The Spatial Efficiency Multiplier and Common Correlated Effects in a Spatial Autoregressive Stochastic Frontier Model

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Abstract

We extend the emerging literature on spatial frontier methods in a number of respects. One contribution includes accounting for unobserved heterogeneity. This involves developing a random effects spatial autoregressive stochastic frontier model which we generalize to a common correlated effects specification to account for correlation between the regressors and the unit specific effects. Another contribution is the introduction of the concept of a spatial efficiency multiplier to show that the efficiency frontiers from the structural and reduced forms of a spatial frontier model differ. To demonstrate various features of the estimators we develop we carry out a Monte Carlo simulation analysis and provide an empirical application. The application is to a state level cost frontier for U.S. agriculture which is a popular case in the efficiency literature and is thus well-suited to highlighting the features of the estimators we propose.

Key words: Direct, indirect and total efficiencies; Efficiency spillovers; Random effects; Time-invariant and time-varying efficiencies; U.S. state agriculture.

JEL Classification: C23; C51; D24; Q10.

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1 Introduction

Omitted variable bias in cross-sectional and panel data modeling due to a misspecification of spillovers from a spatial lag of the dependent variable, which captures what is referred to as spatial autoregressive (SAR) cross-sectional dependence, is well-established. This omitted variable bias was a key motivation for the development of the SAR model in seminal work by Cliff and Ord (1973; 1981). In a stochastic frontier framework, which is characterized by a composed disturbance with idiosyncratic error and inefficiency components, the above misspecification of spillovers leads to a further issue of biased estimates of the efficiency scores. To account for this bias Glass *et al.* (2016) develop a maximum likelihood (ML) estimator of a SAR stochastic frontier model with time-varying inefficiency. Their model, however, which involves employing distributional assumptions to distinguish between the idiosyncratic error and inefficiency components of the composed disturbance, does not account for unobserved heterogeneity. Since spatial frontier modeling is an emerging literature we extend their basic SAR frontier model in a number of ways. Our first extension is to account for unobserved heterogeneity by developing a ML estimator of the SAR stochastic frontier model with random effects. Drawing on Mundlak (1978) and, more recently, Pesaran (2006) we generalize this model to a common correlated effects specification. This generalized specification relaxes the strong assumption underlying the random effects model that the regressors are not correlated with the unit specific effects.

In contrast to other frontier models, which measure either time-varying or time-invariant inefficiency, our second extension is to include both of these inefficiency measures in a single spatial composed error model. The composed error structure for our common correlated effects SAR production frontier has four additive components. These four components include an idiosyncratic error term ($v_{it} \sim N(0, \sigma_v^2)$) and a correlated random effects error term ($\varphi_i = \bar{x}_i'\gamma + \kappa_i$; $\kappa_i \sim N(0, \sigma_\kappa^2)$), which are both assumed to be normally distributed. The other two components are an inefficiency term that is time-invariant ($\eta_i \sim N^+\left(0, \sigma_\eta^2\right)$) and an inefficiency term that is time-varying ($u_{it} \sim N^+\left(0, \sigma_u^2\right)$). For reasons we expand on further in this section, we refer to η_i and u_{it} as net time-invariant and net time-varying inefficiencies (NII and NVI, respectively) to indicate that they are net of time-variance and time-invariance. Rigidities in slow to adjust factors such as fixed assets and the internal organization of production would be sources of NII and concurrent with these rigidities managerial inefficiency would be a source of NVI. Managerial inefficiency will vary over time as there is turnover in managerial staffing possessing different skill sets. We follow much of the frontier production literature by specifying the parametric distribution of both inefficiency terms to be half-normally distributed (e.g., Aigner *et al.*, 1977; Greene, 2005; Horrace and Parmeter, 2015). Our model, however, is sufficiently general to accommodate other distributional assumptions for NII and NVI

from the frontier literature, e.g., the exponential (Meeusen and van den Broeck, 1977) or gamma (Greene, 1990) distribution.[1] Additionally, the multiplicative form of our SAR production frontier allows us to easily convert the estimates of NII and NVI into their efficiency counterparts, which we label as NIE and NVE. We can then calculate a composite measure of efficiency, which we refer to as gross time-varying efficiency ($GVE = NIE * NVE$).[2] We present in detail the set-up of our SAR frontier model with common correlated effects in the next section.

The emerging spatial frontier literature primarily consists of a small number of studies which adopt a different approach to the one we utilize in our study, typically focusing on one-way effects spatial panel models where the efficiencies are based on the unit specific effects. The first such study is Druska and Horrace (2004). By extending the cross-sectional spatial error model in Kelejian and Prucha (1999) they develop a GMM stochastic frontier model with fixed effects. Using the fixed effects they calculate time-invariant efficiency by applying the Schmidt and Sickles (1984) efficiency estimator, which assumes a composed disturbance comprising the idiosyncratic error and time-invariant inefficiency. Glass *et al.* (2013) extended this literature by using the fixed effects from a spatial panel model to calculate time-varying efficiency using the Cornwell *et al.* (1990) time-varying extension of the Schmidt and Sickles estimator.

In contrast to spatial frontier models that compute efficiency using the unit specific effects and thus assume that all the unobserved heterogeneity is inefficiency, the model we present distinguishes not just between unobserved heterogeneity and time-varying inefficiency, which is a common approach in the frontier literature (Greene, 2005; Chen *et al.*, 2014), but also between unobserved heterogeneity and time-invariant inefficiency, which has only been proposed thus far in a non-spatial setting (e.g., Columbi *et al.*, 2011; 2014; Filippini and Greene, 2016). The Columbi *et al.* papers present a one-step (i.e., full information ML) estimator of the non-spatial counterpart of our model, although Filippini and Greene question the tractability of this estimator. ML estimation of a one-way effects SAR model is typically carried out using a sequential approach by estimating the SAR parameter first and then the remaining parameters second. We expand on this further in the paper but for details on this for a one-way effects SAR model see, for example, Elhorst (2009).

Compared to one-way effects SAR models our model set-up is more complex because of the presence of the NVI and NII components, so in the spirit of the sequential

[1] See Parmeter and Kumbhakar (2014) for a detailed discussion of the different distributional assumptions for inefficiency in the frontier literature.

[2] In the corresponding non-spatial frontiers NVE and NIE are referred to as transient and persistent efficiencies in Filippini and Greene (2016) and as short and long run efficiencies in Columbi *et al.* (2014). Both of these studies refer to GVE as overall efficiency. Using each of the own NVE, NIE and GVE estimates we compute three spatial efficiencies which we refer to as direct, indirect and total efficiencies. Rather than use the labels from the corresponding non-spatial frontiers we use the net and gross terminology to avoid the odd labelling of a spatial efficiency as a total overall efficiency.

approach to ML estimation of a one-way effects SAR model we propose an extended ML estimator comprising three steps. In the first step we distinguish between the (two) time-invariant and (two) time-varying components of the composed error by estimating a SAR one-way common correlated effects model. The second step splits the time-varying error component from the first step into the idiosyncratic error term (v_{it}) and the NVI error term (u_{it}^{+}). The third step splits the time-invariant error component from the first step into a correlated random effect (φ_i) and the NII error term (η_i^{+}).

Our third extension involves introducing the concept of a spatial efficiency multiplier, which we use to obtain partitioned efficiencies across space. We can relate the partitioned efficiency spillovers to 1st order neighbors, 2nd order neighbors, etc. The partitioned efficiency spillovers indicate the speed of decay of efficiency spillovers across space. We also use the spatial efficiency multiplier to distinguish between the best practice efficiency frontier from the structural form of a spatial frontier and the best practice efficiency frontier from the reduced form of the model. Following Anselin (2003) in the spatial literature, the structural form of our spatial frontier model includes the SAR variable as a regressor that shifts the frontier technology, whereas the SAR variable does not feature in the reduced form of the model. The upshot is that the structural form of our spatial frontier model accounts for SAR dependence and yields an own efficiency measure that is directly comparable to efficiency from a non-spatial stochastic frontier, which does not include any efficiency spillovers across the system/network. The reduced form of our spatial frontier model, on the other hand, yields a system/network measure of efficiency for a productive unit that includes efficiency spillovers. During the course of this paper we discuss these different efficiency solutions from the structural and reduced forms of a spatial frontier and also their different interpretations.

Putting our spatial frontier model into a general context, we can view a firm's unobserved inefficiency as it appears in the structural form of our model as a placeholder (normalized to have one-sided support) for productivity effects relating to technical efficiency (e.g., Aigner *et al.*, 1977), cost efficiency (Olley and Pakes, 1996), intangible capital (Corrado *et al.*, 2009), unobserved organizational capital (Brynjolfsson and Hitt, 2003), or simply an unobservable factor (Levinsohn and Petrin, 2003). None of these literatures, however, focus on the following spatial issues we address. In particular, we interpret own unobserved efficiency from the structural form of our model as own productivity effects based on frontier production having accounted for SAR interaction. Using the reduced form of our model we compute two asymmetric system/network measures of efficiency for a firm comprising its own productivity/efficiency and the asymmetric productivity/efficiency spillovers that a firm exports and imports to and from other firms.

The remainder of the paper is organized as follows. In section 2 we present our SAR frontier with common correlated effects and develop the extended ML estimation procedure for the model. Section 3 introduces the concept of a spatial efficiency multiplier and discusses how to partition efficiencies across space. We also discuss the different interpretations given to efficiency effects in the structural model and in its reduced form. In the Monte Carlo experiments in section 4, we examine the impact of different types of spatial frontier model misspecification on the finite sample performance of our estimates. The empirical application in section 5 is a state level cost frontier for U.S. agriculture. This is a popular case to study in the efficiency literature and is therefore well-suited to highlight the features of our estimator. Section 6 concludes and suggests some areas for future research.

2 SAR Stochastic Frontier Model with Common Correlated Effects

2.1 Structural Form of the Model

We now focus on formally introducing our model and how it improves on others in the literature. First, we account for both time-invariant and time-varying inefficiency (NII and NVI, respectively) in our new spatial frontier model. Second, we model unobserved heterogeneity via random effects and then we generalize to a common correlated effects specification to relax the strong assumption underpinning the standard random effects model that the regressors are not correlated with the unit specific effects. In the context of a general production, profit, revenue or cost frontier we present the common correlated effects SAR stochastic frontier as it nests the random effects model.

The general structural form of a common correlated effects SAR stochastic panel production frontier model is:

$$y_{it} = \alpha + x'_{it}\beta + \delta \sum_{j=1}^{N} w_{ij} y_{jt} + \varphi_i + v_{it} - \eta_i - u_{it}, \tag{1}$$
$$i = 1, ..., N;\ t = 1, ..., T,$$

where $\varphi_i = \bar{x}'_i\gamma + \kappa_i$, $\kappa_i \sim N(0, \sigma^2_\kappa)$; $\eta_i \sim N^+\left(0, \sigma^2_\eta\right)$; $v_{it} \sim N\left(0, \sigma^2_v\right)$; and $u_{it} \sim N^+\left(0, \sigma^2_u\right)$. y_{it} is the observation for the dependent variable (i.e., output in the case of a production frontier) for the ith unit at time period t, x'_{it} is the $(1 \times K)$ vector of observations for the non-spatial regressors, β is a vector of regression parameters and α is the common intercept. x_{it} will include variables which together with y_{it} represent the frontier technology and x_{it} will also include any variables that shift the frontier. We will consider log linear type functional forms such as the Cobb-Douglas or the translog in our Monte Carlo experiments and empirical example so that the variables in Eq. 1 will enter as logged values of the dependent and independent variables. Our general modeling framework also encompasses other functional forms such as linear, quadratic or generalized Leontief technologies. The interpretation we give to various efficiency measures, however, will differ a bit depending on whether or not the model takes a multiplicative form in levels (which we adopt) or an additive form in levels, which we do not utilize. These different specifications will of course determine how the error terms enter the model and thus determine how they are interpreted.

The composed error structure in Eq. 1 is $\varphi_i + v_{it} - \eta_i - u_{it}$. The difference between our common correlated effects and random effects SAR frontiers relates to the unit specific common correlated effect, $\varphi_i = \bar{x}_i'\gamma + \kappa_i$, where $\bar{x}_i' = \frac{1}{T}\sum_{t=1}^{T} x_{it}'$. Eq. 1 is a generalization of the random effects SAR frontier because by omitting $\bar{x}_i'\gamma$ from Eq. 1 unobserved heterogeneity is accounted for by only the unit specific random effect, κ_i.

The $(N \times N)$ spatial weights matrix, W_N, represents the spatial arrangement of the cross-sectional units and also the strength of the spatial interaction among the units. W_N is specified *a priori* and is a matrix of non-negative constants w_{ij}. Typically in the spatial literature W_N is exogenous, which is also an underlying assumption of Eq. 1. In line with this exogeneity a measure of geographical proximity is frequently used in the spatial literature to specify W_N. The spatial lag of the dependent variable, $\sum_{j=1}^{N} w_{ij} y_{jt}$, shifts the frontier technology, where δ is the SAR scalar parameter. This SAR variable is of course endogenous which we account for in our ML estimator.

