\beta \in (0, 1)$ and values the period-$t$ consumption good through a utility function $u(c_t)$. We assume that $u$ is smooth, strictly increasing, strictly concave, and that the usual Inada conditions hold. There are three different kinds of firms, all owned by the household. In each period, the household chooses how much capital, $k$, to rent in the market at rate $r_t$ and receives all profits resulting from firms' activities. All capital depreciates at the same rate, $\delta \in (0, 1)$.

The final good-producing firm uses capital, $k$, labor, $L$, and energy, $e$, in order to produce output. The labor endowment is normalized to 1 and is supplied inelastically to the firm. In addition, we assume that environmental quality, $\Gamma$, can affect the production process through a damage function $D(\Gamma)$. The final good production function is given by

$$
\begin{aligned}
y_t &\leq A_t \cdot (k_t^g)^{\theta_k} (L_t)^{\theta_L} (e_t)^{1-\theta} = (1 - D_t(\Gamma_t)) \left[ \widetilde{A}_t \cdot (k_t^g)^{\theta_k} (L_t)^{\theta_L} (e_t)^{1-\theta} \right] \\
&= \exp\left[ -\pi_t \left( \Gamma_t - \overline{\Gamma} \right) \right] \widetilde{A}_t \left[ (k_t^g)^{\theta_k} (L_t)^{\theta_L} (e_t)^{1-\theta} \right]
\end{aligned} \tag{1}
$$

where $\widetilde{A}$ is a productivity parameter, $A_t = (1 - D_t(\Gamma)) \widetilde{A}_t$, and $\theta, \theta_k, \theta_L \in (0, 1)$, and $\theta_k + \theta_L = \theta$. Following Golosov, Hassler, Krusell, and Tsyvinski (GHKT, 2011), we assume that

$$
D_t(\Gamma) = 1 - \exp\left[ -\pi_t \left( \Gamma_t - \overline{\Gamma} \right) \right] \tag{2}
$$

where $\overline{\Gamma}$ is the pre-industrial greenhouse gas concentration in the atmosphere and $\pi$ is a random variable which parametrizes the effect of higher greenhouse gas concentrations on the level of damages. Environmental quality evolves according to

$$
\Gamma_t = \sum_{l=0}^{t+T} (1 - d_l) f_{t-l} \tag{3}
$$

where $d_l \in [0, 1]$, and $f_l$ is the fossil fuel use in period $l$. Assuming that a fraction $\varphi_L$ of emitted carbon stays in the atmosphere for ever, while a fraction $(1 - \varphi_0)$ of the remaining emissions exits into the biosphere, and the remaining part decays at geometric rate $\varphi$, we obtain

$$
\begin{aligned}
1 - d_l &= \varphi_L + (1 - \varphi_L)\varphi_0 (1 - \varphi)^l \\
\Gamma_t^p &= \Gamma_{t-1}^p + \varphi_L f_t \\
\Gamma_t^d &= (1 - \varphi)\Gamma_{t-1}^d + (1 - \varphi_L)\varphi_0 f_t \\
\Gamma_t &= \Gamma_t^p + \Gamma_t^d
\end{aligned} \tag{4}
$$

Energy can be produced in two different ways by using a fossil or a renewable source. We assume that the two types of energy are perfect substitutes in the production of the final good.[2] We let $w_t$ denote the available stock of fossil fuel in period $t$, and, as mentioned above, $f_t$ denotes the fossil fuel used in energy production at $t$. Thus, the law of motion for the stock of fossil fuel is: $w_{t+1} \leq w_t - f_t$. The fossil-fuel-derived energy production function is given by $e_t^f \leq (f_t)^{1-\alpha_f} \left(k_t^f\right)^{\alpha_f}$, where $\alpha_f \in (0, 1)$. We assume a competitive sector of renewable energy-producing firms. As these firms are heterogenous, we need to keep track of the identity of each individual firm. The renewable energy production function for firm $j$ is given by $e_{j,t}^r \leq (\mathcal{E}_{j,t})^{1-\alpha_r} \left(k_{j,t}^r\right)^{\alpha_r}$, where $\mathcal{E}_{j,t}$ is a productivity parameter and $\alpha_r \in (0, 1)$. Total capital used in the economy cannot exceed the total supply; i.e., $k_t^g + k_t^f + \int_0^1 k_{j,t}^r dj \leq k_t$, all $t$.

Scrapping some of their existing capital can boost productivity in the renewable energy sector. More precisely, we let $\iota_{j,t}$ denote the scrapping by renewable firm $j$ in period $t$. The cost for firm $j$ from scrapping $\iota_{j,t}$ is $\Psi(\iota_{j,t})k_{j,t}^r$, where $\Psi(\cdot)$ is a convex function. The variables $\iota_{j,t}$ and $k_{j,t}^r$ are chosen *after* the realization of $\mathcal{E}_{j,t}$, in all periods. We also assume that there is a spill-over effect, as aggregate investment affects the productivity of each individual firm. As more firms employ a technology, there is a higher probability of an improvement through new ideas. This creates an externality, implying a discrepancy between equilibrium and desirable levels of investment in renewable energy. We begin by characterizing desirable (efficient) allocations in this environment. More precisely, the productivity of firm $j$ evolves stochastically according to:

$$
\begin{aligned}
\ln \mathcal{E}_{j,t+1} &\leq \gamma + \ln \mathcal{E}_{j,t} + \varepsilon_{j,t} \\
\varepsilon_{j,t} &\sim \left( \int_0^1 \iota_{j,t} k_{j,t}^r dj \Big/ \int_0^1 k_{j,t}^r dj, \sigma^2 \right)
\end{aligned} \tag{5}
$$

Efficient allocations are identical to those solving a social planning problem. The social planner's problem for our economy is as follows:

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[2]Increased substitutability across energy seems a reasonable benchmark assumption in a model like ours, where we concentrate on long-run effects. For similar reasons, our analysis abstracts will abstracts from short-run fluctuations in supply and demand for energy and the corresponding volatility in energy prices.

$$\max_{\{c_t,k_{t+1},k_t^g,k_t^f,w_{t+1}^f,e_t^f,\Gamma_t^p,\Gamma_t^d,\{\iota_{j,t+1},k_{j,t}^r,\mathcal{E}_{j,t+1},e_{j,t}^r\}_{j=0}^1\}_{t=0}^\infty} E\sum_{t=0}^\infty \beta^t u(c_t)$$

$$\text{s.t. } c_t + k_{t+1} + \int_0^1 \Psi(\iota_{j,t})k_{j,t}^r dj \leq A_t \cdot (k_t^g)^{\theta_k}(L_t)^{\theta_L}(e_t)^{1-\theta} + (1-\delta)k_t : \mu_{R,t}$$

$$w_{t+1} \leq w_t - f_t : \mu_{W,t}$$

$$\Gamma_t^p = \Gamma_{t-1}^p + \varphi_L f_t : \mu_{\Gamma_t^p}$$

$$\Gamma_t^d = (1-\varphi)\Gamma_{t-1}^d + (1-\varphi_L)\varphi_0 f_t : \mu_{\Gamma_t^d}$$

$$e_{j,t}^r \leq (\mathcal{E}_{j,t})^{1-\alpha_r}\left(k_{j,t}^r\right)^{\alpha_r} : \mu_{r,t}^j$$

$$e_t^f \leq (f_t)^{1-\alpha_f}\left(k_t^f\right)^{\alpha_f} : \mu_{F,t}$$

$$k_t \geq k_t^g + k_t^f + \int_0^1 k_{j,t}^r dj : \mu_{K,t}$$

$$\ln\mathcal{E}_{t+1}^j \leq \gamma + \ln\mathcal{E}_t^j + \varepsilon_{j,t}\left(\left(\int_0^1 \iota_{j,t}k_{j,t}^r dj \Big/ \int_0^1 k_{j,t}^r dj\right), \sigma\right) : \mu_{\mathcal{E},t}^j$$

