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“Deliberation and Security Design in Bankruptcy”
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Deliberation and Security Design in Bankruptcy\textsuperscript{1}

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Abstract

We consider negotiations among the claimants of a bankrupt firm in which claimants have private information about various operational restructuring alternatives, and can communicate prior to a proposal. Our setup differs from typical bargaining games with incomplete information in two ways. First, the proposals can be made using securities. Second, the negotiations are over two interdependent issues: what to do with the firm and who gets what. In line with Chapter 11 bankruptcy proceedings we first analyze the case in which both issues are negotiated simultaneously. We show that simultaneous negotiation leads to efficient operational restructuring. Moreover, any efficient equilibrium requires that the original senior claimants receive senior securities of the reorganized firm. Next, we analyze the cases in which the two issues are negotiated sequentially. If the first issue is what to do with the firm, then efficient operational restructuring is not possible. In contrast, if the first issue is who gets what, then sequential negotiation is efficient. In comparison to simultaneous negotiation, efficient sequential negotiation may result in junior claimant capturing a larger surplus.
1 Introduction

An ex-post efficient bankruptcy law should facilitate the reorganization of viable firms while liquidating the others. When a firm in the U.S. files for Chapter 11 bankruptcy, negotiations take place among the claimants of the firm to decide what to do with the firm (operational restructuring), and how to split the value (financial restructuring). However, it is often unclear whether the firm is viable or not and even when it is viable, it is not immediate how to best reorganize the firm. Although each claimant may have some information with respect to the relative desirability of various operational restructuring alternatives, it is not expected that a single claimant or a third party has the full information that would enable him to make the optimal decision. The U.S. bankruptcy law also encourages the claimants to share their information through direct communication prior to these negotiations.\footnote{Bankruptcy Rule 2003(a) provides that in a Chapter 11 reorganization case, the United States trustee must call a meeting of creditors to be held no fewer than 20 and no more than 40 days after the order of relief.} Despite the availability of the means for information sharing, conflicts of interests may prevent the claimants to reveal their information truthfully and agree on the efficient operational restructuring. In particular, it is well understood by both the practitioners and the academic literature that junior claimants may have an incentive to reorganize a non-viable firm since they don’t face the downside risk; whereas senior creditors are inclined towards liquidation as they have limited upside potential.

In an influential paper Aghion, Hart and Moore (1992) argue that “there are serious theoretical and practical problems with Chapter 11” and suggest an alternative restructuring mechanism. The most important component of their proposal is the separation of the two decisions so that operational restructuring is decided by the claimants while the financial restructuring is decided by the bankruptcy court.\footnote{The main version of their proposal uses a financial restructuring that is based on Bebchuck (1988).} Specifically, the the-
oretical problems they identify are that (i) “bargaining may break down when agents negotiate over a given pie (e.g. if there is asymmetric information among the agents)”; and that (ii) “matters are merely made worse by having a further conflict of interest over which pie should be chosen.” The first concern is in line with the existing literature on bargaining with asymmetric information. This literature, however, has not considered bargaining using securities so far. In contrast, proposals in Chapter 11 always involve securities in addition to cash. In this paper, we analyze the role of security design in resolving inefficiencies associated with bargaining under asymmetric information. We also address the second concern mentioned above by analyzing whether simultaneous negotiation on two interdependent issues in and of itself causes inefficiency. In addition, we characterize the properties of efficient equilibrium outcomes and evaluate the impact of policy changes on efficiency, operational decisions and recoveries of the claimants.

The model we analyze is a multiple issue incomplete information bargaining game with interdependent valuations. There are two claimants of a bankrupt firm each of whom observes a noisy signal that is informative about the firm value under different operational restructuring alternatives. Claimants can communicate their private information through cheap talk that takes place prior to proposal stage. If an agreement cannot be reached, then the firm is liquidated.

Our key modeling innovations in bargaining theory are twofold. First, proposals can

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3 See, for example, Meirowitz (2007). See Kennan and Wilson (1993) for a review of the earlier literature.

4 Despite the large theoretical literature on bargaining, there are few papers that study bargaining with interdependent values. Exceptions are Evans (1989), Vincent (1989), Bond and Eraslan (2006), Deneckre and Liang (2006).

5 Again, despite the large literature on bargaining theory, there are few papers that incorporate communication in bargaining. Exceptions are Farrell and Gibbons (1989), Crawford (1990), Chakraborty and Harbaugh (2003) and Meirowitz (2007).
involves securities. This is motivated by the fact that in real life bankruptcy negotiations proposals involve securities such as debt, equity, warrants in addition to cash payments. Second, the negotiations are over two interdependent issues: what to do with the firm and who gets what. To provide an analog to the paradigm of the “divide-the-dollar” game in single-issue environments, we may think of our environment as a “divide-a-currency” game where the dollar value of the surplus to be allocated is random (due to uncertainty of the exchange rate) with a distribution that depends on the currency alternative chosen. In this game agents bargain not only over how to divide a currency, but also which currency to divide to begin with. For this environment we are interested in answering the following questions:

- Does security design make a difference?
- Does the sequencing of the issues to be negotiated matter?

Despite having important implications for bankruptcy policy, there is no research on bargaining over multiple interdependent issues or bargaining that involves securities.\(^6\) We aim to fill this gap in the context of a stylized model of bargaining that captures the key elements of the negotiations in Chapter 11 bankruptcy.

In our model, there are three ways an inefficiency can arise if the conflicts of interests among claimants are significant. First, one or both of the claimants might have an incentive to misrepresent his private information in order to manipulate the decisions of the other. Second, the optimal operational restructuring alternative may not be proposed. Third, even when the optimal restructuring alternative is proposed, it may be voted down. All these sources of inefficiency may be present in both simultaneous and sequential negotiations.