To emphasize the contribution of the additional components that we include in our model we relate Eq. 1 to simpler frontier models. For example, when $\delta = 0$ in Eq. 1 the resulting model is the common correlated effects non-spatial frontier in Filippini and Greene (2016). By omitting $\bar{x}_i'\gamma$ from our model and setting $\delta = 0$ we obtain the non-spatial random effects frontier in Colombi *et al.* (2011; 2014). Omitting $\bar{x}_i'\gamma$ together with κ_i and η_i and Eq. 1 collapses to the SAR frontier with time-varying inefficiency in Glass *et al.* (2016), which omits time-invariant inefficiency and does not account for unobserved heterogeneity. Encompassed within Eq. 1 is also the spatial Durbin specification. In this case x will also include spatial lags of the exogenous independent variables which shift the frontier technology. In line with the above simpler frontier models we assume that all the x variables are exogenous, which is what Amsler *et al.* (2016) describe as the typical approach to stochastic frontier modeling.[3]

[3] A worthwhile area for further work would be to extend our approach to the case of endogenous x variables (i.e., where an x variable is correlated with inefficiency or the disturbance). One approach to address this issue would be limited information ML (LIML) estimation of a system of equations consisting of the spatial stochastic frontier and a reduced form equation for each of the endogenous variables which will include a suitable instrument. This LIML estimator will be simpler when the correlation is only between noise from the spatial frontier and the error from each of the reduced form equations (see Kutlu, 2010, for the non-spatial case). When the error from each of the reduced form equations is correlated with inefficiency, which may be in addition to (or instead of) the reduced form errors being correlated with noise from the spatial frontier, the LIML estimator will be more complex. In this case one could model the joint distribution of inefficiency and the reduced form errors by employing a copula. For more details on this in a non-spatial setting see Amsler *et al.* (2016).

In addition to the above assumptions that relate specifically to our stochastic frontier in Eq. 1 such as the distributional assumptions for NII and NVI, our model requires the following general assumptions from the spatial econometrics literature (e.g., Kelejian and Prucha, 2004), which are based on standard normalizations and regularity conditions.

Assumption 1 (A1): *W_N is non-stochastic and fixed over time with the elements on the main diagonal set to zero.*

Assumption 2 (A2): *The matrix $(I_N - \delta W_N)$ is non-singular for all $\delta \in (1/r_{\min}, 1/r_{\max})$, where $r_{\min}$ and $r_{\max}$ are the most negative and most positive real characteristic roots of W_N.*

Assumption 3 (A3): *N and T are large.*

Assumption 4 (A4): *The row and column sums of W_N before normalization, $\widetilde{W}_N$, are uniformly bounded in absolute value as $N \to \infty$, and for all δ the row and column sums of $\left(I_N - \delta \widetilde{W}_N\right)^{-1}$ are uniformly bounded in absolute value as $N \to \infty$.*

Assumption 5 (A5): *The $(NT \times 2K)$ regressor matrix $Z = \left[X, \overline{X}\right]$ has full column rank of $2K$ and the elements of Z are non-stochastic and are uniformly bounded in absolute value in N and T. Also, $\lim_{N\to\infty} (1/NT)\, Z'\Omega^{-1}Z$ exists and is non-singular, where Ω is the variance-covariance matrix.*[4]

Setting all the elements on the main diagonal of W_N equal to zero in A1 is a normalization rule which ensures that no unit can be viewed as its own neighbor. A2 ensures that the reduced form of Eq. 1 exists and if W_N is asymmetric it may have complex roots so in this case $r_{\min}$ is the most negative pure real characteristic root. For all the specifications of W_N that we employ in the application section of the paper the normalizations of $\widetilde{W}_N$ yield $r_{\max} = 1$. A3 defines the asymptotic setting of our estimator that allows us to consistently estimate the slope parameters, the variances, the common correlated effects coefficients and the unit specific efficiencies. If only unbiased estimates of unit specific efficiencies and effects are required then T does not need to be large. See Battese and Coelli (1988) and Schmidt and Sickles (1984) for a more detailed discussion of this issue. A4 ensures that the spatial process of the dependent variable has a fading memory to limit this spatial process to a manageable degree (Kelejian and Prucha, 2001). A5 rules out perfect collinearity.

[4]In the estimation $\overline{x}$ is treated as vector of auxiliary regressors, hence the $(NT \times 2K)$ dimension of the regressor matrix.

2.2 Maximum Likelihood Estimation and Net and Gross Efficiencies

The ML estimation procedure we set out can also be the basis for the development of similar ML procedures for other spatial stochastic frontier models with random or common correlated effects such as a higher order SAR frontier or a dynamic SAR frontier.[5] Since in the estimation we treat $\overline{x}$ as a vector of auxiliary regressors we can rewrite Eq. 1 as:

$$y_{it} = \alpha + x'_{it}\beta + \delta\sum_{j=1}^{N} w_{ij}y_{jt} + \bar{x}'_i\gamma + \kappa_i + v_{it} - \eta_i - u_{it}, \tag{2}$$

where the time-invariant component of the composed error is $\varepsilon_i = \kappa_i - \eta_i$ and the time-varying component is $\varepsilon_{it} = v_{it} - u_{it}$.

Estimating the regression parameters of Eq. 2 involves first estimating the SAR parameter and second estimating the parameters for the other regressors. In the spirit of this sequential approach we add two further steps to this estimation procedure, which relate to the estimation of NVI and NII. To explain the steps involved in our estimation procedure we find it useful to rewrite the frontier model in Eq. 2 as the one-way effects SAR panel model:

$$y_{it} = \alpha^{\circ} + x'_{it}\beta + \bar{x}'_i\gamma + \delta\sum_{j=1}^{N} w_{ij}y_{jt} + \varepsilon_i^{\circ} + \varepsilon_{it}^{\circ}, \tag{3}$$

where we use the following reparameterizations of the intercept and the negatively skewed time-invariant and time-varying errors, ε_i and ε_{it}: $\alpha^{\circ} = \alpha - \mu_{\varepsilon_i} - \mu_{\varepsilon_{it}}$; $\varepsilon_i^{\circ} = \kappa_i - \eta_i + \mu_{\varepsilon_i}$; and $\varepsilon_{it}^{\circ} = v_{it} - u_{it} + \mu_{\varepsilon_{it}}$, where $\mu_{\varepsilon_{it}} = \mathrm{E}(u_{it})$ and $\mu_{\varepsilon_i} = \mathrm{E}(\eta_i)$, and we note that ε_i° and ε_{it}° satisfy the zero-mean condition. In order for our estimation procedure to split the time-invariant and time-varying error components into their constituent parts, we also find it useful to reparameterize the variance terms as: $\sigma^2_{\eta\kappa} = \sigma^2_{\eta} + \sigma^2_{\kappa}$ and $\lambda_{\eta\kappa} = \sigma_{\eta}/\sigma_{\kappa}$, and $\sigma^2_{uv} = \sigma^2_u + \sigma^2_v$ and $\lambda_{uv} = \sigma_u/\sigma_v$. Therefore, $\sigma^2_{\eta} = \sigma^2_{\eta\kappa}/\left(1 + \lambda^2_{\eta\kappa}\right)$; $\sigma^2_{\kappa} = \sigma^2_{\eta\kappa}\lambda^2_{\eta\kappa}/\left(1 + \lambda^2_{\eta\kappa}\right)$; $\sigma^2_u = \sigma^2_{uv}/\left(1 + \lambda^2_{uv}\right)$; and $\sigma^2_v = \sigma^2_{uv}\lambda^2_{uv}/\left(1 + \lambda^2_{uv}\right)$.

Next let the superscript * denote the transformations of y_{it}, x_{it}, $\bar{x}_i$, $\sum_{j=1}^{N} w_{ij}y_{jt}$ and thus ε_{it}° into the quasi-differenced forms:

$$y^*_{it} = y_{it} - (1-\theta)\,\frac{1}{T}\sum_{t=1}^{T} y_{it}, \tag{4}$$

[5] With higher order SAR models a decision must be made about the nature of the higher order dimension. This involves choosing between including two or more SAR variables that are constructed using different specifications of W, which is the most common form of a higher order SAR model, or including higher order polynomials in the different specifications of W (Elhorst *et al.*, 2012). With regard to the dynamic SAR frontier, we have in mind a model with lagged dependent and SAR variables rather than a specification where the dynamics relate to inefficiency (e.g., Tsionas, 2006).

$$x_{it}^{*} = x_{it} - (1-\theta)\frac{1}{T}\sum_{t=1}^{T} x_{it}, \tag{5}$$

$$\bar{x}_{i}^{*} = \bar{x}_{i} - (1-\theta)\,\bar{x}_{i}, \tag{6}$$

$$\left(\sum_{j=1}^{N} w_{ij} y_{jt}\right)^{*} = \sum_{j=1}^{N} w_{ij} y_{jt} - (1-\theta)\frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{N} w_{ij} y_{jt}, \tag{7}$$

$$\varepsilon_{it}^{*} = y_{it}^{*} - x_{it}^{*\prime}\beta - \bar{x}_{i}^{*\prime}\gamma - \delta\left(\sum_{j=1}^{N} w_{ij} y_{jt}\right)^{*}, \tag{8}$$

where θ denotes the weight attached to the cross-sectional component of the data and $0 < \theta^2 = \sigma_{uv}^2 / \left(T\sigma_{\eta\kappa}^2 + \sigma_{uv}^2\right) \leq 1$.[6]

Step 1 first specifies the log-likelihood function for Eq. 3 in terms of the parameters σ^2, β, γ and δ:

$$LL = -\frac{NT}{2}\log\left(2\pi\sigma^2\right) + T\log|I_N - \delta W_N| - \frac{1}{2\sigma^2}\sum_{i=1}^{N}\sum_{t=1}^{T}\left[y_{it}^{*} - x_{it}^{*\prime}\beta - \bar{x}_{i}^{*\prime}\gamma - \delta\left(\sum_{j=1}^{N} w_{ij} y_{jt}\right)^{*}\right]^{2}, \tag{9}$$

where the term $T\log|I_N - \delta W_N|$ represents the contribution to the log-likelihood from the Jacobian of the transformation from ε_{it}^{*} to y_{it}^{*} and where this transformation accounts for the endogeneity of the SAR variable (Anselin, 1988, pp. 63; Elhorst, 2009).

The parameters are estimated in **step 1** in the following way. We obtain the estimate of δ using the concentrated log-likelihood function:

$$LL_C = \varpi - \frac{NT}{2}\log\left[\left(e_0^{*} - \delta e_1^{*}\right)'\left(e_0^{*} - \delta e_1^{*}\right)\right] + T\log|I_N - \delta W_N|, \tag{10}$$

where ϖ is a constant that does not depend on δ, and e_0^{*} and e_1^{*} denote the OLS residuals from regressing y^{*} and $(I_T \otimes W_N)y^{*}$ on $Z^{*} = \left[X^{*}, \bar{X}^{*}\right]$. Here the residuals and observations for the variables are denoted in terms of stacked cross-sections for $t = 1, .., T$; I_T denotes the $(T \times T)$ identity matrix; and $\otimes$ denotes the Kronecker product. Before maximizing Eq. 10 we follow Pace and Barry (1997) for linear spatial models by calculating $\log|I_N - \delta W_N|$ for a vector of values of δ over the interval $(1/r_{\min}, 1)$. As they suggest, we calculate $\log|I_N - \delta W_N|$ for values of δ based on 0.001 increments over the above feasible range for δ. Using ς to collectively denote the β and γ parameter vectors

[6] If $\theta = 0$ Eq. 3 collapses to the one-way fixed effects SAR model.

and given the estimate of δ, the estimator for ς is:

$$\widehat{\varsigma} = b_0 - \widehat{\delta} b_1 = \left(Z^{*\prime} Z^*\right)^{-1} Z^{*\prime}[y^* - \widehat{\delta}\left(I_T \otimes W_N\right) y^*], \tag{11}$$

where b_0 and b_1 are the OLS estimates from regressing y^* and $(I_T \otimes W_N)y^*$ on Z^*.

Following LeSage and Pace (2009) we obtain the standard errors using a mixed analytical-numerical Hessian, where all the second order derivatives are computed analytically, with the exception of $\partial^2 LL/\partial\delta^2$, which is evaluated numerically. Evaluating the second order derivatives of the log-likelihood function analytically rather than numerically is less sensitive to badly scaled data, and numerical rather than analytical evaluation of $\partial^2 LL/\partial\delta^2$ avoids computational difficulties associated with the evaluation of a large spatial multiplier matrix, $(I_N - \delta W_N)^{-1}$.

Given β, γ, δ and σ^2, the concentrated log-likelihood function we use to compute θ is:

$$LL_C(\theta) = -\frac{NT}{2} \log\left[e(\theta)' e(\theta)\right] + \frac{N}{2} \log \theta^2, \tag{12}$$

where

$$e(\theta)_{it} = y^*_{it} - x^{*\prime}_{it}\beta - \bar{x}^{*\prime}_{i}\gamma - \delta\left(\sum_{j=1}^{N} w_{ij} y_{jt}\right)^*. \tag{13}$$

Step 2 estimates λ_{uv} using $\widehat{\varepsilon}^{\circ}_{it}$ from **step 1**. This is done by maximizing the concentrated log-likelihood function in Eq. 14:

$$LL_C(\lambda_{uv}) = -NT \ln \widehat{\sigma}_{uv} + \sum_{i=1}^{N}\sum_{t=1}^{T} \ln\left[1 - \Phi\left(\frac{\widehat{\varepsilon}^{\circ}_{it}\lambda_{uv}}{\sigma_{uv}}\right)\right] - \frac{1}{2\widehat{\sigma}^2_{uv}} \sum_{i=1}^{N}\sum_{t=1}^{T} \widehat{\varepsilon}^{\circ 2}_{it}, \tag{14}$$

where Φ is the standard normal cumulative distribution function and

$$\widehat{\sigma}_{uv} = \left(\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left[y^*_{it} - x^{*\prime}_{it}\beta - \bar{x}^{*\prime}_{i}\gamma - \delta\left(\sum_{j=1}^{N} w_{ij} y_{jt}\right)^*\right]^2 / \left[1 - 2\lambda^2_{uv}/\pi\left(1 + \lambda^2_{uv}\right)\right]\right)^{\frac{1}{2}}. \tag{15}$$

Maximizing $LL_C(\lambda_{uv})$ yields the ML estimate $\widehat{\lambda}_{uv}$. By substituting $\widehat{\lambda}_{uv}$ into Eq. 15 we obtain the ML estimate $\widehat{\sigma}^2_{uv}$. The consistent estimator of the constant term is then scaled by the value of $\widehat{\mu}_{\varepsilon_{it}}\left(\widehat{\lambda}_{uv}, \widehat{\sigma}^2_{uv}\right)$.[7]

We follow Battese and Coelli (1988), who in the spirit of Jondrow *et al.* (1982), predict u_{it} conditional on ε_{it}, where we refer to u_{it} as net time-varying inefficiency, NVI_{it}.