$$e_t \leq e_t^f + \int_0^1 e_{j,t}^r dj : \mu_{E,t}$$

$$0 \leq f_t : \mu_{f_t}$$

$$k_{t+1} \geq 0, w_{t+1} \geq 0, \text{ all } t$$

$$k_0 > 0, \mathcal{E}_{j,t} > 0, w_0 > 0, \text{ given} \tag{6}$$

The FOCs for the planner´s problem, which are also sufficient in this model, are:[3]

$$\partial c_t : \beta^t u'(c_t) = \mu_{R,t} \tag{7}$$

$$\partial k_{t+1} : -\mu_{R,t} + (1-\delta)E_t\mu_{R,t+1} + E_t\mu_{K,t+1} = 0 \tag{8}$$

$$\partial w_{t+1} : \mu_{W,t} = E_t\mu_{W,t+1} \tag{9}$$

$$\partial\Gamma_t^p : \mu_{\Gamma_t^p} - E_t\mu_{\Gamma_{t+1}^p} - \pi y_t \mu_{R,t} = 0 \tag{10}$$

$$\partial\Gamma_t^d : \mu_{\Gamma_t^d} - (1-\varphi)E_t\mu_{\Gamma_{t+1}^d} - \pi y_t \mu_{R,t} = 0 \tag{11}$$

$$\partial f_t : -\mu_{W,t} + \mu_{F,t}(1-\alpha)\left(\frac{k_t^f}{f_t}\right)^{\alpha_f} - \mu_{\Gamma_t^p}\varphi_L - \mu_{\Gamma_t^d}(1-\varphi_L)\varphi_0 + \mu_{f_t} = 0 \tag{12}$$

---

[3]We implicitly assume an upper bound for $\iota_0$. Otherwise, in the presence of scrapping costs, the optimal policy would involve setting $k_0^r$ arbitrarily small, and $\iota_0$ arbitrarily large.

Notice that the above implies that

$$\partial e_{jt}^r : \mu_{r,t}^j = \mu_{E,t} dj \text{ or } \int_0^1 \mu_{r,t}^j dj = \mu_{E,t} \tag{13}$$

Also note that the marginal utility of having firm $i$ producing an extra infinitesimal amount of energy should be equal to the marginal utility of having firm $j$ producing an extra infinitesimal amount of energy; i.e., $\mu_{r,t}^j = \mu_{r,t}^i$, for almost all $j$ and $i$. We have:

$$\partial e_t^f : -\mu_{F,t} + \mu_{E,t} = 0 \tag{14}$$

$$\partial e_t : \mu_{R,t} (1 - \theta) A_t \frac{(k_t^g)^{\theta_k} (L_t)^{\theta_L}}{(e_t)^\theta} = \mu_{E,t} \tag{15}$$

$$\partial k_t^g : \mu_{R,t} \theta_k A_t (k_t^g)^{\theta_k - 1} (L_t)^{\theta_L} (e_t)^{1-\theta} = \mu_{K,t} \tag{16}$$

$$\partial k_t^f : \mu_{F,t} \alpha_f \left( \frac{f_t}{k_t^f} \right)^{1-\alpha_f} = \mu_{K,t} \tag{17}$$

$$\partial k_{j,t}^r : \int_0^1 \mu_{\mathcal{E},t}^i \left( \frac{\iota_{j,t} \int_0^1 k_{j,t}^r dj - \int_0^1 \iota_{j,t} k_{j,t}^r dj}{\left( \int_0^1 k_{j,t}^r dj \right)^2} dj \right) di + \alpha_s \mu_{r,t}^j \left( \frac{\mathcal{E}_{jt}}{k_{jt}^r} \right)^{1-\alpha_s} - \mu_{R,t} \Psi (\iota_{j,t}) dj = \mu_{K,t} dj$$

or

$$\partial k_{j,t}^r : \left( \frac{\iota_{j,t} - \overline{\iota_t}}{\overline{k_t^r}} dj \right) \int_0^1 \mu_{\mathcal{E},t}^i di + \alpha_s \mu_{r,t}^j \left( \frac{\mathcal{E}_{jt}}{k_{jt}^r} \right)^{1-\alpha_s} - \mu_{R,t} \Psi (\iota_{j,t}) dj = \mu_{K,t} dj \tag{18}$$

where $\overline{k_t^r} = \int k_{t,i}^r di$, and $\overline{\iota_t} = \int_0^1 \iota_{j,t} k_{j,t}^r dj / \int_0^1 k_{j,t}^r dj$.

$$\partial k_{j,t}^r : \left( \frac{\iota_{j,t} - \overline{\iota_t}}{\overline{k_t^r}} \right) \cdot \int_0^1 \mu_{\mathcal{E},t}^i di + \alpha_r \mu_{E,t} \left( \frac{\mathcal{E}_{jt}}{k_{jt}^r} \right)^{1-\alpha_r} - \mu_{R,t} \Psi (\iota_{j,t}) = \mu_{K,t} \tag{19}$$

From (13) and (19), note that $\iota_{j,t}$ is a function of $\frac{\mathcal{E}_{jt}}{k_{jt}^r}$ only. In addition,

$$\partial \iota_{j,t} : \mu_{R,t} \Psi'(\iota_{j,t}) k_{j,t}^r dj = \frac{k_{j,t}^r}{\overline{k_t^r}} dj \int_0^1 \mu_{\mathcal{E},t}^i di \tag{20}$$

which implies

$$\mu_{R,t} \Psi'(\iota_{j,t}) = \left( \overline{k_t^r} \right)^{-1} \int_0^1 \mu_{\mathcal{E},t}^i di \tag{21}$$

$$\partial \mathcal{E}_{j,t+1} : \mu_{\mathcal{E},t}^j = E_t \mu_{\mathcal{E},t+1}^j + \mu_{r,t+1}^j (1 - \alpha_r) (\mathcal{E}_{jt+1})^{1-\alpha_r} (k_{jt+1}^r)^{\alpha_r} \tag{22}$$

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Taking together (7), (8) and (16) we have:

$$u'(c_t) = (1-\delta)\beta u'(c_{t+1}) + \theta_k \beta u'(c_{t+1}) A_{t+1} \left(k_{t+1}^g\right)^{\theta_k - 1} (L_{t+1})^{\theta_L} (e_{t+1})^{1-\theta} \tag{23}$$

From (13) and (14),

$$\mu_{F,t} = \int_0^1 \mu_{r,t}^j dj = \mu_{E,t} \tag{24}$$

From (15),

$$\frac{\mu_{R,t}}{\mu_{E,t}} = \left[ (1-\theta) A_t \frac{\left(k_t^g\right)^{\theta_k} (L_t)^{\theta_L}}{(e_t)^{\theta}} \right]^{-1} \tag{25}$$

From (17) and (19),

$$\frac{\left(\int_0^1 \mu_{\mathcal{E},t}^i di\right)}{\mu_{F,t}} \left(\frac{\iota_{j,t} - \overline{\iota_t}}{\overline{k_t^r}}\right) + \frac{\mu_{E,t}}{\mu_{F,t}} \alpha_r \left(\frac{\mathcal{E}_{jt}}{k_{jt}^r}\right)^{1-\alpha_r} - \frac{\mu_{R,t}}{\mu_{F,t}} \Psi \left(\iota_{j,t}\right) = \alpha_f \left(\frac{f_t}{k_t^f}\right)^{1-\alpha_f} \tag{26}$$

Now from (21),

$$\frac{\left(\int_0^1 \mu_{\mathcal{E},t}^i di\right)}{\mu_{F,t}} \left(\frac{\iota_{j,t} - \overline{\iota_t}}{\overline{k_t^r}}\right) - \frac{\mu_{R,t}}{\mu_{F,t}} \Psi \left(\iota_{j,t}\right) = \left[\Psi' \left(\iota_{j,t}\right) \left(\iota_{j,t} - \overline{\iota_t}\right) - \Psi \left(\iota_{j,t}\right)\right] \frac{\mu_{R,t}}{\mu_{F,t}} \tag{27}$$