[7] For stochastic production, revenue and profit frontiers a productive unit is assumed to maximize the objective variable so for these technologies the constant is scaled down by $\widehat{\mu}_{\varepsilon_{it}}\left(\widehat{\lambda}_{uv}, \widehat{\sigma}^2_{uv}\right)$. Conversely, for a stochastic cost frontier the constant is scaled up by $\widehat{\mu}_{\varepsilon_{it}}\left(\widehat{\lambda}_{uv}, \widehat{\sigma}^2_{uv}\right)$.

$$\widehat{u}_{it} = \mathrm{E}\left(u_{it}|\varepsilon_{it}\right) = \frac{\sigma_u \sigma_v}{\sigma_{uv}} \left(\frac{\phi_{it}}{1 - \Phi_{it}} - \frac{\varepsilon_{it} \lambda_{uv}}{\sigma_{uv}} \right), \tag{16}$$

where $\Phi_{it} = \Phi\left(\varepsilon_{it}\lambda_{uv}/\sigma_{uv}\right)$, $\phi_{it} = \phi\left(\varepsilon_{it}\lambda_{uv}/\sigma_{uv}\right)$, Φ is as previously defined and ϕ is the probability density function for the standard normal distribution.

Step 3 uses the same approach to compute $\widehat{\eta}_i$ as we use in **step 2** to calculate $\widehat{u}_{it}$, where we refer to η_i as net time-invariant inefficiency, NII_i. Accordingly, in **step 3** the corresponding ML estimator to that in **step 2** (see Eq. 14) is used to estimate $\lambda_{\eta\kappa}$ using $\widehat{\varepsilon}_i^{\circ}$ from **step 1**. For a stochastic production, revenue or profit frontier the intercept is scaled down further by $\widehat{\mu}_{\varepsilon_i}\left(\widehat{\lambda}_{\eta\kappa}, \widehat{\sigma}^2_{\eta\kappa}\right)$, while for a stochastic cost frontier the intercept is scaled up further by this term. We then use the Battese and Coelli (1988) estimator to predict $\widehat{\eta}_i = \mathrm{E}(\eta_i|\varepsilon_i)$.

If Eq. 2 is in log form, the multiplicative form of our common correlated effects SAR frontier is:

$$\mathbf{y}_{it} = \exp\left(\alpha\right) * \mathbf{x}_{it}^{\beta} * \bar{\mathbf{x}}_i^{\gamma} * \left(\sum_{j=1}^{N} w_{ij} \mathbf{y}_{jt} \right)^{\delta} * \exp\left(-\eta_i\right) * \exp(-u_{it}) * \exp\left(v_{it} + \kappa_i\right), \tag{17}$$

where we use bold notation to highlight the level form of the data and everything else is as previously defined for Eq. 2. Productive units may of course lie below the concave production, profit or revenue frontier or above the convex cost frontier because they are inefficient. Lower NVI_{it} (NII_i) will push net time-varying efficiency, NVE_{it} (net time-invariant efficiency, NIE_i), which is bounded in the interval $[0, 1]$, closer to the upper bound. To estimate efficiency we adopt the widely used Battese and Coelli (1988) estimator, which involves using the multiplicative form of a stochastic frontier and computing a unit's efficiency by taking the exponential of the unit's distance below (above) the concave (convex) frontier. For a convex frontier this distance is measured by inefficiency and for a concave frontier it is measured by the negative of inefficiency. From the multiplicative form of our concave spatial frontier in Eq. 17 we recognize that the estimates of the net time-varying and net time-invariant efficiencies are $NVE_{it} = \exp\left(-\widehat{u}_{it}\right)$ and $NIE_i = \exp\left(-\widehat{\eta}_i\right)$. Using the estimates of NVE_{it} and NIE_i and again with recourse to the multiplicative form of our model, we compute the estimate of combined efficiency, $GVE_{it} = \exp\left[-\left(\widehat{\eta}_i + \widehat{u}_{it}\right)\right] = NIE_i * NVE_{it}$, which we refer to as gross time-varying efficiency.

3 The Spatial Efficiency Multiplier and the Reduced Form of a SAR Frontier

We relate the general spatial literature (e.g., LeSage and Pace, 2009) to all the productivity oriented functional forms of our model including flexible functional forms (e.g., the translog function) as a marginal effect is a function not just of the relevant slope parameter but also the SAR parameter.[8] For our model we obtain the marginal effects by following the general spatial literature which involves computing the direct, indirect and total marginal effects. If Eq. 2 is in log form then these marginal effects are elasticity measures. In the case of flexible functional forms with mean adjusted data the first order marginal effects are elasticities at the sample mean because at the sample mean the interaction and squared terms are zero.

For a SAR stochastic frontier a direct elasticity is interpreted in the same way as the corresponding elasticity from a non-spatial frontier, although a direct elasticity takes into account feedback effects. This feedback is the effect of a change in an independent variable for a particular unit which as a result of the spatial multiplier, $(I_N - \delta W_N)^{-1}$, partially rebounds back to the same unit's dependent variable via the effect on the dependent variables of its 1st and higher order neighbors. The rebound is only partial because of the fading memory across space (Assumption 4 above). An indirect elasticity from a SAR stochastic frontier can be calculated in two ways yielding the same numerical value. This leads to two interpretations of an indirect elasticity: (i) average change in the dependent variables of all other units following a change in an independent variable for one particular unit; or (ii) average change in the dependent variable for a particular unit following a change in an independent variable for all the other units. A total elasticity from a SAR stochastic frontier is the sum of the relevant direct and indirect elasticities. To compute the direct, indirect and total elasticities we differentiate the following reduced form of Eq. 2:

$$y_t = (I_N - \delta W_N)^{-1} \left(\alpha \iota + x_t' \beta + \bar{x}_t' \gamma + \kappa + v_t - \eta - u_t \right), \tag{18}$$

where now the model is in terms of vectors or matrices of stacked cross-sections, ι is an $(N \times 1)$ vector of ones and everything else is as previously defined. We use Monte Carlo simulation of the distributions of the direct, indirect and total elasticities to compute the standard errors which is a widely used approach in the spatial literature. This approach involves drawing $1,000$ parameter combinations of $\left(\widehat{\delta}, \widehat{\beta}, \widehat{\gamma}, \widehat{\sigma}^2\right)$ from the variance-covariance matrix, where each parameter has a random component drawn from $N(0,1)$.

Using the reduced form of their basic SAR stochastic frontier that, among other things,

[8] In the case of the spatial Durbin specification of Eq. 2 a marginal effect will also be a function of the relevant coefficient on the spatial lag of the exogenous regressor.

does not account for unobserved heterogeneity, Glass *et al.* (2016) develop an approach to compute direct, asymmetric indirect and asymmetric total efficiencies relative to the best performing unit in the sample in each period using the Schmidt and Sickles (1984) (SS from hereon) estimator. Relative direct efficiency is interpreted in a similar way to own efficiencies from a non-spatial frontier or a structural spatial frontier such as Eq. 2. Relative direct efficiency, however, also takes into account efficiency feedback. An example of such feedback is the effect of a change in an independent variable for a particular unit which affects the dependent variables and the efficiencies of the unit's 1st and higher order neighbors, and then through the spatial multiplier matrix this effect partially reverberates back to the dependent variable and the efficiency of the unit which initiated the change. Asymmetric relative indirect efficiencies are relative efficiency spillovers that a unit exports (imports) to (from) all the other units in the sample. Since combining the relative direct efficiency and a relative indirect efficiency yields a relative total efficiency, the asymmetric relative indirect efficiencies lead to asymmetric relative total efficiencies.

Calculating relative direct, asymmetric relative indirect and asymmetric relative total efficiencies using the aforementioned SS estimator is very informative as it constructs appropriate direct, indirect and total efficiency frontiers. This is important because the relative total efficiency frontiers will differ from the efficiency frontier from the structural form of the SAR frontier in Eq. 2. This is because the frontier from this structural form is the efficiency benchmark for each unit's own efficiency. In contrast, the relative total efficiency frontiers are benchmarks for a unit's efficiency across a network/system. We know that own efficiency from the structural form of the SAR frontier in Eq. 2 and relative direct efficiency from the reduced form of the SAR frontier in Eq. 18 are all measures of substantive economic performance. We do not know, however, whether the relative asymmetric indirect efficiencies from the reduced form of a SAR frontier represent substantive economic performance spillovers. To illustrate, all units in a sample can have high relative indirect efficiencies but economic performance spillovers can be negligible because the absolute indirect efficiencies are all small. To establish whether relative indirect efficiencies relate to substantive economic performance spillovers we propose using relative indirect efficiencies alongside absolute indirect efficiencies.

We now turn to the method to compute absolute efficiencies. This is a rather complex exercise as Eq. 2 has time-varying and time-invariant inefficiency components. From the reduced form of the SAR frontier in Eq. 18 we recognize that $(I_N - \delta W_N)^{-1} \eta = NII_M^{Tot}$ and $(I_N - \delta W_N)^{-1} u_t = NVI_M^{Tot}$, where NII_M^{Tot} and NVI_M^{Tot} denote $(N \times 1)$ vectors of absolute total NII and absolute total NVI. The subscript M denotes that the absolute inefficiency spillovers used in the calculation of these absolute total inefficiency vectors are the inefficiency spillovers which the ith unit implicitly imports from all the jth units for $i \neq j$. Drawing on the multiplicative form of our model in Eq. 17 the efficiencies that correspond directly to the inefficiencies NII_M^{Tot} and NVI_M^{Tot} are $(I_N - \delta W_N)^{-1} \exp(-\eta) =$

NIE_M^{Tot} and $(I_N - \delta W_N)^{-1} \exp(-u_t) = NVE_M^{Tot}$, where NIE_M^{Tot} and NVE_M^{Tot} denote $(N \times 1)$ vectors of absolute total NIE and absolute total NVE.

Since we have established from the multiplicative form of our model that $GVE = NIE * NVE = \exp[-(\widehat{\eta} + \widehat{u}_t)]$ is the $(N \times 1)$ vector of own GVE, the $(N \times 1)$ vector of absolute total GVE that corresponds directly to this own GVE vector is $GVE_M^{Tot} = (I_N - \delta W_N)^{-1} \exp[-(\eta + u_t)]$. GVE_M^{Tot} can be written in the following form and similar expressions can be used to represent NIE_M^{Tot} and NVE_M^{Tot}.

$$(I_N - \delta W_N)^{-1} \begin{pmatrix} GVE_1 \\ \vdots \\ GVE_N \end{pmatrix}_t = \begin{pmatrix} GVE_{11}^{Dir} & + & \cdots & + & GVE_{1N}^{Ind} \\ \vdots & + & \ddots & + & \vdots \\ GVE_{N1}^{Ind} & + & \cdots & + & GVE_{NN}^{Dir} \end{pmatrix}_t = \begin{pmatrix} GVE_{M,1}^{Tot} \\ \vdots \\ GVE_{M,N}^{Tot} \end{pmatrix}_t, \tag{19}$$

where GVE_{ij}^{Dir} on the main diagonal is the direct GVE of a unit, GVE_{ij}^{Ind} is the indirect GVE spillover to the ith unit from the jth unit for $i \neq j$ and $GVE_M^{Ind} = \sum_{j=1}^{N} GVE_{ij}^{Ind}$ is the sum of the absolute indirect GVE spillovers to the ith unit from all the jth units for $i \neq j$.

The column sums of the components in Eq. 19 is the $(1 \times N)$ absolute total GVE vector that we denote $GVE_X^{Tot'} = \left(GVE_{X,1}^{Tot}, GVE_{X,2}^{Tot}, ..., GVE_{X,N}^{Tot}\right)$. The subscript X denotes that the absolute indirect GVE spillovers used in the calculation of $GVE_X^{Tot'}$ are the GVE spillovers that the ith unit implicitly exports to the jth unit for $i \neq j$. $GVE_X^{Ind} = \sum_{i=1}^{N} GVE_{ij}^{Ind}$ is the sum of these absolute indirect GVE spillovers to the jth unit from all the ith units for $i \neq j$. In terms of interpretation GVE_M^{Tot} and GVE_X^{Tot} measure a unit's absolute GVE across a system/network.

In the spatial Monte Carlo simulations that we introduce in the next section W_N is symmetric, which as we highlight in that section is the typical approach in the spatial econometrics literature. In empirical applications, however, W_N is often asymmetric. If W_N is asymmetric $(I_N - \delta W_N)^{-1}$ will be asymmetric resulting in $GVE_{ij}^{Ind} \neq GVE_{ji}^{Ind}$ in Eq. 19 indicating that there are asymmetric absolute indirect GVE spillovers to and from a unit. Since direct, indirect and total NIE, NVE and GVE from the reduced form in Eq. 18 all contain some form of efficiency spillover they represent different performance metrics to own NIE, NVE and GVE from the structural form of the model in Eq. 2. An own NIE, NVE and GVE frontier is not therefore the appropriate benchmark for the corresponding direct, indirect and total NIE, NVE and GVE from the reduced form of our model.

Own NIE, NVE and GVE from a non-spatial frontier or the structural form of a SAR frontier are of course all bounded in the interval $[0, 1]$. The lower bound of absolute direct, indirect and total NIE, NVE and GVE from the reduced form of a SAR frontier will also be 0. Other than that absolute direct, indirect and total NIE, NVE and GVE are unbounded.[9] Absolute direct, indirect and total NIE, NVE and GVE, however, can easily be interpreted because they are percentages as they are scaled own NIE, NVE and GVE. The magnitude of the scaling relates to the magnitude of the efficiency spillover that partially/entirely makes up the absolute direct, indirect and total NIE, NVE and GVE. If the magnitude of the efficiency spillover is sufficiently large, absolute direct/indirect/total NIE, NVE or GVE will be greater than 1. In this case the efficiency spillover has pushed the unit beyond the best practice frontier for own efficiency from the structural form of the SAR frontier.

As a result of the fading memory property of $(I_N - \delta W_N)^{-1}$ efficiency spillovers to and from a unit die out across space. To examine the speed of the decay of efficiency spillovers across space we recognize that $(I_N - \delta W_N)^{-1} \exp(-\eta)$, $(I_N - \delta W_N)^{-1} \exp(-u_t)$ and $(I_N - \delta W_N)^{-1} \exp[-(\eta + u_t)]$ are the spatial absolute efficiency multipliers for NIE_M^{Tot}, NVE_M^{Tot} and GVE_M^{Tot}, respectively. Expansion of GVE_M^{Tot} across space involves the following infinite series expansion of $(I_N - \delta W_N)^{-1}$, where the spatial expansions of GVE_X^{Tot}, NIE_M^{Tot}, NIE_X^{Tot}, NVE_M^{Tot} and NVE_X^{Tot} have the same form.

$$(I_N - \delta W_N)^{-1} \exp[-(\eta + u_t)] = \left(I_N + \delta W_N + \delta^2 W_N^2 + \delta^3 W_N^3 + ...\right) \exp[-(\eta + u_t)]. \tag{20}$$

Using Eq. 20 we partition GVE_{ij}^{Dir}, GVE_M^{Ind} and GVE_M^{Tot} into own GVE (pertaining to I_N) and GVE feedback and spillovers which come to the ith unit from 1st order neighbors (pertaining to δW_N), 2nd order neighbors (pertaining to $\delta^2 W_N^2$) and so on and so forth up to pre-specified higher order neighbors determined by the upper limit that is placed on the order of W_N.

4 Monte Carlo Simulations

We next turn to a Monte Carlo simulation analysis of the random effects and common correlated effects SAR frontier estimators that we propose. Our simulation analysis focuses on the sensitivity of the statistical performance to: (i) if and how unobserved heterogeneity is modeled; and (ii) whether and how SAR dependence is accounted for. To this end we use three experimental designs. The data generating process (DGP) for design 1 is the random effects non-spatial frontier and the DGP for design 2 is the random effects SAR frontier when W is based on Rook contiguity.[10] The DGP for design 3 is the common

[9] For details on using the aforementioned SS method to transform absolute direct, indirect and total efficiencies into relative efficiencies that are bounded in the interval $[0, 1]$ see Glass *et al.* (2016).

[10] Rook contiguity defines a pair of units as neighbors if they share a common edge.

correlated effects SAR frontier, where once again W is based on Rook contiguity. For the DGPs for designs 2 and 3 the Rook contiguous spatial weights are row-normalized. We discuss normalization of W in more detail in the application section but note at this juncture that a row-normalized contiguous W is often used in empirical work (e.g., Fredriksson and Millimet, 2002; Autant-Bernard and LeSage, 2011) and is consistently used in Monte Carlo experiments in the spatial econometrics literature (e.g., Baltagi *et al.*, 2003; 2007).

The key objectives of the three experimental designs are as follows. Design 1 analyzes the statistical performance of our random effects SAR frontier under a non-spatial frontier DGP to explore the implications of failing to omit the SAR variable. Design 2 analyzes how the extent of the misspecification of W impacts on the statistical performance of our random effects SAR frontier. This involves departing from the row-normalized Rook contiguous W in the DGP and considering the effect of incorrectly assuming that the row-normalized W is based on Queen or Bishop contiguity.[11] In addition, under a random effects SAR frontier DGP that includes NVI and NII, design 2 analyzes the impact of a SAR frontier that does not model unobserved heterogeneity that is not interpreted as inefficiency and which also omits the NII error component. Design 3 assesses the statistical performance of various estimators under a common correlated effects SAR frontier DGP. This allows us to examine the impact of incorrectly omitting the SAR variable while accounting for unobserved heterogeneity. It also allows us to evaluate the impact of omitting common correlated effects and NII from a SAR frontier model with time-varying inefficiency but no time-invariant heterogeneity.

We list the DGP and estimator combinations in table 1 and in line with this list the simulation results are for models $1-12$. For all three distinct DGPs the spatial arrangement of the units is based on a perfect square board of dimension $\sqrt{N} \geq 3$, which ensures a common form of connectivity between the units across the sample sizes and is in keeping with Monte Carlo set-ups used in the spatial econometrics literature (see, for example, Yu and Lee, 2010, and Tao and Yu, 2012). The sample sizes we consider are all comparable to those used in Monte Carlo experiments in the stochastic frontier literature (see, for example, Chen *et al.*, 2014). Further details of our Monte Carlo set-up are provided in the Supplementary document that accompanies the paper.[12] Our extensive Monte Carlo simulation results can also be viewed in the Supplementary document as space limitations prevent us from displaying them here so in the remainder of this section we summarize these results.

When the model is correctly specified (models 1, 4 and 11) the simulation results for

[11] Bishop contiguity defines a pair of units as neighbors if they share a common vertex. Queen contiguity incorporates both the Rook and Bishop definitions of contiguity and defines a pair of units as neighbors if their boundaries share at least one common point (i.e., the units have a common edge or vertex).

[12] The enclosed Supplementary document does not form part of the submitted paper. In due course the Supplementary document will be made available online.

Table 1: Design of the Monte Carlo experiments

	Model estimators	Size of experiments	Total number of unique simulation designs
Design 1 DGP: Random effects non-spatial frontier	1) Random effects non-spatial frontier 2) Random effects SAR frontier when W is based on Rook contiguity	All combinations of $N = \{9, 16, 49, 100, 225, 529\}$ and $T = \{5, 10, 50\}$ for both estimators	$2 \times 18 = 36$
Design 2 DGP: Random effects SAR frontier when W is based on Rook contiguity	3) Random effects non-spatial frontier 4) Random effects SAR frontier when W is based on Rook contiguity 5) Random effects SAR frontier when W is based on Bishop contiguity 6) Random effects SAR frontier when W is based on Queen contiguity 7) SAR frontier that does not account for unobserved heterogeneity and omits NII, where W is based on Rook contiguity	All combinations of $N = \{9, 16, 49, 100, 225, 529\}$ and $T = \{5, 10, 50\}$ for estimators $3-6$ and for estimator **7** the above combinations of N and T with the exception of $\{N = 100, 225, T = 50\}$ and $\{N = 529, T = 10, 50\}$. See the note below why we do not consider these combinations of N and T for estimator **7**.	$(4 \times 18) + 14 = 86$
Design 3 DGP: Common correlated effects SAR frontier when W is based on Rook contiguity	8) Random effects non-spatial frontier 9) Common correlated effects non-spatial frontier 10) Random effects SAR frontier when W is based on Rook contiguity 11) Common correlated effects SAR frontier when W is based on Rook contiguity 12) SAR frontier that does not account for unobserved heterogeneity and omits NII, where W is based on Rook contiguity	All combinations of $N = \{9, 16, 49, 100, 225, 529\}$ and $T = \{5, 10, 50\}$ for estimators $8-11$ and the same combinations of N and T for estimator 12 as for the same estimator in design 2 (model **7**).	$(4 \times 18) + 14 = 86$
		Total number of unique simulations	$36 + 86 + 86 = 208$
		Number of replications	$1,000$
		Total number of simulations	$208,000$

Note
The estimator for model **7** (which is model 12 under a different DGP) is for pooled data so the dimension of W is $(NT \times NT)$. The estimators for all the other models are for panel data so the dimension of W is $(N \times N)$ in each period. For the pooled estimator we do not consider $\{N = 100, 225, T = 50\}$ and $\{N = 529, T = 10, 50\}$ because for this estimator in Stata the dimensions of W are prohibitively large for these combinations of N and T.

the β and δ estimates are quite good, as we would expect, in terms of bias and mean squared error (MSE). The statistical performance of the β estimator, however, for the nine misspecified models is quite varied. For the misspecified models 5 and 6 the simulation results for δ are in line with our expectations. We find that the statistical performance of the δ estimator is much better when the misspecification of W is less marked as W in model 5 is based on Bishop contiguity which represents a bigger departure from the Rook contiguity under the design 2 DGP than the Queen contiguous W in model 6. This is because Queen contiguity incorporates Rook contiguity while Bishop contiguity does not. For model 5 the δ estimate exhibits substantial negative bias for all combinations of N and T, whereas for model 6 the δ estimate performs better for all sample sizes and particularly for the larger sample sizes as the bias is relatively small compared to the small samples.

We also analyze the effect of three additional sources of misspecification on the performance of the δ estimator. These include modeling SAR dependence when it is not present (model 2); failing to account for unobserved heterogeneity by omitting, first, random effects and, second, common correlated effects (models 7 and 12, respectively); and ignoring correlation between x and the unit specific effects (model 10). The results for model 2 indicate that our random effects SAR frontier estimate of δ properly infers that there is no SAR dependence in the data. When we fail to account for unobserved heterogeneity via random effects or common correlated effects there is a downward bias in the estimate of δ (models 7 and 12). We also find that the estimate of δ for model 10 is quite similar to the prediction from model 11, where the latter model reflects the design 3 DGP when there is correlation between x and the unit specific effects.

Turning to the Monte Carlo simulation results for the estimates of $\mathrm{E}(u_{it}|\varepsilon_{it})$ and $\mathrm{E}(\eta_i|\varepsilon_i)$, we find that the bias and MSE are smaller for larger values of T and N. The performance of the estimate of $\mathrm{E}(u_{it}|\varepsilon_{it})$ is noticeably worse for two of the twelve models (models 7 and 12), both of which omit η_i. Thus compared to other forms of model misspecification we consider, we find that failing to account for unobserved heterogeneity by omitting random effects or common correlated effects will accentuate the bias of the estimate of $\mathrm{E}(u_{it}|\varepsilon_{it})$. We find this is particularly the case when common correlated effects are erroneously omitted (model 12). Interestingly and in contrast to the other misspecified models, the bias for the estimates of $\mathrm{E}(u_{it}|\varepsilon_{it})$ is negative for models 7 and 12.

5 Application to State Agriculture in the U.S.

We next present an empirical illustration of our new random effects and common correlated effects estimators. The general structural form of the SAR production, profit and revenue frontiers in Eq. 1 modeled technical inefficiency as a factor that moved the firm or producing unit, which is a state here, inside its frontier. Our application involves a cost frontier in which technical inefficiency causes observed costs to be greater than their efficient levels. Thus the inefficiency terms (with positive support) are added to instead of subtracted from the cost function to represent the excess costs of inefficient production.

We estimate random effects and common correlated effects SAR and spatial Durbin frontiers using agricultural data for the contiguous states in the U.S. The spatial Durbin frontier belongs to the general class of SAR models as it is the SAR model augmented with spatial lags of the exogenous variables as additional regressors that shift the frontier technology. We also estimate non-spatial cost frontiers with random effects and common correlated effects. SARF, SDF and NSF are used to denote the SAR, spatial Durbin and non-spatial frontiers and a subscript R or C is used to distinguish between the random effects and common correlated effects specifications.

We adopt the flexible translog cost technology and because the SDF_C nests the SARF_C, NSF_C, SDF_R, SARF_R and NSF_R we present these six model specifications using only the following SDF_C, where all variables here are logged.

$$c_{it} = \alpha + \tau_t + TL\left(y_{it}, p_{it}, t\right) + \sum_{j=1}^{N} w_{ij} TL\left(y_{jt}, p_{jt}, t\right) + \delta \sum_{j=1}^{N} w_{ij} c_{jt} +$$
$$\rho' \bar{y}_i + \varrho' \bar{p}_i + \zeta' \sum_{j=1}^{N} w_{ij} \bar{y}_j + \psi' \sum_{j=1}^{N} w_{ij} \bar{p}_j + \kappa_i + v_{it} + \eta_i + u_{it}. \quad (21)$$

Here $TL\left(y_{it}, p_{it}, t\right)$ represents the translog cost technology and is a quadratic function in y_{it}, p_{it} and t. c_{it} is normalized total cost for the ith state at time t, y_{it} is a vector of outputs, p_{it} is a vector of normalized input prices, t is a time trend, α is the common intercept and τ_t is a time period effect.[13] Returns to scale are variable over the sample so the cost efficiency measures differ due to the different scales of agricultural operations across the U.S. states. The error structure, $\varepsilon_{it} + \varepsilon_i = \kappa_i + v_{it} + \eta_i + u_{it}$, differs from that for Eq. 2 because Eq. 21 is a stochastic cost frontier.

$\sum_{j=1}^{N} w_{ij} c_{jt}$ is the spatial lag of the dependent variable, δ is the SAR parameter, $\sum_{j=1}^{N} w_{ij} TL\left(y_{jt}, p_{jt}, t\right)$ is the spatial lag of $TL\left(y_{it}, p_{it}, t\right)$ and W_N is a matrix of non-negative constant spatial weights w_{ij}. When W_N is row-normalized, as is the case here, $W_N t$ and $W_N t^2$ must be omitted for reasons of perfect collinearity because $W_N t = t$ and $W_N t^2 = t^2$. No variables shift the cost frontier technology in the NSF models and $\sum_{j=1}^{N} w_{ij} c_{jt}$ shifts the cost frontier technology in the SARF models. In the SDF models $\sum_{j=1}^{N} w_{ij} TL\left(y_{jt}, p_{jt}, t\right)$ and $\sum_{j=1}^{N} w_{ij} c_{jt}$ shift the cost frontier technology. The vectors

[13]We follow the spatial decomposition of aggregate total factor productivity (TFP) growth for European countries by Glass *et al.* (2013) and capture the effects of time in Eq. 21 in two ways. We specify a non-linear time trend to capture the trend of technical change over the study period by including t, t^2 and interactions with t. We also include time period dummy variables to capture common departures from the non-linear trend in a particular year due to, for example, common macroeconomic shocks.

of means $\bar{y}_i$ and $\bar{p}_i$ are included in the NSF and SARF specifications to account for correlation between the unit specific effects and the first order outputs and first order normalized input prices. This is for parsimony and because from a behavioral perspective one can argue that a productive unit only considers the values of the first order variables in its decision making and not the values of the higher order variables and interactions between the first order variables. To be consistent with this approach and because often the spatial lags of the exogenous regressors are found to be important determinants in applications of the spatial Durbin model, in the SDF specifications we also include $\sum_{j=1}^{N} w_{ij}\bar{y}_j$ and $\sum_{j=1}^{N} w_{ij}\bar{p}_j$. In the NSF and SARF models $\sum_{j=1}^{N} w_{ij}TL(y_{jt}, p_{jt}, t)$ does not appear so the terms $\sum_{j=1}^{N} w_{ij}\bar{y}_j$ and $\sum_{j=1}^{N} w_{ij}\bar{p}_j$ drop out. The vectors of parameters associated with the aforementioned means are ρ', ϱ', ζ' and ψ'.