Using (24), (25), and (27) in (26) we obtain

$$\alpha_r \left(\frac{\mathcal{E}_{jt}}{k_{jt}^r}\right)^{1-\alpha_r} + \left[\Psi' \left(\iota_{j,t}\right) \left(\iota_{j,t} - \overline{\iota_t}\right) - \Psi \left(\iota_{j,t}\right)\right] \left[(1-\theta) A_t \frac{\left(k_t^g\right)^{\theta_k} (L_t)^{\theta_L}}{(e_t)^{\theta}}\right]^{-1} = \alpha_f \left(\frac{f_t}{k_t^f}\right)^{1-\alpha_f} \tag{28}$$

The above equation characterizes the optimal $\iota$ in our model. Equations (15) and (16) imply

$$\frac{\mu_{R,t} \theta_k A_t \left(k_t^g\right)^{\theta_k - 1} (e_t)^{1-\theta}}{\mu_{R,t} (1-\theta) A_t \frac{\left(k_t^g\right)^{\theta_k}}{(e_t)^{\theta}}} = \frac{\mu_{K,t}}{\mu_{E,t}} \tag{29}$$

Using (24) and (17) in the equation above, we have

$$\frac{\theta_k}{1-\theta} \left(\frac{e_t}{k_t^g}\right) = \alpha_f \left(\frac{f_t}{k_t^f}\right)^{1-\alpha_f} \tag{30}$$

Using (7), (14) and (15), we obtain:

$$\begin{aligned} &\beta^t u'(c_t) \left\{MPF_t - \pi y_t \Phi_1\right\} + \mu_{\Gamma_t^d} \Phi_2 \\ =\ & \beta^{t+1} E_t u'(c_{t+1}) MPF_{t+1} \end{aligned} \tag{31}$$

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where $MPF_t \equiv \left[ (1-\theta) A_t \frac{\left(k_t^g\right)^{\theta_k} (L_t)^{\theta_L}}{(e_t)^\theta} \right] \left[ (1-\alpha) \left( \frac{k_t^f}{f_t} \right)^{\alpha_f} \right]$, $\Phi_1 \equiv \left( \frac{\varphi_L(1-\varphi) + (1-\varphi_L)\varphi_0}{(1-\varphi)} \right)$ and
$\Phi_2 \equiv \frac{\varphi(1-\varphi_L)\varphi_0}{1-\varphi}$. Next, we replace $\mu_{\Gamma_t^d}$ in the expression above to obtain:

$$\beta^t u'(c_t) \{ MPF_t - \pi_t y_t \Phi_1 \} + \sum_{j=0}^\infty \beta^{t+j} (1-\varphi)^j E_t u'(c_{t+j}) \pi_{t+j} y_{t+j} \Phi_2 \qquad (32)$$
$$= \beta^{t+1} E_t u'(c_{t+1}) MPF_{t+1}$$

The following Proposition greatly simplifies our analysis. It asserts that, although renewable energy-producing firms are heterogeneous, efficiency implies that they will choose identical levels of investment, $\iota$.

**Proposition 1.** *The optimal allocation implies* $\frac{k_{i,t}^r}{\mathcal{E}_{i,t}} = \frac{k_t^r}{\mathcal{E}_t}$ *and* $\iota_{i,t} = \iota_t$, *all* $i$.

*Proof.* From (21) $\iota_{i,t} = \iota_t$, for all $j \in [0,1]$. From (19), $\frac{\mathcal{E}_{jt}}{k_{jt}^r}$ is a function of $\iota_{j,t}$ only. Since $\iota_{i,t} = \iota_t$ then $\frac{\mathcal{E}_{jt}}{k_{jt}^r} = \frac{\mathcal{E}_t}{k_t^r}$. QED $\qquad\qquad \square$

We next discuss long run growth. As the next Proposition demonstrates, in the long run, the economy is fueled exclusively by renewable energy. In other words, the use of fossil fuel stops before the reserves are exhausted.[4]

**Proposition 2.** *For sufficiently large levels of* $K_t$ *and* $\mathcal{E}_t$, *efficiency implies that* $\lim_{t \to \infty} f_t = 0$.

*Proof.* A sufficient condition for stopping the use of fossil fuel is:

$$-\pi \left( \varphi_L + (1-\varphi_L)\varphi_0 \right) \cdot y_t + (1-\theta) \frac{y_t}{e_t} \cdot (1-\alpha_f) \cdot \left( \frac{k_t^f}{f_t} \right)^{\alpha_f} \leq 0$$

Equating the marginal product of capital in the fossil fuel sector and final good sector implies

$$\left\{ \alpha_f \cdot \left( \frac{f_t}{k_t^f} \right)^{1-\alpha_f} \right\} \cdot (1-\theta) \frac{y_t}{e_t} = \theta_k \cdot \frac{y_t}{k_t^g}$$

Hence,

$$\frac{k_t^f}{f_t} = \left\{ \alpha_f \frac{1-\theta}{\theta_k} \cdot \frac{k_t^g}{e_t} \right\}^{\frac{1}{1-\alpha_f}}$$

---

[4] Although fossil fuel is not used in the long run, the endogenous technological progress in renewables implies that a higher initial endowment of fossil fuel allows for less intensive use of capital in renewable energy production. Thus, the growth path of an "oil-rich" economy lies above that of an otherwise identical economy with a lower $w$.

Plugging the above into the stopping condition we obtain:

$$(1-\theta)\frac{1}{e_t}\cdot(1-\alpha_f)\cdot\left\{\alpha_f\frac{1-\theta}{\theta_k}\cdot\frac{k_t^g}{e_t}\right\}^{\frac{\alpha_f}{1-\alpha_f}}\leq\pi\left(\varphi_L+(1-\varphi_L)\varphi_0\right)$$

or equivalently

$$\frac{1}{e_t}\cdot\left\{\frac{k_t^g}{e_t}\right\}^{\frac{\alpha_f}{1-\alpha_f}}\leq\frac{1}{1-\alpha_f}\left(\frac{1}{1-\theta}\right)^{\frac{1}{1-\alpha_f}}\left(\frac{\theta_k}{\alpha_f}\right)^{\frac{\alpha_f}{1-\alpha_f}}\cdot\pi\left(\varphi_L+(1-\varphi_L)\varphi_0\right)$$

The left-hand-side of the above expression can be written as:

$$\frac{1}{e_t}\cdot\left\{\frac{k_t^g}{e_t}\right\}^{\frac{\alpha_f}{1-\alpha_f}}=\left\{\frac{1}{\mathcal{E}^{1-\alpha_r}\cdot K_t^{\alpha_r-\alpha_f}}\cdot\frac{\left(1-\frac{k_t^r}{K_t}\right)^{\alpha_f}}{\left(\frac{k_t^r}{K_t}\right)^{\alpha_r}}\right\}^{\frac{1}{1-\alpha_f}}$$

Notice that $\frac{k_t^r}{K_t}$ asymptotically converges to a fixed value given by

$$\begin{aligned}\frac{k^r}{K}&=\frac{k^r}{k^r+k^g}\\&=\frac{\frac{k^r}{y}}{\frac{k^r}{y}+\frac{k^g}{y}}\\&=\frac{\frac{\alpha_r(1-\theta_k-\theta_L)}{\frac{g^\sigma}{\beta}-(1-\delta)+\psi(\iota)}}{\frac{\alpha_r(1-\theta_k-\theta_L)}{\frac{g^\sigma}{\beta}-(1-\delta)+\psi(\iota)}+\frac{\theta_k}{\frac{g^\sigma}{\beta}-(1-\delta)}}\\&=\frac{\alpha_r(1-\theta_k-\theta_L)\left[\frac{g^\sigma}{\beta}-(1-\delta)\right]}{\alpha_r(1-\theta_k-\theta_L)\left[\frac{g^\sigma}{\beta}-(1-\delta)\right]+\theta_k\left[\frac{g^\sigma}{\beta}-(1-\delta)+\psi(\iota)\right]}\end{aligned}$$

Since $\mathcal{E}$ and $K$ are both growing and $\alpha_r>\alpha_f$, we have $\lim_{t\to\infty}\frac{1}{e_t}\cdot\left\{\frac{k_t^g}{e_t}\right\}^{\frac{\alpha_f}{1-\alpha_f}}=0$.QED   $\square$