5.1 Data and Spatial Weights Matrices

The data is publicly available from the U.S. Department of Agriculture's (USDA) productivity database and often used in the productivity literature (e.g., Karagiannis and Mergos, 2000; Morrison Paul *et al.*, 2004; Morrison Paul and Nehring, 2005).[14] In particular, the data set is a balanced panel for the 48 contiguous states in the U.S. for the period $1960 - 2004$.[15] Balanced panel data is the case in the majority of spatial panel studies because the asymptotic properties of spatial estimators for unbalanced panels become problematic unless the reason why data are missing is known (Elhorst, 2009). From a methodological perspective unbalanced spatial panels have been considered by assuming that observations are missing at random (MAR) (Pfaffermayr, 2013) or are missing for the edge units in a spatial structure (Kelejian and Prucha, 2010). These assumptions about the nature of missing spatial data, however, will not be valid in many empirical settings. The MAR assumption for instance is justified when the observed values of the dependent and independent variables can be used to predict the unobserved data to balance the panel. In a productivity setting this assumption will not be valid if, as can be the case, data tends to be missing for the less productive units in the sample.

The three outputs are measured in 000s of 1996 U.S. dollars and are the implicit quantities of livestock and products (y_1), crops (y_2) and farm related output (y_3).[16] Input prices are indices for capital services (p_1), labor services (p_2), total intermediate inputs (p_3) and land service flows (p_4).[17] The total cost relative (c) is the sum of the four factor input expenditure relatives, where the data for $p_1 - p_4$ and c are relative to the value for Alabama in 1996 and p_1 is the normalizing factor for c and $p_2 - p_4$.[18] Descriptive

[14] For a comprehensive discussion of the construction of the data we use see USDA (2014).

[15] The study period ends in 2004 due to the discontinuation of key data sources.

[16] Farm related output refers to goods and services from non-agricultural activities (e.g., processing and packaging of agricultural products) and secondary activities (e.g., machine services for hire).

[17] Total intermediate inputs include, for example, energy and agricultural chemicals.

[18] The dataset is constructed on the basis that each state is one large farm. We do not therefore include

statistics for the level variables are in table 2. In order to interpret the first order direct, indirect and total marginal effects as elasticities at the sample mean the logged variables are mean adjusted.

Table 2: Descriptive statistics for the level variables

	Variable	Mean	St. Dev.	Min	Max
Total cost relative (relative to the value for Alabama in 1996)	$\mathbf{c}$	3,105,805.49	3,498,059.11	23,561.12	24,027,241.07
Implicit quantity of livestock and products output (000s of 1996 U.S. dollars)	$\mathbf{y_1}$	1,677,446.55	1,588,017.95	9,100.68	8,497,604.24
Implicit quantity of crop output (000s of 1996 U.S. dollars)	$\mathbf{y_2}$	1,940,240.71	2,292,047.29	21,671.75	19,386,468.33
Implicit quantity of farm related output (000s of 1996 U.S. dollars)	$\mathbf{y_3}$	202,458.76	268,807.66	798.60	2,660,367.45
Capital services input price index excluding land (Alabama in 1996 is 1)	$\mathbf{p_1}$	0.64	0.37	0.13	1.24
Labor services input price index (Alabama in 1996 is 1)	$\mathbf{p_2}$	0.44	0.33	0.05	2.11
Total intermediate inputs price index (Alabama in 1996 is 1)	$\mathbf{p_3}$	0.89	0.38	0.22	2.02
Land service flows input price index (Alabama in 1996 is 1)	$\mathbf{p_4}$	0.61	0.58	0.01	3.63

As in Eq. 17 bold notation is used to denote level variables

All the specifications of W we employ are either based on Rook contiguous states (denoted W_{Cont}) or the inverse distances between a state centroid and the centroids of its nearest $3-7$ neighboring states (denoted $W_{3Near} - W_{7Near}$). By using specifications of W that are based on geographical proximity we are following the majority of the spatial literature and as a result the spatial weights are exogenous, which is an underlying assumption of our random effects and common correlated effects SAR frontiers. Replacing the exogenous W in our models with an endogenous W based on economic factors would involve replacing our first step estimator with an extension of the SAR ML based approach that Qu and Lee (2015) propose. Although their estimator accounts for an endogenous W it is for cross-sectional data and would therefore need to be extended to our panel data setting, which we do not undertake in this paper.

Since all our specifications of W have an assumed cut-off ($3-7$ nearest neighbors) or a natural cut-off (Rook contiguous neighbors) they are referred to as sparse as they contain many zeros. By using sparse formulations of W, which often feature in the spatial literature (e.g., Fredriksson and Millimet, 2002; Autant-Bernard and LeSage, 2011), partitioning of a spatial efficiency multiplier across space can be demonstrated much more

average farm size in a state as a variable that shifts the cost frontier technology to account for farm scale effects. This is because these scale effects are accounted for by c and y. Also, for reasons of collinearity we do not include additional covariates that shift the cost frontier technology to account for input quality (e.g., farmer education and weather covariates to account for the quality of the labor and land inputs, respectively). This is because the input prices will reflect the quality of the inputs, among other things. Moreover, the input quantities used to calculate c are quality adjusted using hedonic methods. For more details on this quality adjustment see USDA (2014).

clearly than would otherwise be the case.[19] This is because, for example, partitioned asymmetric indirect efficiency spillovers to (from) a state can be easily interpreted as being from (to) 1st order neighbors, 2nd order neighbors, etc. In contrast, when W is dense because it has only a few or no zeros such as a W based on inverse distances between each pair of state centroids, the partitioned spatial efficiency multiplier has a rather opaque interpretation. The reason is because with a W that has a cut-off (e.g., $W_{3Near} - W_{7Near}$) a unit's 1st order, 2nd order, etc. neighborhood sets do not contain the same units, whereas with a W based on the inverse distances between each pair of units in the sample a unit's 1st order, 2nd order, etc. neighborhood sets all contain the same units.

In total we use eleven normalized specifications of W. Six of these specifications are denoted W^{Row}_{Cont} and $W^{Row}_{3Near} - W^{Row}_{7Near}$, where the superscript denotes that the matrix is row-normalized. Using a row-normalized specification of W preserves the scaling of the data across space because for a particular state the SAR variable will be a weighted average of the dependent variable for the states in its neighborhood set. When an inverse distance based W is row-normalized spillovers are inversely related to the relative distances between the units. On one hand this is reasonable because distance can be viewed as a relative measure which will vary from state-to-state depending on how remote a state is from other states. As a result, the same absolute distance from a comparatively remote state will be relatively shorter than from a state in a much more accessible location. This is intuitive because, everything else being equal, agents in a comparatively remote state will be more accustomed to traveling and transporting goods further within the U.S. On the other hand it could be argued that a relative measure of distance is unreasonable because the information on absolute distances between states is lost by row-normalizing. To address this issue the five remaining specifications of W, which are denoted $W^{Eig}_{3Near} - W^{Eig}_{7Near}$, are normalized by the largest eigenvalue of W. This normalization does not change the proportional relationship between the spatial weights so spillovers are inversely related to the absolute distances between the states.

5.2 Estimated Models

Following the spatial analysis in Pfaffermayr (2009) model selection is based on the Akaike information criterion (AIC). Additionally we inform model selection using the Schwarz/Bayesian information criterion (SIC). Of the 23 random effects frontiers (NSF_R, and eleven SARF_R and eleven SDF_R models pertaining to W^{Row}_{Cont}, $W^{Row}_{3Near} - W^{Row}_{7Near}$ and $W^{Eig}_{3Near} - W^{Eig}_{7Near}$), the AIC and SIC both point to the W^{Row}_{Cont} SDF_R. Of the further 23

[19] Sparse specifications of W are commonplace because of the entirely reasonable prior view that a unit's neighborhood set is a small subset of the other units in the sample. A unit is assumed to be explicitly linked to the units in its neighborhood set and via the spatial multiplier matrix a unit is implicitly linked to other units' neighborhood sets.

common correlated effects frontiers (NSF$_{\mathrm{C}}$, and eleven SARF$_{\mathrm{C}}$ and eleven SDF$_{\mathrm{C}}$ models pertaining to W^{Row}_{Cont}, $W^{Row}_{3Near} - W^{Row}_{7Near}$ and $W^{Eig}_{3Near} - W^{Eig}_{7Near}$), the AIC and SIC both prefer the W^{Row}_{Cont} SDF$_{\mathrm{C}}$. Between the W^{Row}_{Cont} SDF$_{\mathrm{R}}$ and W^{Row}_{Cont} SDF$_{\mathrm{C}}$ the AIC and SIC both favor the W^{Row}_{Cont} SDF$_{\mathrm{C}}$. This is further supported by a likelihood ratio (LR) test of the W^{Row}_{Cont} SDF$_{\mathrm{R}}$ against the W^{Row}_{Cont} SDF$_{\mathrm{C}}$. The test rejects the null that all the coefficients on the mean variables in the W^{Row}_{Cont} SDF$_{\mathrm{C}}$ are zero at the 0.1% level and thus highlights the importance of accounting for correlation between the exogenous regressors and the unit specific effects in this application. From the fitted W^{Row}_{Cont} SDF$_{\mathrm{C}}$ in table 3 we can see that a number of these mean variables are significant ($\bar{y}_1$, $\bar{y}_2$ and $\sum_{j=1}^{N} w_{ij}\bar{y}_3$). The estimation results for the W^{Row}_{Cont} SDF$_{\mathrm{C}}$ also emphasize the importance of the local spatial variables as a number of these variables are also significant (e.g., those pertaining to Wy_1, Wp_2 and Wp_4). In table 4 we present the corresponding fitted NSF$_{\mathrm{C}}$ and W^{Row}_{Cont} SARF$_{\mathrm{C}}$ and in tables 5 and 6 we present the marginal effects from the W^{Row}_{Cont} SDF$_{\mathrm{C}}$ and W^{Row}_{Cont} SARF$_{\mathrm{C}}$.[20]

The spillover elasticities from the W^{Row}_{Cont} SARF$_{\mathrm{C}}$ and W^{Row}_{Cont} SDF$_{\mathrm{C}}$ are the indirect marginal effects which are a function of, among other things, δ (LeSage and Pace, 2009). δ itself, however, has a meaningful interpretation as it represents the degree of SAR dependence. From the W^{Row}_{Cont} SDF$_{\mathrm{C}}$ and W^{Row}_{Cont} SARF$_{\mathrm{C}}$ it is evident from tables 3 and 4 that the estimates of δ are significant at the 0.1% level and are of the order of 0.390 and 0.315, respectively, which in both cases represents substantial positive SAR dependence. At the 5% level the estimate of δ from the W^{Row}_{Cont} SDF$_{\mathrm{C}}$ is significantly larger than that from the W^{Row}_{Cont} SARF$_{\mathrm{C}}$.

From tables $4 - 6$ we can see that each of the own/direct first order time trend parameters from the NSF$_{\mathrm{C}}$, W^{Row}_{Cont} SARF$_{\mathrm{C}}$ and W^{Row}_{Cont} SDF$_{\mathrm{C}}$ are significant at the 0.1% level. As we would expect, each of these time trend parameters is negative which for the sample average state indicates cost diminution due to technical progress. From the same tables we can see that all the own/direct output and input price elasticities at the sample mean from the NSF$_{\mathrm{C}}$, W^{Row}_{Cont} SARF$_{\mathrm{C}}$ and W^{Row}_{Cont} SDF$_{\mathrm{C}}$ are positive and therefore satisfy the monotonicity property of the translog cost function. The own/direct output elasticities at the sample mean from the NSF$_{\mathrm{C}}$, W^{Row}_{Cont} SARF$_{\mathrm{C}}$ and W^{Row}_{Cont} SDF$_{\mathrm{C}}$ also suggest diseconomies of scale of the order of 0.63, 0.74 and 0.88, respectively, which suggests that economies of scale are understated when either SAR dependence is overlooked or local spatial dependence together with SAR dependence is not accounted for. Finding that the cost technology for the U.S. agricultural sector is characterized by diseconomies of scale is in line with historical evidence (e.g., Ray, 1982), which together with our findings

[20] The estimated parameters and standard errors for the time period dummies from the NSF$_{\mathrm{C}}$, W^{Row}_{Cont} SARF$_{\mathrm{C}}$ and W^{Row}_{Cont} SDF$_{\mathrm{C}}$ are available on request. A large number of the time period dummies from these models are significant thereby justifying their inclusion to capture significant departures from the non-linear time trend.

Table 3: Estimation results for the preferred common correlated effects spatial Durbin stochastic cost frontier model

	W_{Cont}^{Row} SDF_C							
	Coeff	t-stat		Coeff	t-stat		Coeff	t-stat
y_1	0.327***	3.89	t^2	8.98×10^{-5}	1.33	$W\,(y_1y_2)$	-0.146***	-5.87
y_2	0.707***	7.01	y_1t	-0.001	-1.86	$W\,(y_1y_3)$	-0.032	-1.60
y_3	-0.006	-0.09	y_2t	-0.002***	-3.73	$W\,(y_2y_3)$	0.052**	2.74
p_2	0.189***	26.28	y_3t	0.004***	6.07	$W\,(y_1p_2)$	-0.017	-0.86
p_3	0.417***	13.77	p_2t	-0.001	-0.55	$W\,(y_1p_3)$	-0.082	-1.92
p_4	0.041***	3.64	p_3t	0.013***	4.50	$W\,(y_1p_4)$	-0.027	-1.76
y_1^2	0.042***	6.10	p_4t	-5.99×10^{-5}	-0.09	$W\,(y_2p_2)$	0.092***	3.85
y_2^2	0.009	1.35	$\overline{y}_1$	0.156***	3.80	$W\,(y_2p_3)$	-0.035	-0.71
y_3^2	-0.023***	-4.19	$\overline{y}_2$	0.337***	4.93	$W\,(y_2p_4)$	0.019	1.10
p_2^2	0.020	1.52	$\overline{y}_3$	-0.090	-1.24	$W\,(y_3p_1)$	-0.065**	-2.81
p_3^2	0.235***	3.95	$\overline{p}_2$	-0.297	-1.26	$W\,(y_3p_3)$	0.096*	2.02
p_4^2	-0.018**	-3.18	$\overline{p}_3$	0.041	0.14	$W\,(y_3p_4)$	0.027	1.55
p_2p_3	-0.010	-0.25	$\overline{p}_4$	-0.117	-1.87	$W\,(y_1t)$	0.003**	2.64
p_2p_4	0.048***	4.53	Wy_1	0.590***	3.64	$W\,(y_2t)$	-0.004**	-3.18
p_3p_4	-0.098***	-3.73	Wy_2	-0.161	-0.81	$W\,(y_3t)$	0.003**	2.95
y_1y_2	-0.091***	-8.33	Wy_3	0.089	0.78	$W\,(p_2t)$	0.009***	4.15
y_1y_3	0.004	0.42	Wp_2	-0.061***	-4.33	$W\,(p_3t)$	-0.002	-0.32
y_2y_3	0.040***	4.21	Wp_3	-0.009	-0.17	$W\,(p_4t)$	2.54×10^{-4}	0.22
y_1p_2	-0.038***	-4.00	Wp_4	0.058***	3.71	$W\overline{y}_1$	0.075	0.98
y_1p_3	0.052*	2.49	Wy_1^2	0.058***	3.85	$W\overline{y}_2$	0.089	0.52
y_1p_4	0.012	1.51	Wy_2^2	0.056***	4.01	$W\overline{y}_3$	-0.320*	-2.10
y_2p_2	0.005	0.47	Wy_3^2	-0.017	-1.44	$W\overline{p}_2$	-0.742	-1.12
y_2p_3	0.047	1.72	Wp_2^2	-0.160***	-5.76	$W\overline{p}_3$	0.499	0.81
y_2p_4	0.051***	5.65	Wp_3^2	-0.151	-1.41	$W\overline{p}_4$	-0.224	-1.57
y_3p_1	0.032**	2.59	Wp_4^2	-0.048***	-4.38	*Constant*	4.423*	2.35
y_3p_3	-0.053	-1.94	$W\,(p_2p_3)$	0.189*	2.38	Wc	0.390***	16.15
y_3p_4	-0.049***	-5.36	$W\,(p_2p_4)$	-0.053**	-2.61	$\log_{10}\theta$	-2.530***	-21.79
t	-0.011***	-10.33	$W\,(p_3p_4)$	0.016	0.36			
σ_v	0.2785	σ_u	0.5147		σ_κ	0.1008	σ_η	0.0482
σ_{uv}^2	0.3425	λ_{uv}	1.8483		$\sigma_{\eta\kappa}^2$	0.0125	$\lambda_{\eta\kappa}$	0.4785
LL				3144.61				
AIC				-6039.2				
BIC				-5329.5				

Notes
SDF_C denotes the common correlated effects spatial Durbin stochastic frontier model
*, ** and *** denote statistical significance at the 5%, 1% and 0.1% levels, respectively

Table 4: Estimated non-spatial and spatial autoregressive stochastic cost frontier models with common correlated effects

	NSF$_C$					W_{Cont}^{Row} SARF$_C$				
	Coeff	t-stat		Coeff	t-stat		Coeff	t-stat		Coeff	t-stat
y_1	0.412***	4.58	y_2p_3	0.092**	3.17	y_1	0.420***	5.09	y_2p_3	0.074**	2.82
y_2	1.133***	10.41	y_2p_4	0.064***	6.17	y_2	0.879***	8.67	y_2p_4	0.044***	4.68
y_3	0.051	0.73	y_3p_1	0.020	1.39	y_3	0.015	0.23	y_3p_1	0.028*	2.22
p_2	0.206***	25.29	y_3p_3	-0.063*	-2.11	p_2	0.199***	26.90	y_3p_3	-0.059*	-2.18
p_3	0.515***	17.17	y_3p_4	-0.045***	-4.50	p_3	0.456***	16.64	y_3p_4	-0.048***	-5.30
p_4	0.030**	3.27	t	-0.011***	-15.56	p_4	0.047***	5.64	t	-0.010***	-14.82
y_1^2	0.032***	4.05	t^2	2.28×10^{-5}	0.48	y_1^2	0.033***	4.48	t^2	-2.43×10^{-5}	-0.56
y_2^2	0.002	0.25	y_1t	-0.001	-1.91	y_2^2	0.012	1.69	y_1t	-0.001	-1.55
y_3^2	-0.031***	-4.62	y_2t	-0.001	-1.85	y_3^2	-0.032***	-5.30	y_2t	-0.001	-1.21
p_2^2	0.001	0.07	y_3t	0.004***	6.07	p_2^2	0.033*	2.32	y_3t	0.003***	4.90
p_3^2	0.334***	5.37	p_2t	0.001	0.95	p_3^2	0.305***	5.40	p_2t	-4.41×10^{-4}	-0.40
p_4^2	-0.042***	-6.77	p_3t	0.016***	5.50	p_4^2	-0.030***	-5.24	p_3t	0.016***	5.93
p_2p_3	0.030	0.67	p_4t	-0.003***	-4.11	p_2p_3	-0.031	-0.75	p_4t	-0.002**	-3.23
p_2p_4	0.060***	4.91	$\overline{y}_1$	0.197***	4.71	p_2p_4	0.056***	5.04	$\overline{y}_1$	0.123	1.91
p_3p_4	-0.182***	-6.88	$\overline{y}_2$	0.247***	3.79	p_3p_4	-0.138***	-5.70	$\overline{y}_2$	0.171	1.71
y_1y_2	-0.092***	-7.22	$\overline{y}_3$	-0.158**	-2.61	y_1y_2	-0.098***	-8.45	$\overline{y}_3$	-0.015	-0.16
y_1y_3	0.021*	2.03	$\overline{p}_2$	-0.045	-0.20	y_1y_3	0.026**	2.67	$\overline{p}_2$	-0.122	-0.35
y_2y_3	0.030**	2.72	$\overline{p}_3$	-0.229	-0.88	y_2y_3	0.031**	3.06	$\overline{p}_3$	0.816*	2.03
y_1p_2	-0.039***	-3.56	$\overline{p}_4$	-0.104	-1.94	y_1p_2	-0.033***	-3.36	$\overline{p}_4$	-0.031	-0.38
y_1p_3	0.025	1.05	*Constant*	2.665**	2.68	y_1p_3	0.021	0.97	*Constant*	1.661	1.68
y_1p_4	0.008	0.91	θ	0.780***	20.81	y_1p_4	0.015*	1.97	Wc	0.315***	18.03
y_2p_2	0.007	0.55				y_2p_2	-0.002	-0.19	$\log_{10}\theta$	-3.030***	-26.05
σ_v	0.0525		σ_κ	0.1482		σ_v	0.2772		σ_κ	0.1404	
σ_u	0.0661		σ_η	0.1733		σ_u	0.5141		σ_η	0.2042	
σ_{uv}^2	0.0071		$\sigma_{\eta\kappa}^2$	0.0520		σ_{uv}^2	0.3412		$\sigma_{\eta\kappa}^2$	0.0614	
λ_{uv}	1.2588		$\lambda_{\eta\kappa}$	1.1693		λ_{uv}	1.8546		$\lambda_{\eta\kappa}$	1.4539	
LL			2756.78			LL			2912.43		
AIC			-5343.55			AIC			-5652.86		
BIC			-4860.93			BIC			-5164.56		

Notes
NSF$_C$ denotes the common correlated effects non-spatial stochastic frontier model
SARF$_C$ denotes the common correlated effects spatial autoregressive stochastic frontier model
*, ** and *** denote statistical significance at the 5%, 1% and 0.1% levels, respectively

Table 5: Marginal effects from the preferred common correlated effects spatial Durbin stochastic cost frontier model

		W^{Row}_{Cont} SDF_C					
		Marginal Eff.	t-stat			Marginal Eff.	t-stat
y_1	*Direct*	0.407***	5.10	y_1p_2	*Direct*	-0.042***	-4.33
	Indirect	1.107***	4.63		*Indirect*	-0.049	-1.55
	Total	1.514***	5.21		*Total*	-0.091**	-2.60
y_2	*Direct*	0.727***	6.31	y_1p_3	*Direct*	0.045*	2.22
	Indirect	0.174	0.61		*Indirect*	-0.099	-1.63
	Total	0.901**	2.71		*Total*	-0.054	-0.83
y_3	*Direct*	0.006	0.10	y_1p_4	*Direct*	0.009	1.24
	Indirect	0.131	0.78		*Indirect*	-0.037	-1.66
	Total	0.137	0.70		*Total*	-0.028	-1.09
p_2	*Direct*	0.190***	27.22	y_2p_2	*Direct*	0.018	1.54
	Indirect	0.018	0.85		*Indirect*	0.144***	4.20
	Total	0.208***	8.96		*Total*	0.162***	4.05
p_3	*Direct*	0.440***	15.80	y_2p_3	*Direct*	0.043	1.51
	Indirect	0.232***	3.43		*Indirect*	-0.026	-0.40
	Total	0.673***	9.71		*Total*	0.017	0.23
p_4	*Direct*	0.052***	4.81	y_2p_4	*Direct*	0.057***	6.29
	Indirect	0.117***	5.87		*Indirect*	0.060*	2.31
	Total	0.169***	9.01		*Total*	0.117***	3.95
y_1^2	*Direct*	0.051***	6.77	y_3p_1	*Direct*	0.023*	1.98
	Indirect	0.115***	5.17		*Indirect*	-0.083*	-2.14
	Total	0.166***	6.51		*Total*	-0.060	-1.34
y_2^2	*Direct*	0.015*	2.19	y_3p_3	*Direct*	-0.042	-1.78
	Indirect	0.091***	3.94		*Indirect*	0.120	1.89
	Total	0.106***	4.07		*Total*	0.078	1.06
y_3^2	*Direct*	-0.027***	-4.26	y_3p_4	*Direct*	-0.048***	-5.14
	Indirect	-0.039*	-1.96		*Indirect*	0.012	0.40
	Total	-0.066**	-2.80		*Total*	-0.036	-1.02
p_2^2	*Direct*	0.003	0.21	t	*Direct*	-0.011***	-10.23
	Indirect	-0.230***	-5.55		*Indirect*	-0.006***	-9.71
	Total	-0.227***	-4.97		*Total*	-0.018***	-10.98
p_3^2	*Direct*	0.230***	4.10	t^2	*Direct*	9.27×10^{-5}	1.42
	Indirect	-0.096	-0.69		*Indirect*	5.25×10^{-5}	1.42
	Total	0.134	0.92		*Total*	1.45×10^{-4}	1.42
p_4^2	*Direct*	-0.025***	-4.69	y_1t	*Direct*	-0.001	-1.39
	Indirect	-0.086***	-5.52		*Indirect*	0.003*	2.36
	Total	-0.111***	-6.52		*Total*	0.003	1.81
p_2p_3	*Direct*	0.010	0.24	y_2t	*Direct*	-0.003***	-4.21
	Indirect	0.283**	2.63		*Indirect*	-0.007***	-3.87
	Total	0.293*	2.39		*Total*	-0.010***	-4.78
p_2p_4	*Direct*	0.041***	3.67	y_3t	*Direct*	0.004***	6.09
	Indirect	-0.052	-1.75		*Indirect*	0.008***	3.75
	Total	-0.011	-0.30		*Total*	0.012***	5.05
p_3p_4	*Direct*	-0.096***	-4.31	p_2t	*Direct*	3.71×10^{-4}	0.30
	Indirect	-0.035	-0.61		*Indirect*	0.012***	4.58
	Total	-0.131*	-2.14		*Total*	0.013***	3.95
y_1y_2	*Direct*	-0.112***	-8.91	p_3t	*Direct*	0.013***	4.70
	Indirect	-0.276***	-6.87		*Indirect*	0.005	0.65
	Total	-0.388***	-8.41		*Total*	0.018*	2.46
y_1y_3	*Direct*	3.46×10^{-4}	0.04	p_4t	*Direct*	6.10×10^{-5}	0.08
	Indirect	-0.048	-1.72		*Indirect*	3.75×10^{-4}	0.26
	Total	-0.048	-1.45		*Total*	4.36×10^{-4}	0.26
y_2y_3	*Direct*	0.048***	4.87				
	Indirect	0.104***	3.41				
	Total	0.152***	4.54				

Notes
SDF_C denotes the common correlated effects spatial Durbin stochastic frontier model
*, **, *** denote statistical significance at the 5%, 1% and 0.1% levels, respectively

Table 6: Marginal effects from a common correlated effects spatial autoregressive stochastic cost frontier model