An intertemporal condition for the optimal allocation of capital in the production of the final good is that its marginal productivity, $MPK_{t+1}^g$, is such that $u'(c_t)=\beta\left[MPK_{t+1}+1-\delta\right]u'(c_{t+1})$. Assuming utility function $u(c)=(c^{1-\sigma}-1)/(1-\sigma)$, on the balanced growth path we must have $\frac{u'(c_t)}{u'(c_{t+1})}=g^\sigma$ and, therefore,

$$g^\sigma=\beta\left[\theta_k\frac{y_{t+1}}{k_{t+1}^g}+1-\delta\right] \tag{33}$$

Since on the balance growth path the ratios of $\frac{y}{k^g}$ and $\frac{y}{k^r}$ are constant, we have

$$\frac{k^g}{y}=\theta_k\left[\frac{g^\sigma}{\beta}-(1-\delta)\right]^{-1} \tag{34}$$

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Similarly, by the equality of the rate of return of capital across the final goods sector and renewable energy sector, we have

$$\frac{k^r}{y} = \alpha_r(1 - \theta_k - \theta_L)\left[\frac{g^\sigma}{\beta} - (1 - \delta) + \psi(\iota)\right]^{-1} \tag{35}$$

By substituting these balanced growth path ratios into the output equation we obtain

$$
\begin{aligned}
y_t &= \left[A_t \cdot (L_t)^{\theta_L} \cdot (\mathcal{E}_t)^{(1-\alpha_r)(1-\theta)}\right] \cdot (k_t^g)^{\theta_k} \cdot (k_t^r)^{(1-\theta)\alpha_r} \\
&= \left[A_t \cdot (L_t)^{\theta_L} \cdot (\mathcal{E}_t)^{(1-\alpha_r)(1-\theta)} \cdot \left\{\frac{\theta_k}{\frac{g^\sigma}{\beta} - (1 - \delta)}\right\}^{\theta_k} \cdot \left\{\frac{\alpha_r(1 - \theta_k - \theta_L)}{\frac{g^\sigma}{\beta} - (1 - \delta) + \psi(\iota)}\right\}^{(1-\theta)\alpha_r}\right]^{\frac{1}{1-\theta_k-\alpha_r(1-\theta_k-\theta_L)}}
\end{aligned}
\tag{36}
$$

Asymptotically, as all energy will be produced by the renewable sector, the capital will be allocated between goods sector and renewable sector. Denote the asymptotic growth rate by $g^*$. Output, $y_t$, consumption, $c_t$, capital used in the final good sector, $k_t^g$, and total capital used in the renewable sector, $k_t^r$, should all grow at the same rate, $g^*$, on a balanced growth path. Note that $y_t = A_t (k_t^g)^{\theta_k} (L_t)^{\theta_L} \left(\mathcal{E}_t^{1-\alpha_r} (k_t^r)^{\alpha_r}\right)^{1-\theta}$, which implies that on a balanced growth path we should have $g^* = g_g \cdot (g^*)^{\theta_k} (\exp(\gamma + \iota))^{(1-\alpha_r)(1-\theta)} (g^*)^{\alpha_r(1-\theta)}$, where $g_g$ is the exogenous growth rate of the aggregate productivity $A_t$ and $\exp(\gamma + \iota)$ is the rate of technological progress in the renewable sector, which includes both exogenous progress, $\gamma$, and endogenous progress, $\iota$.

Finally the optimal decision for scraping is given by

$$\left(\overline{k_t^r}\right)\Psi'(\iota_{j,t}) = (1 - \alpha_r) E_t \sum_{\tau=1}^{\infty} \frac{\beta^\tau u'(c_{t+\tau})}{u'(c_t)}(1 - \theta) A_{t+\tau}\left(\frac{(k_{t+\tau}^g)^{\theta_k}(L_{t+\tau})^{\theta_L}}{(e_{t+\tau})^\theta}\right)\int_0^1 \mathcal{E}_{i,t+\tau}\left(\frac{k_{i,t+\tau}^r}{\mathcal{E}_{i,t+\tau}}\right)^{\alpha_r} di \tag{37}$$

which on the balanced growth path with $u(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$ implies

$$\psi'(\iota) = (1 - \theta)(1 - \alpha_r) \cdot \frac{\beta(g^*)^{1-\sigma}}{1 - \beta(g^*)^{1-\sigma}} \cdot \frac{y}{k^r} \tag{38}$$

Notice that we have four asymptotic variables $g^*$, $\iota$, $\frac{k^g}{y}$, and $\frac{k^r}{y}$, which are pinned down by four equations (??),(??),(??), and (38). More specifically combining them the optimal endogenous parameters of the technological progress in the renewable technology, $\iota$, is determined

by

$$\psi'(\iota) = (1-\theta)(1-\alpha_r) \cdot \frac{\beta \left\{ g_g(g_L)^{\theta_L}(\exp(\gamma+\iota)^{(1-\alpha_r)(1-\theta_L-\theta_k)} \right\}^{\frac{1-\sigma}{1-\theta_k-\alpha_r(1-\theta)}}}{1-\beta\left\{ g_g(g_L)^{\theta_L}(\exp(\gamma+\iota)^{(1-\alpha_r)(1-\theta_L-\theta_k)} \right\}^{\frac{1-\sigma}{1-\theta_k-\alpha_r(1-\theta)}}} \cdot \frac{y}{k^r}$$

$$= \frac{(1-\alpha_r)}{\beta\alpha_r} \cdot \frac{\beta\left( (g_g)^{\frac{1}{1-\theta_k-\alpha_r(1-\theta)}} \cdot \exp\left(\gamma+\iota\right)^{\frac{(1-\alpha_r)(1-\theta)}{1-\theta_k-\alpha_r(1-\theta)}} \right)^{1-\sigma}}{1-\beta\left( (g_g)^{\frac{1}{1-\theta_k-\alpha_r(1-\theta)}} \cdot \exp\left(\gamma+\iota\right)^{\frac{(1-\alpha_r)(1-\theta)}{1-\theta_k-\alpha_r(1-\theta)}} \right)^{1-\sigma}} \cdot$$

$$\cdot \left\{ \left\{ g_g(g_L)^{\theta_L}(\exp(\gamma+\iota)^{(1-\alpha_r)(1-\theta_L-\theta_k)} \right\}^{\frac{\sigma}{1-\theta_k-\alpha_r(1-\theta)}} + \beta\left(\psi(\iota)-(1-\delta)\right) \right\}$$

The next section discusses a decentralized version of our model and characterizes a competitive equilibrium for the corresponding economy.

# 3   Equilibrium and Policy

Here we solve for a competitive equilibrium of the above economy and demonstrate that, as argued earlier, there is discrepancy between equilibrium and optimal allocations. We also discuss the role of policy in restoring efficiency.

The household's problem is given by

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{s.t. } \sum_{t=0}^{\infty} p_t \left[ c_t + k_{t+1} - (1-\delta)k_t \right] \leq \sum_{t=0}^{\infty} p_t \left[ r_t k_t + w_{L,t} L_t + p_t^f f_t + \pi_t^g + \pi_t^f + \int_0^1 \pi_{j,t}^r dj \right] : \lambda$$

$$w_{t+1} \leq w_t - f_t : \mu_t \tag{39}$$

where $p_t$ is the Arrow-Debreu price of the period-$t$ final good, $r_t$ is the rental price of capital at $t$, $p_t^f$ is the price of fossil fuel in period $t$, and $\pi$ stands for the respective firms' profits. The FOC, which are also sufficient for a maximum, are

$$\partial c_t : \beta^t u'(c_t) = \lambda p_t \tag{40}$$

$$\partial k_{t+1} : \lambda\left[ -p_t + (1-\delta+r_{t+1})p_{t+1} \right] = 0 \tag{41}$$

$$\partial w_{t+1} : \mu_t = \mu_{t+1} \tag{42}$$

$$\partial f_t : -\mu_t + \lambda p_t p_t^f = 0 \tag{43}$$

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These equations can be rewritten as

$$\frac{p_{t+1}}{p_t} = \frac{p_t^f}{p_{t+1}^f} \tag{44}$$

$$\beta^t \frac{u'(c_t)}{p_t} = \lambda \tag{45}$$

$$1 - \delta + r_{t+1} = \frac{p_t}{p_{t+1}} \tag{46}$$

$$\beta^t u'(c_t) p_t^f = \mu_t = \mu \tag{47}$$

The final-good firm's problem is

$$\max \left[ A_t \cdot (k_t^g)^{\theta_k} (L_t)^{\theta_L} (e_t)^{1-\theta} - r_t k_t^g - w_{L,t} L_t - p_t^e e_t \right] \tag{48}$$

The first order conditions are

$$\partial k_t^g : \theta_k A_t (k_t^g)^{\theta_k - 1} (L_t)^{\theta_L} (e_t)^{1-\theta} = r_t \tag{49}$$

$$\partial L_t : \theta_L A_t (k_t^g)^{\theta_k} (L_t)^{\theta_L - 1} (e_t)^{1-\theta} = w_{L,t} \tag{50}$$

$$\partial e_t : (1 - \theta) A_t \frac{(k_t^g)^{\theta_k} (L_t)^{\theta_L}}{e_t^\theta} = p_t^e \tag{51}$$

The Fossil-Fuel Firm's Problem is:

$$\max \left[ p_t^e (f_t)^{1-\alpha_f} \left( k_t^f \right)^{\alpha_f} - r_t k_t^f - p_t^f f_t \right] \tag{52}$$

$$\partial k_t^f : p_t^e \alpha_f \left( \frac{f_t}{k_t^f} \right)^{1-\alpha_f} = r_t \tag{53}$$

$$\partial f_t : p_t^e (1 - \alpha_f) \left( \frac{k_t^f}{f_t} \right)^{\alpha_f} = p_t^f \tag{54}$$

The renewable firm $j$'s problem is:

$$\max_{\{\iota_{j,t}, k_{j,t}^r\}_{t=0}^\infty} \sum_{\tau=0}^\infty \beta^\tau \frac{u'(c_{t+\tau})}{u'(c_t)} \left[ p_{t+\tau}^e (\mathcal{E}_{j,t+\tau})^{1-\alpha_r} \left( k_{j,t+\tau}^r \right)^{\alpha_r} - r_{t+\tau} k_{j,t}^r - \Psi(\iota_{j,t+\tau}) k_{j,t+\tau}^r \right]$$

$$\text{s.t. } \ln \mathcal{E}_{t+1}^j \leq \gamma + \ln \mathcal{E}_t^j + \varepsilon_{j,t} \left( \left( \int_0^1 \iota_{j,t} k_{j,t}^r dj / \int_0^1 k_{j,t}^r dj \right), \sigma \right) : \lambda_{\mathcal{E},t}^j$$

$$\Psi(\iota_{j,t+\tau}) k_{j,t}^r \leq k_{j,t}^r$$

$$\iota_{j,t} \geq 0, \text{ and } \mathcal{E}_0 \text{ given} \tag{55}$$

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Here, $\Psi(\iota_{j,t})$ is a convex function, with $\Psi(0) = 0$, $\Psi' > 0$, $\Psi'' > 0$, $\lim_{x \to 0} \Psi'(x) = 0$. The FOC are

$$\partial k_{j,t}^r : p_t^e \alpha_r \left( \frac{\mathcal{E}_{j,t}}{k_{j,t}^r} \right)^{1-\alpha_r} = r_t + \Psi(\iota_{j,t}) \tag{56}$$

$$\partial \iota_{j,t} : \Psi'(\iota_{j,t}) k_{j,t}^r = o\lambda_{\mathcal{E},t}^j = 0 \tag{57}$$

Clearly, condition (57) implies that in a competitive equilibrium without policy intervention the first best is not achievable. We summarize this in the following.

**Proposition 3.** *In competitive equilibrium, $\iota_{j,t} = 0$, for all $j$, $t$*

*Proof.* The unique solution to equation (57) is $\iota_{j,t} = 0$.QED $\qquad\qquad\qquad\square$

We next discuss implementation of optimal policy. Policy needs to take into account two distortions. The first, relates to under-scrapping in $\iota$ due to spillover effects. The second involves the social costs associated with the environmental externality. The next Proposition demonstrates that these two distortions can be fully accommodated through the use of two policy instruments. First, a policy that taxes firms in proportion to their under-investment in $\iota$ restores optimal investment by making firms indifferent when they choose between paying the tax or pursuing the optimal level of scrapping. As in GHKT (2012), under the special assumptions of log utility and 100% depreciation of capital, the tax of fossil fuel firms does not depend on the growth rate of the economy.

**Proposition 4.** *The optimal allocation can be supported by a combination of a revenue-neutral policy $\Phi_t(k, \iota_{j,t}) = \Psi'(\iota_t^*) (\iota_{j,t} - \iota_t^*) k_{j,t}^r$ imposed on renewable firms, together with a Pigouvian tax on the usage of fossil fuel: $\tau_t^f = \sum_{j=0}^{\infty} \beta^j \frac{u'(c_{t+j}^*)}{u'(c_t^*)} \cdot \pi_{t+j} \cdot y_{t+j}^*(1 - d_j)$, where $\{c_t^*, y_t^*\}_{t=0}^{\infty}$ is the solution to planners' problem, and $1 - d_j = \varphi_L + (1 - \varphi_L)\varphi_0(1 - \varphi)^j$;*

*Proof.* The firm $j$'s problem becomes

$$\max_{\{\iota_{j,t}\}_{t=0}^{\infty}} \sum_{\tau=0}^{\infty} \beta^\tau \frac{u'(c_{t+\tau})}{u'(c_t)} \left[ p_{t+\tau}^e \left( \mathcal{E}_{j,t+\tau} \right)^{1-\alpha_r} \left( k_{j,t}^r \right)^{\alpha_r} - r_{t+\tau} k_{j,t}^r - \Psi(\iota_{j,t+\tau}) k_{j,t}^r + \Phi_{t+\tau}^j \right]$$

s.t. $\ln \mathcal{E}_{t+1}^j \leq \gamma + \ln \mathcal{E}_t^j + \varepsilon_{j,t} \left( \left( \int_0^1 \iota_{j,t} k_{j,t}^r dj / \int_0^1 k_{j,t}^r dj \right), \sigma \right) : \lambda_{\mathcal{E},t}^j$

$\iota_{j,t} \geq 0$, and $\mathcal{E}_0$ given

The FOC are

$$\partial k_{j,t}^r : p_t^e \cdot \alpha_r \left( \frac{\mathcal{E}_{j,t}}{k_{j,t}^r} \right)^{1-\alpha_r} + \Psi'(\iota_t^*) (\iota_{j,t} - \iota_t^*) = r_t + \Psi(\iota_{j,t}) \tag{58}$$

$$\partial \iota_t^i : \Psi'(\iota_{j,t}) = \Psi'(\iota_t^*) \tag{59}$$

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From (59) we get $\iota_{j,t} = \iota_t^*$ and (58) implies:

$$\alpha_r \left(\frac{\mathcal{E}_{jt}}{k_{jt}^r}\right)^{1-\alpha_r} + \left[\Psi'\left(\iota_t^*\right)\left(\iota_{j,t} - \iota_t^*\right) - \Psi\left(\iota_{j,t}\right)\right]\left[(1-\theta)A_t\left(\frac{(k_t^g)^{\theta_k}(L_t)^{\theta_L}}{(e_t)^\theta}\right)\right]^{-1} = \alpha_f \left(\frac{f_t}{k_t^f}\right)^{1-\alpha_f} \tag{60}$$

Equation (60) is identical to condition (28) of the social planner's problem. Thus, the optimal levels for $\frac{\mathcal{E}_{jt}}{k_{jt}^r}$ and $\iota_{j,t}$, are implemented with $\Phi_t\left(k, \iota_{j,t}\right)$.