		W_{Cont}^{Row} SARF$_C$					
		Marginal Eff.	t-stat			Marginal Eff.	t-stat
y_1	*Direct*	0.430***	5.97	y_1p_2	*Direct*	-0.033***	-3.36
	Indirect	0.182***	5.26		*Indirect*	-0.014**	-3.23
	Total	0.612***	5.84		*Total*	-0.048***	-3.35
y_2	*Direct*	0.911***	7.94	y_1p_3	*Direct*	0.021	1.00
	Indirect	0.385***	7.95		*Indirect*	0.009	0.99
	Total	1.296***	8.24		*Total*	0.030	1.00
y_3	*Direct*	0.018	0.26	y_1p_4	*Direct*	0.016*	2.28
	Indirect	0.008	0.26		*Indirect*	0.007*	2.23
	Total	0.026	0.26		*Total*	0.022*	2.28
p_2	*Direct*	0.205***	28.00	y_2p_2	*Direct*	-3.18×10^{-4}	-0.03
	Indirect	0.087***	11.63		*Indirect*	-1.12×10^{-4}	-0.02
	Total	0.291***	22.37		*Total*	-4.30×10^{-4}	-0.02
p_3	*Direct*	0.475***	17.58	y_2p_3	*Direct*	0.074**	2.58
	Indirect	0.201***	11.94		*Indirect*	0.031*	2.49
	Total	0.676***	17.57		*Total*	0.105*	2.57
p_4	*Direct*	0.050***	5.79	y_2p_4	*Direct*	0.047***	5.04
	Indirect	0.021***	5.00		*Indirect*	0.020***	4.91
	Total	0.071***	5.61		*Total*	0.066***	5.07
y_1^2	*Direct*	0.034***	4.32	y_3p_1	*Direct*	0.027*	2.23
	Indirect	0.014***	4.15		*Indirect*	0.011*	2.20
	Total	0.048***	4.31		*Total*	0.038*	2.23
y_2^2	*Direct*	0.012	1.63	y_3p_3	*Direct*	-0.057*	-2.48
	Indirect	0.005	1.59		*Indirect*	-0.024*	-2.41
	Total	0.017	1.62		*Total*	-0.082*	-2.47
y_3^2	*Direct*	-0.033***	-5.22	y_3p_4	*Direct*	-0.051***	-5.53
	Indirect	-0.014***	-4.99		*Indirect*	-0.021***	-5.09
	Total	-0.047***	-5.23		*Total*	-0.072***	-5.47
p_2^2	*Direct*	0.033*	2.26	t	*Direct*	-0.010***	-13.92
	Indirect	0.014*	2.22		*Indirect*	-0.004***	-9.45
	Total	0.047*	2.25		*Total*	-0.014***	-13.04
p_3^2	*Direct*	0.315***	5.56	t^2	*Direct*	-3.07×10^{-5}	-0.71
	Indirect	0.133***	5.23		*Indirect*	-1.30×10^{-5}	-0.71
	Total	0.448***	5.56		*Total*	-4.37×10^{-5}	-0.71
p_4^2	*Direct*	-0.031***	-5.81	y_1t	*Direct*	-0.001	-1.62
	Indirect	-0.013***	-5.68		*Indirect*	-3.39×10^{-4}	-1.59
	Total	-0.044***	-5.88		*Total*	-0.001	-1.62
p_2p_3	*Direct*	-0.034	-0.75	y_2t	*Direct*	-0.001	-1.29
	Indirect	-0.014	-0.77		*Indirect*	-3.75×10^{-4}	-1.28
	Total	-0.048	-0.76		*Total*	-0.001	-1.29
p_2p_4	*Direct*	0.054***	4.72	y_3t	*Direct*	0.003***	4.88
	Indirect	0.023***	4.39		*Indirect*	0.001***	4.91
	Total	0.078***	4.67		*Total*	0.005***	4.96
p_3p_4	*Direct*	-0.137***	-6.34	p_2t	*Direct*	-4.05×10^{-4}	-0.31
	Indirect	-0.058***	-5.55		*Indirect*	-1.79×10^{-4}	-0.33
	Total	-0.194***	-6.20		*Total*	-0.001	-0.32
y_1y_2	*Direct*	-0.101***	-7.86	p_3t	*Direct*	0.016***	5.97
	Indirect	-0.043***	-6.80		*Indirect*	0.007***	5.40
	Total	-0.144***	-7.74		*Total*	0.023***	5.90
y_1y_3	*Direct*	0.026**	2.81	p_4t	*Direct*	-0.002**	-2.98
	Indirect	0.011**	2.73		*Indirect*	-0.001**	-2.83
	Total	0.037**	2.80		*Total*	-0.003**	-2.95
y_2y_3	*Direct*	0.033**	3.00				
	Indirect	0.014**	2.95				
	Total	0.047**	3.00				

Notes
SARF$_C$ denotes the common correlated effects spatial autoregressive stochastic frontier model
*, **, *** denote statistical significance at the 5%, 1% and 0.1% levels, respectively

suggests that operating beyond minimum efficient scale is a persistent feature of the U.S. agricultural sector. Wald tests reveal that the own/direct returns to scale from the NSF_C and W_{Cont}^{Row} SARF_C are significantly different from 1 at the 5% level or lower. Interestingly, however, for the direct returns to scale from the W_{Cont}^{Row} SDF_C we cannot reject the null of constant direct returns. The difference between the direct returns to scale Wald test results for the W_{Cont}^{Row} SARF_C and W_{Cont}^{Row} SDF_C highlights the importance of including in our preferred W_{Cont}^{Row} SDF_C specification spatial lags of the exogenous variables and the means of the spatial lags of the first order outputs and input prices.

Additionally, differences in the magnitudes of the own/direct technical progress and returns to scale from the NSF_C, W_{Cont}^{Row} SARF_C and W_{Cont}^{Row} SDF_C can lead to differences in the predictions of own/direct TFP growth. To see this consider TFP growth as calculated in Eqs. 11 and 12 in Sickles (1985). Although the NSF_C and W_{Cont}^{Row} SARF_C both suggest that the sample average rate of own/direct TFP growth is 0.7%, the W_{Cont}^{Row} SDF_C points to a much higher rate of 1.0%.

Looking at the marginal effects of the first order outputs and input prices in more detail. The NSF_C, W_{Cont}^{Row} SARF_C and W_{Cont}^{Row} SDF_C yield own/direct elasticities for farm related output, y_3, which are not significant and significant own/direct elasticities for the core outputs (livestock and products, y_1, and crops, y_2). We classify y_3 as non-core because it refers to goods and services from non-agricultural activities (e.g., processing and packaging of agricultural products) and secondary activities (e.g., machine services for hire). Of the indirect first order output and input price elasticities only the y_1, p_3 and p_4 elasticities are significant. For y_2 and p_2 the significant direct elasticity from the W_{Cont}^{Row} SDF_C dominates the corresponding insignificant indirect elasticity. Consequently, all the total first order output and input price elasticities from the W_{Cont}^{Row} SDF_C are significant with the exception of the total y_3 elasticity.

5.3 Cost Efficiency Results

In our discussion of the efficiency results we emphasize how the reduced forms of our random effects and common correlated effects spatial frontiers yield a rich set of direct, asymmetric indirect and asymmetric total efficiency estimates. These direct, asymmetric indirect and asymmetric total efficiencies are partially/entirely made up of efficiency spillovers. In contrast, own efficiencies from the non-spatial frontier and the structural random effects and common correlated effects spatial frontiers omit these spillovers. As a result, the own efficiency benchmarks from the non-spatial frontier and the structural random effects and common correlated effects spatial frontiers differ from the direct, indirect and total efficiency benchmarks from the reduced form spatial frontiers.

5.3.1 Own Net Time-Invariant, Own Net Time-Varying and Own Gross Time-Varying Cost Efficiencies

In table 7 we present the average own NIE, NVE and GVE scores for the sample from the structural form of our preferred W_{Cont}^{Row} SDF_C and also from the NSF_C and the structural W_{Cont}^{Row} SARF_C. We also report in table 7 average own NIE, NVE and GVE scores and the corresponding efficiency rankings for selected states. The states are selected on the basis of their average own GVE ranking from the structural W_{Cont}^{Row} SDF_C and comprise: the three states with the highest average own GVE ranking (1. North Dakota, 2. Rhode Island and 3. New Jersey); four states with a mid-ranking average own GVE (23. California, 24. South Dakota, 25. Kansas and 26. Minnesota); and the three states with the lowest average own GVE ranking (46. Vermont, 47. Delaware and 48. Alabama). Here we only summarize the sample average efficiencies due to space limitations. For the discussion of the average own efficiencies for individual states see the Supplementary document that accompanies the paper.

The sample average own NIE (NVE) scores in table 7 from the NSF_C, W_{Cont}^{Row} SARF_C and W_{Cont}^{Row} SDF_C are 0.87 (0.95), 0.85 (0.67) and 0.96 (0.67), respectively. This indicates that controlling for SAR dependence and, in the case of the W_{Cont}^{Row} SDF_C controlling also for local spatial dependence, leads to a substantial change in the magnitude of the gap between the sample average own NIE and the corresponding NVE. The presence of non-negligible average own NVI for the sample from the W_{Cont}^{Row} SARF_C and W_{Cont}^{Row} SDF_C is entirely reasonable. Overlooking SAR dependence in the NSF_C specification, however, leads to a small sample average own NVI score, which is counterintuitive. In addition, as the substantial rigidities in agricultural assets and the internal organization of agricultural production that we associate with non-negligible own NII are unlikely to persist for the duration of our 45-year sample, as we would expect, the W_{Cont}^{Row} SDF_C yields a small sample average NII score. From the W_{Cont}^{Row} SARF_C, however, the average NII for the sample is much more marked than from the W_{Cont}^{Row} SDF_C. The presence though of average NII for the sample from the W_{Cont}^{Row} SARF_C and W_{Cont}^{Row} SDF_C, even though in the latter case the estimate is small, provides support for using spatial frontier specifications in this application with both NVI and NII components.

To test for the presence of the error components we conduct the test in Gourieroux *et al.* (1982) of the null $\widehat{\sigma}_H^2 = 0$ against $\widehat{\sigma}_H^2 > 0$ for $H \in \{\kappa, \upsilon, \eta, u\}$. The asymptotic distribution of the test statistic is a mixture of chi-squared distributions, $\frac{1}{2}\chi_0^2 + \frac{1}{2}\chi_1^2$. For the NSF_C, W_{Cont}^{Row} SARF_C and W_{Cont}^{Row} SDF_C we reject the null at the 1% level for κ, υ and u. For the NSF_C and W_{Cont}^{Row} SARF_C we also reject the null at the 1% level for η but for the W_{Cont}^{Row} SDF_C we fail to reject the absence of η. Interestingly, this highlights the effect of the specification of the spatial frontier on the statistical evidence of the presence of η. In the case of the W_{Cont}^{Row} SDF_C it does not follow that because the absence of η has

Table 7: Average own net time-invariant, net time-varying and gross time-varying cost efficiency scores and rankings

Panel A: NSF_C						
State	Av NIE Score	Av NIE Rank	Av NVE Score	Av NVE Rank	Av GVE Score	Av GVE Rank
Alabama	0.89	22	0.95	4	0.85	20
California	0.83	37	0.95	31	0.78	36
Delaware	0.94	1	0.95	13	0.90	1
Kansas	0.81	40	0.95	27	0.77	40
Minnesota	0.79	46	0.95	19	0.75	46
New Jersey	0.94	2	0.95	5	0.89	2
North Dakota	0.83	34	0.95	32	0.79	34
Rhode Island	0.91	13	0.94	46	0.86	15
South Dakota	0.85	31	0.95	16	0.80	31
Vermont	0.92	10	0.95	8	0.87	10
Sample	**0.87**		**0.95**		**0.82**	

Panel B: W_{Cont}^{Row} $SARF_C$						
State	Av NIE Score	Av NIE Rank	Av NVE Score	Av NVE Rank	Av GVE Score	Av GVE Rank
Alabama	0.89	14	0.47	47	0.42	45
California	0.73	45	0.67	21	0.49	38
Delaware	0.93	5	0.47	48	0.44	43
Kansas	0.91	10	0.67	24	0.61	16
Minnesota	0.85	30	0.65	26	0.55	27
New Jersey	0.93	4	0.81	6	0.75	1
North Dakota	0.77	42	0.88	1	0.67	8
Rhode Island	0.85	26	0.84	2	0.72	3
South Dakota	0.85	28	0.66	25	0.56	23
Vermont	0.79	39	0.48	46	0.38	48
Sample	**0.85**		**0.67**		**0.56**	

Panel C: W_{Cont}^{Row} SDF_C						
State	Av NIE Score	Av NIE Rank	Av NVE Score	Av NVE Rank	Av GVE Score	Av GVE Rank
Alabama	0.96	27	0.47	47	0.46	48
California	0.96	26	0.67	21	0.65	23
Delaware	0.97	2	0.47	48	0.46	47
Kansas	0.96	37	0.67	24	0.64	25
Minnesota	0.96	30	0.65	26	0.63	26
New Jersey	0.97	3	0.81	5	0.79	3
North Dakota	0.96	36	0.87	1	0.84	1
Rhode Island	0.95	43	0.84	2	0.80	2
South Dakota	0.97	20	0.66	25	0.64	24
Vermont	0.97	19	0.48	46	0.46	46
Sample	**0.96**		**0.67**		**0.64**	

Notes
NSF_C denotes the common correlated effects non-spatial stochastic frontier model
$SARF_C$ denotes the common correlated effects spatial autoregressive stochastic frontier model
SDF_C denotes the common correlated effects spatial Durbin stochastic frontier model
NIE, NVE and GVE denote net time-invariant, net time-varying and gross time-varying cost efficiencies

not been rejected that $\eta = 0$. This is indicated for the W_{Cont}^{Row} SDF$_C$ by using a sample average NIE of less than 1 in the calculation of the sample average GVE.