Next, suppose sellers of fossil fuel face a linear tax at rate

$$\tau_t^f = \sum_{j=0}^{\infty} \beta^j \frac{u'(c_{t+j}^*)}{u'(c_t^*)} \cdot \pi_{t+j} \cdot y_{t+j}^*(1-d_j) \tag{61}$$

where $\{c_t^*, y_t^*\}_{t=0}^{\infty}$ solve the planner's problem, and

$$1 - d_j = \varphi_L + (1 - \varphi_L)\varphi_0(1-\varphi)^j$$

Facing such taxes, fossil fuel sellers' optimal intertemporal substitution implies that

$$u'(c_t)\left\{p_t^f - \tau_t^f\right\} = \beta \cdot u'(c_{t+1})\left\{p_{t+1}^f - \tau_{t+1}^f\right\}$$

Using (51) and (54) for the price of fossil fuel and (61) for the tax, we obtain

$$u'(c_t)\left\{MPF_t - \pi_t \cdot y_t^*\left(\varphi_L + (1-\varphi_L)\varphi_0\right)\right\} + \sum_{j=1}^{\infty} \beta^j u'(c_{t+j}^*) \cdot \pi_{t+j} \cdot y_{t+j}^*((1-\varphi_L)\varphi_0(1-\varphi)^{j-1}\varphi)$$

$$= \beta \cdot u'(c_{t+1})\left\{MPF_{t+1}\right\}$$

which is equal to (32).QED $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

The implied policy has some interesting implications. The tax on renewable energy firms generates no revenue, but it reduces households' profits from the renewable sector, as a result of inducing additional scraping. The Pigouvian tax reduces households' profits from the fossil fuel sector. However, households receive a lump-sum transfer of equal magnitude and, thus, their budget constraint remains unchanged. Finally, there is no interaction between these two schemes, since the total effect on households' budget is the same as the resource cost of scrapping in the planner's problem.

# 4   Calibration and Policy Scenarios

In this section we report on some preliminary findings where we are ignoring the environmental costs from fossil fuel use. This can be considered as a benchmark case where the environmental externality is not internalized in any way. We calibrate the model using

the following parameters. We assume that a period equals one year. Regarding preferences, we assume that the annual discount rate is $\beta = 0.96$, and the utility function is $u(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$, with a coefficient of relative risk aversion, $\sigma = 2$. Moving to the technology side, we assume that the share of capital in the final good production is $\theta_k = (1/3) * 0.95$, while the share of capital in the final good production is $\theta_l = (2/3) * 0.95$. Like before, $1 - \theta = 1 - \theta_k - \theta_l$ is the share of energy in the goods production. Annual depreciation is $\delta = 0.1$. The productivity growth rate in the final goods sector is calibrated to $g_g = \exp(0.02)$. The share of capital in the renewable sector is $\alpha_r = 1/2$, and the share of capital in the fossil sector is $\alpha_f = 1/3$. For the renewable technology we set $\Psi(\iota) = \frac{1}{\psi}[\exp(\psi\iota) - 1]$, with $\psi = 100$. This implies that $\Psi'(\iota) = \exp(\psi\iota)$. Finally we set the exogenous growth rate of the renewable productivity to $\gamma = 0$.

To calculate estimated total reserves of fossil fuel and the resulting emissions, we used data and conversions from the *Energy Information Administration*, the *Survey of Energy Resources,* and the *World Energy Sources*.[5] We assume total reserves of coal equal to 18.050922 x (10^18) BTU, oil equal to 104.26 x (10^18) BTU, natural gas equal to 6.433647 x (10^18) BTU, and methane hydrates equal to 719.6 x (10^18) BTU.[6] Using conversion factors for $CO_2$ emissions (in lbs per 100,000 BTU) given by 30.29 for coal, 17.5 for oil, and 14.12 for natural gas, we estimate total emissions from using all these resources to equal 57256.4986918168784 Gt of $CO_2$.

So far we have considered three scenarios in the absence of the environmental externality. We consider the next 175 years. In all scenarios, capital is allocated optimally across activities and all energy is produced through renewable fuel asymptotically. The benchmark scenario has the investment in renewables, $\iota$, fixed at the long run optimal level, while the consumption of fossils is fixed for the next 100 years, after which it declines for 75 years. The vertical axis is normalized such that current usage = 1. Growth requires an increasing amount of energy use. Fossil fuel use declines after about 2120 and it is overtaken by renewable energy by around 2145.

Scenario 2 considers the case of doubling $\iota$ in the first 100 years, then reducing $\iota$ to the same level as in scenario 1 for the following 75 years. As a result of increased investment, renewables take over faster, around 2110. This has a positive effect on economic growth, as indicated by the total amount of energy used relative to the benchmark scenario.

Scenario 3 assumes "under-investment" in renewables, with $\iota$ being half that in the benchmark scenario in the first 100 years, then increasing $\iota$ to reach the same level as in scenario 1 for the following 75 years. The growth in renewable energy is slower than in the benchmark. Renewables begin to grow significantly only after 2100, and they overtake fossil fuel production only after 2160. As indicated by the significantly lower total energy use, overall growth is lower than the benchmark in this scenario. Indeed, growth stagnates around the time that renewables overtake fossil fuel and it takes off again only after $\iota$ recovers and renewables take

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[5]See also Hartley, Medlock, Temzelides, and Zhang (2012).

[6]While energy from methane hydrates (effectively frozen natural gas on the bottom of deep oceans) is not commercially viable using today's technology, experiments suggest that these resources will become available in the medium to long run.

over completely.

Energy prices are increasing in all three scenarios as the economy is growing. However, energy prices grow slower when $\iota$ is high initially. To summarize, our quantitative analysis so far suggests that investment in renewables is an important variable that may have significant long-term effects on the composition of energy use, the allocation of capital in the economy, and economic growth. However, one should also be aware of the possibility of over investing in a field where technological progress is fast. The highest growth path is not the one involving the highest investment in the renewable sector. Investing too early in a fast-changing technology can be detrimental to growth.

# 5    Conclusions

We studied the adaptation of new technologies by renewable energy-producing firms in a dynamic general equilibrium model where energy is an input in the production of final goods. Spillovers lead to under-scrapping in the productivity of renewable energy. Efficiency requires a policy which promotes adaptation of new technologies by subsidizing investment in renewables. Our theoretical analysis argues that it is not optimal to make large investments in new technologies when technological progress is fast. Our quantitative results suggest that investment in renewables has significant long-term positive effects on the composition of energy use, the allocation of capital in the economy, and overall economic growth. However, one should also be aware of the possibility of over investing in a field where technological progress is fast. The highest growth path is not the one involving the highest investment in the renewable sector. Taken together, these results suggest that short-run investment in renewables is desirable. At the same time, the focus of this investment should take into consideration that technological progress comes at a cost of making part of the existing capital obsolete. Investing in R&D, a common way to increase future productivity, appears to be more effective that investing in capital that will need to be partly scrapped before long.
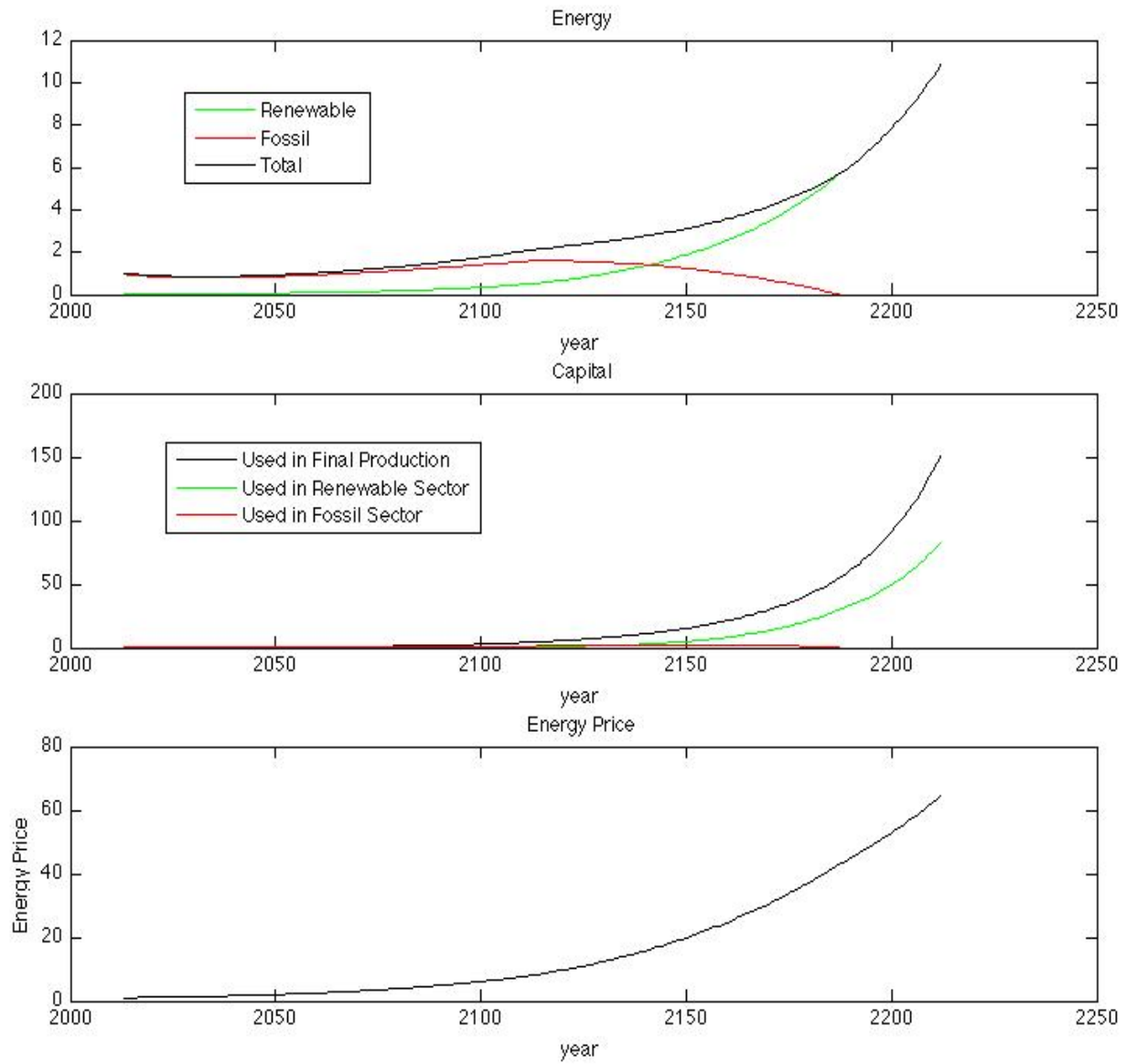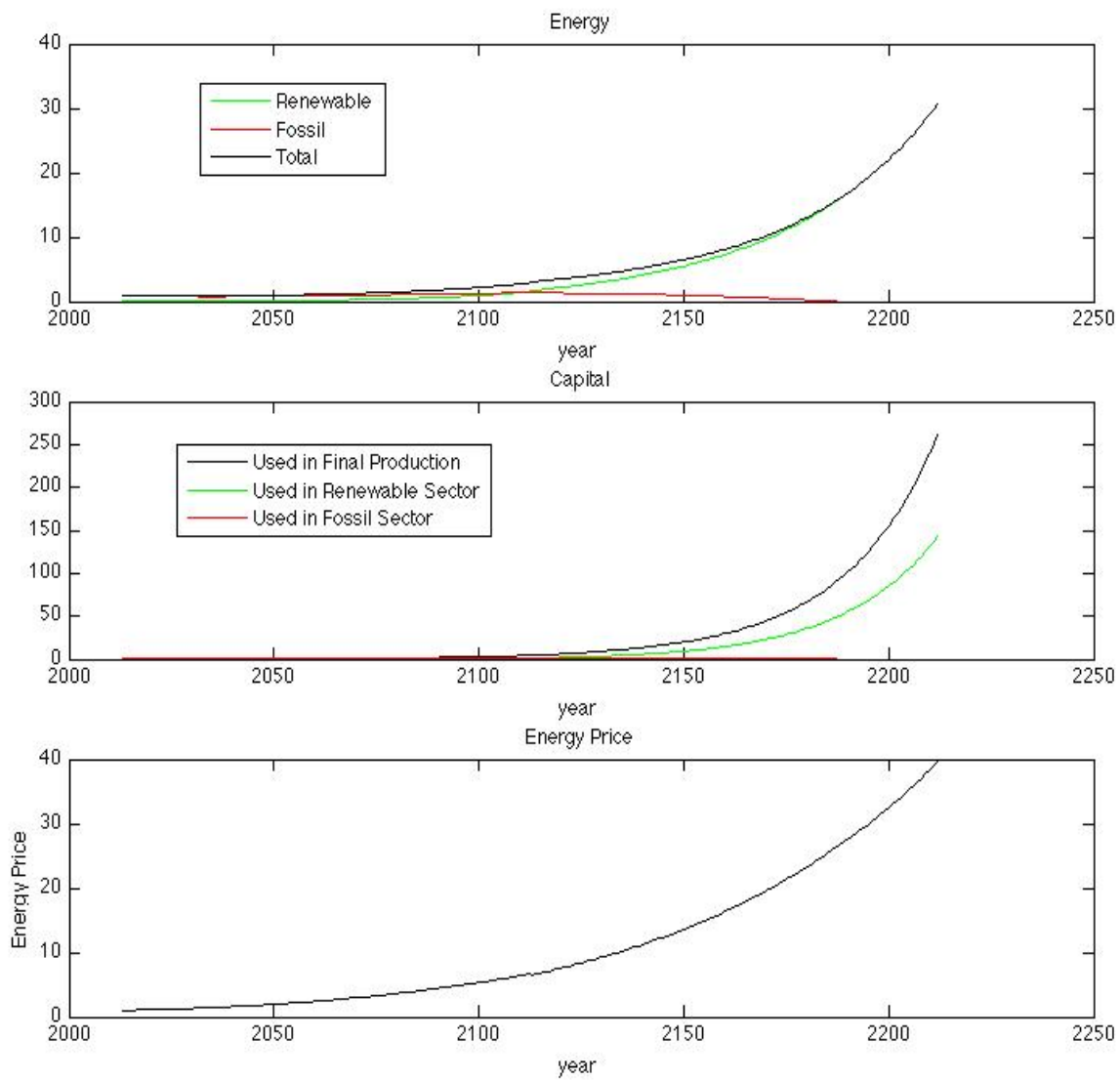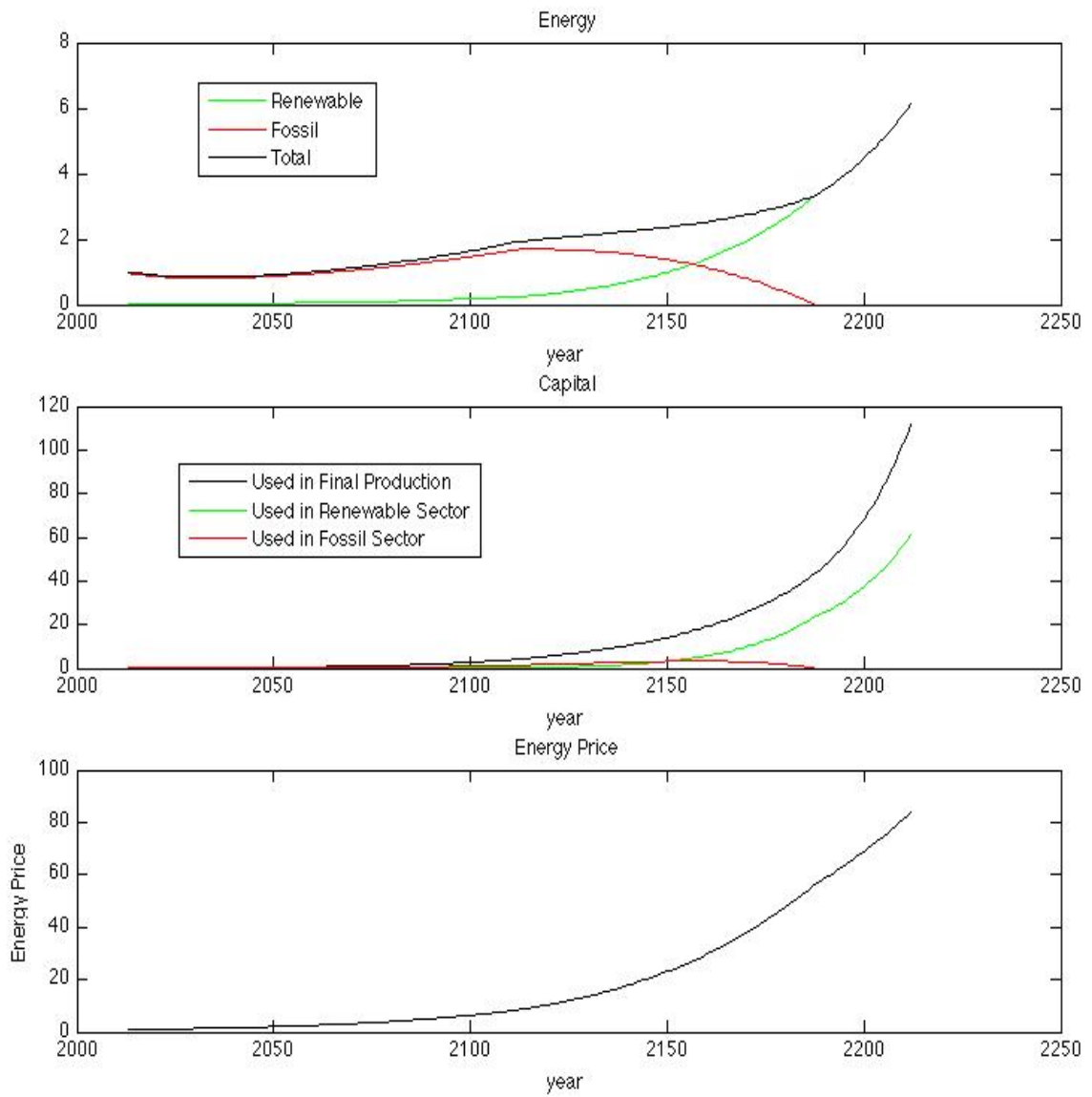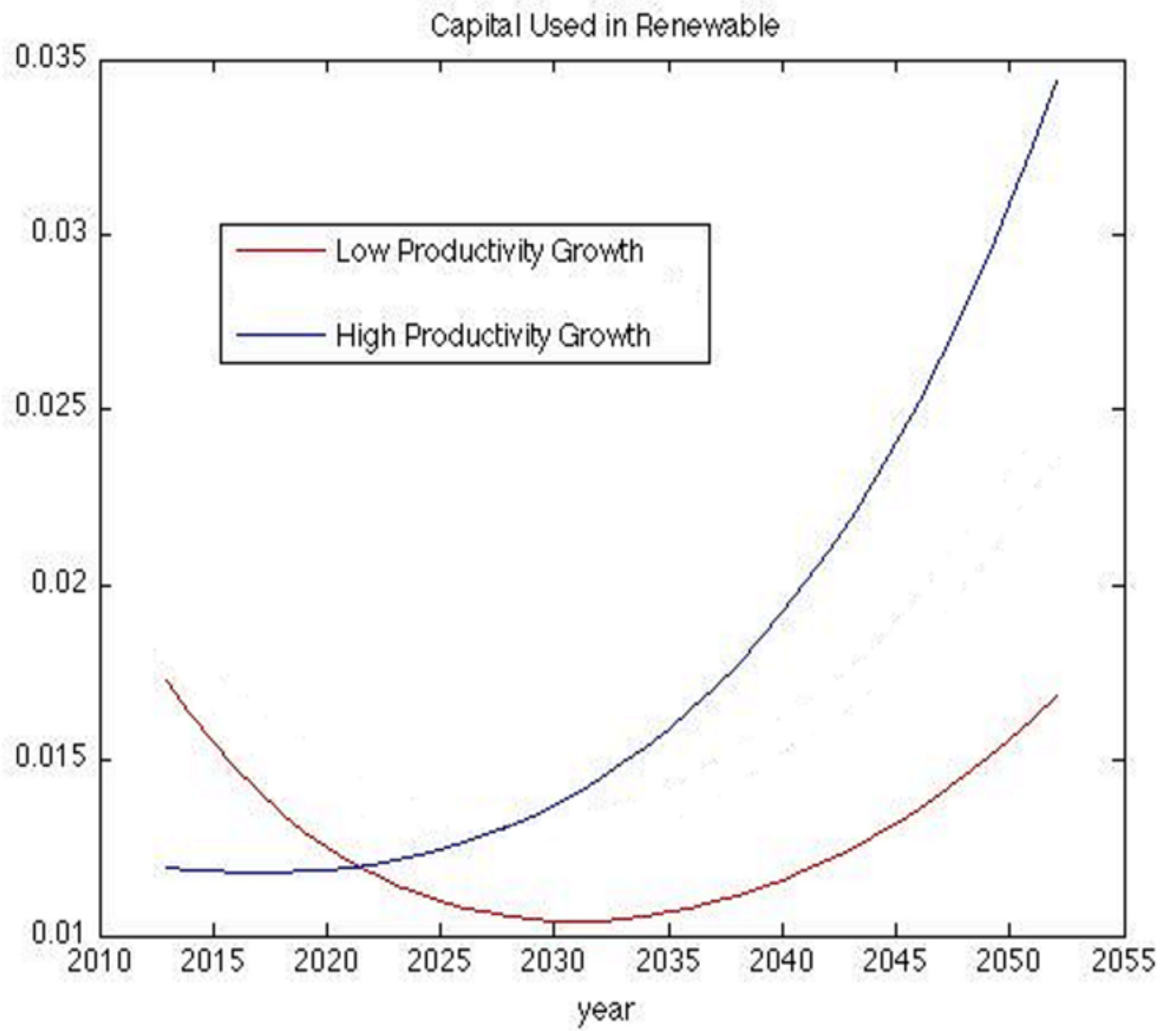
Figure 1: Regime 1