Own GVE is revealing because it provides a complete picture of performance as it is own NVE and NIE combined. The sample average own GVE scores from the NSF$_C$, W_{Cont}^{Row} SARF$_C$ and W_{Cont}^{Row} SDF$_C$ are 0.82, 0.56 and 0.64, respectively. The difference between this score from the NSF$_C$ and the corresponding scores from the W_{Cont}^{Row} SARF$_C$ and W_{Cont}^{Row} SDF$_C$ again emphasize the importance of not overlooking spatial dependence in this application. Further differences and similarities between the own efficiencies from the W_{Cont}^{Row} SARF$_C$, W_{Cont}^{Row} SDF$_C$ and NSF$_C$ are evident from the kernel densities in figure 1.

5.3.2 Direct, Indirect and Total Cost Efficiencies

The direct NIE, NVE and GVE scores from the reduced forms of our spatial frontiers are the own NIE, NVE and GVE scores from the structural forms of the models plus efficiency feedback. Efficiency feedback is the component of a unit's direct efficiency that passes through neighboring units and partially rebounds back to the unit via the mechanics of the spatial efficiency multiplier. The indirect NIE, NVE and GVE scores from the reduced forms of our spatial frontiers are efficiency spillovers that a unit implicitly exports (imports) to (from) the other units in the sample. Summing the direct and indirect NIE, NVE and GVE scores yields total NIE, NVE and GVE estimates. Here we only summarize our direct, indirect and total NIE, NVE and GVE results from the reduced forms of the W_{Cont}^{Row} SARF$_C$ and W_{Cont}^{Row} SDF$_C$ due to space limitations. For a more detailed coverage of these results see the Supplementary document that accompanies the paper, wherein we present the average direct, indirect, and total NIE, NVE and GVE scores and the corresponding efficiency rankings for the same selected states as in table 7.

Since we find that the sample average absolute direct NIE score from the reduced form W_{Cont}^{Row} SDF$_C$ is 1.00 and from the above discussion in 5.3.1 the sample average own NIE score from the structural W_{Cont}^{Row} SDF$_C$ is 0.96, we can conclude for the sample average state that the efficiency feedback component of direct NIE from this model is 4%. Although this efficiency feedback component is not big its presence is sufficient to push the sample average state onto the own best practice frontier from the structural W_{Cont}^{Row} SDF$_C$.

The average absolute indirect NIE, NVE and GVE scores for individual states from the reduced forms of our spatial frontiers are asymmetric. This indicates that the efficiency spillovers which a state exports and imports to and from all the other states differ. The asymmetric absolute indirect NIE, NVE and GVE scores for individual states, however, by construction yield symmetric sample average absolute indirect NIE,

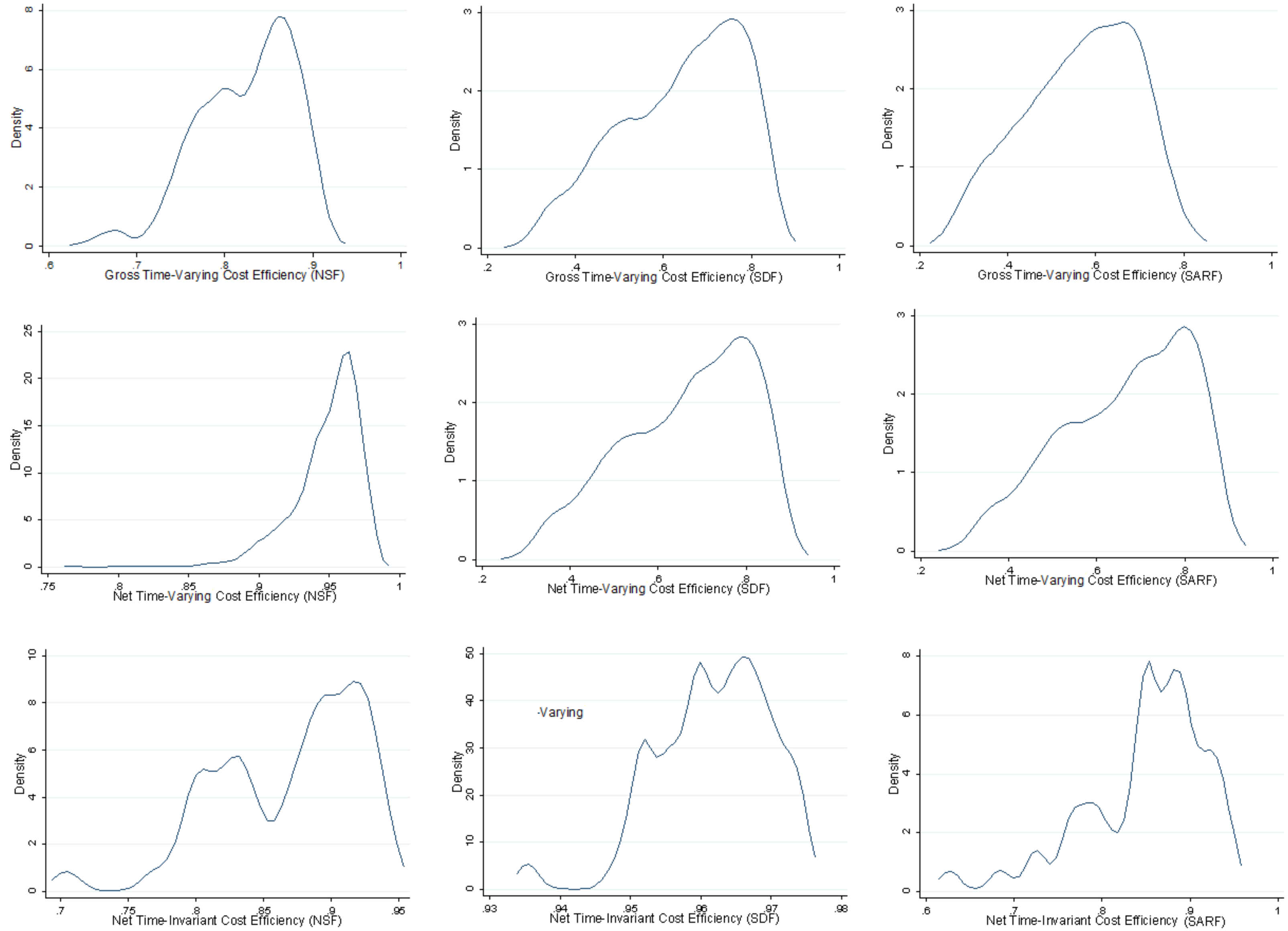

Figure 1: Kernel densities of the own net time-varying, net time-invariant and gross time-varying cost efficiencies

NVE and GVE measures. From the reduced form of the W_{Cont}^{Row} SDF_C (W_{Cont}^{Row} SARF_C) the symmetric sample average absolute indirect NIE, NVE and GVE scores are 0.57 (0.37), 0.39 (0.29) and 0.38 (0.24), respectively. For the sample average state this clearly demonstrates that these reduced form models yield non-negligible absolute indirect NIE, NVE and GVE spillovers.

It follows from the average absolute indirect NIE, NVE and GVE scores that the average absolute total NIE, NVE and GVE measures for the sample are symmetric and for individual states they are asymmetric. The absolute total NIE, NVE and GVE scores for the sample average state from the reduced form W_{Cont}^{Row} SDF_C (W_{Cont}^{Row} SARF_C) are 1.58 (1.24), 1.09 (0.97) and 1.05 (0.82), respectively. As a number of these efficiencies are greater than 1 this shows that non-negligible indirect NIE, NVE and GVE spillovers can locate a unit well beyond the corresponding own efficiency best practice frontier. It is therefore clear that because own and total efficiencies are very different efficiency metrics, the own NIE, NVE and GVE frontiers are not the appropriate benchmarks for absolute total NIE, NVE and GVE.

5.3.3 Partitioned Direct, Indirect and Total Cost Efficiencies Across Space

We now partition the absolute direct, indirect and total NIE, NVE and GVE scores into own efficiencies (pertaining to W^0) and efficiencies that relate to 1st−4th order neighbors (pertaining to $W^1 - W^4$). Since GVE provides a more complete picture of a state's economic performance we focus here on the partitioned absolute direct, indirect and total GVE scores from the reduced form W_{Cont}^{Row} SDF_C. Here we only summarize our partitioned absolute GVE results due to space constraints. For a full presentation of these results for the sample average state and selected individual states and further discussion see the Supplementary document that accompanies this paper.

Absolute direct W^0 NIE, NVE and GVE scores omit the efficiency feedback component and are therefore own efficiencies from the structural spatial frontier. Absolute direct $W^1 - W^4$ NIE, NVE and GVE scores are partitioned efficiency feedback components which have rebounded back to a state from its 1st−4th order neighbors. By construction absolute direct W^1 NIE, NVE and GVE scores are zero because efficiency feedback is a 2nd or higher order neighbor phenomenon. We find that the sample average absolute direct $W^1 - W^4$ NIE, NVE and GVE scores are small or even zero, which indicates that nearly all of the sample average unpartitioned absolute direct NIE, NVE and GVE scores are due to W^0 (i.e., own) efficiencies.

The absolute indirect $W^1 - W^4$ NIE, NVE and GVE scores measure the efficiency spillovers that a state implicitly exports (imports) to (from) its 1st−4th order neighbors. By construction absolute indirect W^0 NIE, NVE and GVE scores are zero because indirect efficiency spillovers are a 1st or higher order neighbor phenomenon. From the reduced forms of the W_{Cont}^{Row} SARF$_{\text{C}}$ and W_{Cont}^{Row} SDF$_{\text{C}}$ the picture is much the same for the partitioned absolute indirect and total NIE, NVE and GVE scores as we noted above for the corresponding unpartitioned efficiencies in the discussion in 5.3.2. This is because partitioned absolute indirect and total NIE, NVE and GVE scores for individual states are asymmetric and the corresponding sample average efficiencies are symmetric. For both the W_{Cont}^{Row} SARF$_{\text{C}}$ and W_{Cont}^{Row} SDF$_{\text{C}}$ we find that the large unpartitioned absolute indirect NIE, NVE and GVE spillovers are primarily due to substantial absolute indirect W^1 NIE, NVE and GVE spillovers. This is because the partitioned absolute indirect NIE, NVE and GVE spillovers die out across space quite quickly. To illustrate, of the sample average unpartitioned absolute indirect GVE spillover of 0.38 from the W_{Cont}^{Row} SDF$_{\text{C}}$, 0.25 is a partitioned indirect W^1 GVE spillover, whereas the indirect W^4 GVE spillover is just 0.01.

6 Concluding Remarks and Further Work

Our paper extends the emerging literature on spatial stochastic frontier methods in three respects. Our first extension is to account for various forms of unobserved heterogeneity. We do so by developing a ML estimator of a random effects SAR stochastic frontier model, which we generalize to a common correlated effects specification to relax the strong underlying assumption of the standard random effects treatment that the regressors are uncorrelated with the unit specific effects. Our second extension is to incorporate two stochastic frontier inefficiency measures into a single SAR frontier. Having computed the time-invariant and time-varying efficiency estimates, we show how they can be used to compute a composite time-varying measure of efficiency. Our third extension is to introduce the concept of a spatial efficiency multiplier, which can be used to partition a unit's direct, asymmetric indirect and asymmetric total efficiencies across space. This involves partitioning asymmetric indirect efficiency spillovers into efficiency spillovers that a unit implicitly exports (imports) to (from) other units at different points in space (i.e., efficiency spillovers to/from 1st order neighbors, 2nd order neighbors and so on and so forth up to the pre-specified higher order neighborhood set).

Using the spatial efficiency multiplier we also show that the own efficiency best practice frontier from the structural spatial frontier model is not the appropriate benchmark for the direct, asymmetric indirect and asymmetric total efficiencies from the reduced form model. This is because the direct, asymmetric indirect and asymmetric total efficiencies are partially/entirely made up of efficiency spillovers, whereas the own efficiency metric omits these spillovers.

To demonstrate various features of our random effects and common correlated effects SAR frontier estimators we first carried out a series of Monte Carlo experiments which considered the impact of different types of spatial frontier model misspecification on the finite sample statistical performance of an estimator. Second, we demonstrated our two estimators in an empirical application to a state level cost frontier for U.S. agriculture. This is a popular case in the efficiency literature and is therefore well-suited to highlighting the features of our estimators. Further interesting cases where the asymmetric efficiency spillovers that we propose are highly relevant are clusters of hi-tech firms e.g., Silicon Valley. The locations of such firms are often clustered together so they can take full advantage of knowledge and technological diffusion. For a cluster of hi-tech firms asymmetric efficiency spillovers can be used to identify which firms are net generators of efficiency spillovers and which are net recipients.

With regard to potential extensions of our analysis, despite a few recent studies on spatial frontier methods this body of literature remains underdeveloped. One important area for further work that emerges from our paper is the development of an alternative estimator for the SAR fixed effects stochastic frontier model, where distributional assumptions are made to distinguish between inefficiency and the idiosyncratic error. Our extended estimation procedure relies on the error components being independently distributed but this will not be the case for the one-way fixed effects SAR model in the first step of our estimation routine due to correlation between the fixed effects and the time-varying errors. This therefore rules out a similar extended estimation procedure for a SAR fixed effects stochastic frontier to the one we develop here.

The likely starting point for the development of an alternative estimator for the SAR fixed effects stochastic frontier model is the recent estimator that Chen *et al.* (2014) propose. Their estimator is for the fixed effects stochastic frontier in the absence of SAR dependence and is with recourse to the closed skew normal distribution. That said, in contrast to our random effects and common correlated effects SAR stochastic frontiers, which include time-varying and time-invariant inefficiencies in a single model, extending the Chen *et al.* approach as it stands to model SAR dependence would involve modeling only time-varying inefficiency.

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