Figure 2: Regime 2
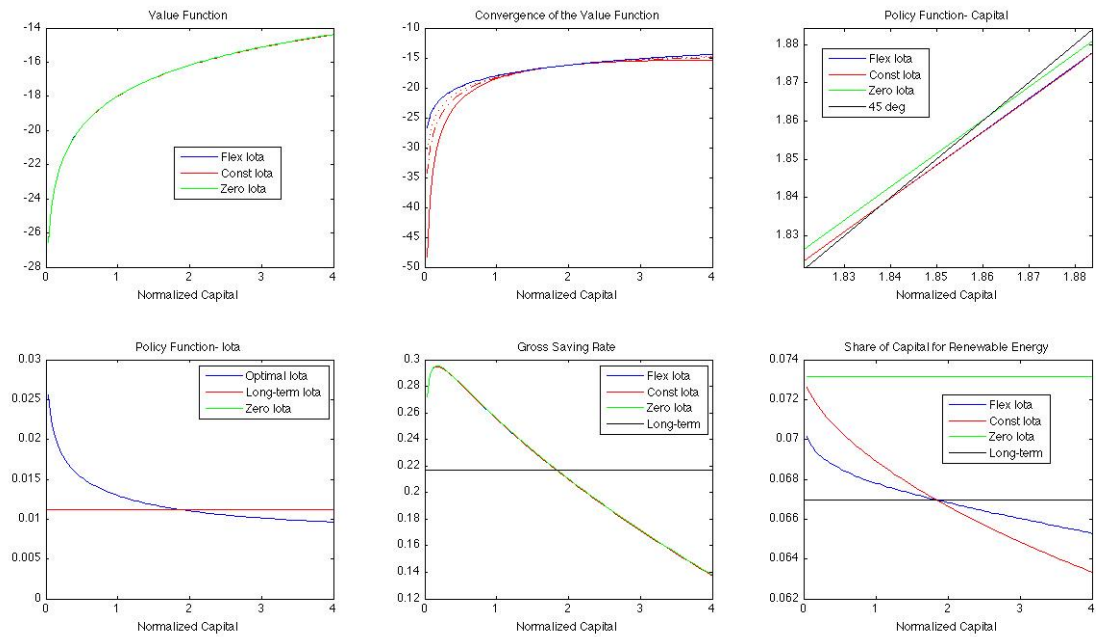
Figure 3: Regime 3

Figure 4: Capital use in renewables

Figure 5: Value and Policy Functions

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