\(^6\)Existing studies on multi-issue bargaining focus on the case where each issue involves a division of a different dollar, and the interaction between the issues are imposed exogenously through utility functions of the players. See, for example, Fershtman (1990, 2000).
We first analyze the benchmark case that corresponds to the arrangement in Chapter 11 in that the two issues are negotiated simultaneously. Our analysis yields the following results. If financial restructuring is not allowed, then the claimants cannot credibly reveal their information. For example, senior claimants will always report messages that are biased towards liquidation because they do not gain from the upside potential but they bear the downside risk. The lack of meaningful communication leads to inefficient liquidation or operational restructuring decisions. Likewise, absent financial restructuring, there is no equilibrium in which operational restructuring is efficient. Moreover, security design plays a crucial role. In particular, all efficient equilibria require issuance of multiple securities and allocation of all of the newly issued senior security to senior claimants. This completely eliminates the claimants’ incentive to misrepresent information and to veto efficient operational restructuring. Such a financial restructuring is also consistent with the empirical evidence (see, for example, Franks and Torous (1989)).

Given the criticism of Chapter 11 mentioned above, a natural question to ask is to what extent the sequencing of the issues matters in obtaining efficient outcomes. To answer this question, we next analyze the effect of a policy change which involves the two issues being negotiated sequentially. As before, efficient equilibrium does not exist if either communication or financial restructuring is not allowed. However, allowing both communication and security design is not sufficient for efficiency. In particular, the order in which the issues are negotiated matters. If the first issue involves operational restructuring decision, then there is never an efficient equilibrium. This is because, once the claimants agree on an operational restructuring alternative, there is no incentive to transfer wealth through financial restructuring. Anticipating the lack of financial restructuring, the claimants do not have the incentive to share information in the first stage. But then, it is not possible to have an efficient equilibrium. In contrast, if the first issue involves the financial restructuring decision, then an efficient equi-
librium exists. By choosing a financial restructuring that maximizes each claimant’s payoffs when the optimal operational restructuring is implemented, it is possible to induce information sharing. Our results imply that simultaneous negotiation on two interdependent issues in and of itself does not cause \textit{ex post} inefficiency. In contrast, sequential negotiation of the two issues may result in \textit{ex post} inefficiency.\footnote{Note that our analysis exclusively focuses on \textit{ex post} efficiency. Policy proposals by Bebchuck (1988) and Aghion, Hart and Moore (1992) are mainly aimed at attaining \textit{ex ante} efficiency by designing a financial restructuring in a way that preserves the \textit{ex ante} entitlements against the firm. Cornelli and Felli (1997) provide a framework for distinguishing between ex-ante and ex-post efficiency.} In addition to its efficiency implications, the order of negotiations has distributional consequences as well.

Our result on efficiency also makes a contribution to the mechanism design literature on bilateral bargaining. In their seminal work, Myerson and Satterthwaite (1983) show that in a bilateral bargaining problem with independent private values, there exists no \textit{ex post} efficient mechanism without outside subsidies. Cramton, Bulow, Klemperer (1987) generalize this bilateral bargaining problem to the case where there are many agents, each of whom is endowed with a fraction of an asset that may be traded among them.\footnote{When one agent owns the 100\% of the good, the problem reduces to the bilateral bargaining case of Myerson and Satterthwaite (1983).} They show that it is possible to achieve \textit{ex post} efficient trade provided that no agent owns too large a share. Our setup differs in that the claimants have interdependent values. In this case, it is much harder to achieve efficient trade since the information revealed \textit{ex post} is always bad news due to winner’s or loser’s curse reminiscent of the no-trade theorems of Grossman and Stiglitz (1980) and Milgrom and Stokey (1982) in pure common values case. Fieseler, Kittsteiner and Moldovanu (2003) show that even if initial shares are equal, it is not always possible to dissolve a partnership efficiently when there is an arbitrarily small common values in valuations. In our model, first best is an equilibrium outcome despite the presence
of interdependent payoffs, inherent conflicts of interests, and the absence of commit-
ment. This is because it is possible to propose payments in securities which enables
the claimants to credibly reveal their information in the first place.

This paper also makes a contribution to the theoretical literature on bargaining and
voting in bankruptcy. So far theoretical studies mostly focused on bargaining within
complete information framework (see, for example, Baird and Picker (1991), Bebchuck
and Chang (1992) and Eraslan (2006)). An exception is Kordana and Posner (1999) but
they do not look at the information aggregation issues. The papers within bankruptcy
literature that are most closely related this work are, Bond and Eraslan (2006), which
analyzes the optimal voting arrangements among claimants when the proposals are
given endogenously; and Maug and Yılmaz (2002), which shows that two class voting
may be optimal when there are conflicts of interest among claimants (but takes the
proposals to be exogenous).

The rest of the paper is organized as follows. In the next section, we describe our
model. In Section 3 we present our analysis: in Section 3.1 we analyze the case where
operational restructuring and financial restructuring is decided simultaneously, and in
Section 3.2 we analyze the sequential cases. Section 4 discusses potential extensions.
Section 5 concludes. Appendix provides the omitted proofs.

2 The Model

We consider a firm that is in Chapter 11 bankruptcy. There are two claimants of the
firm. The creditor, denoted by $c$, owns a debt contract against the firm with face value
$F > 0$, and the equityholder, denoted by $e$, is the residual claimant. For expositional
clarity, the creditor is male and the equityholder is female. Both of the claimants
are risk-neutral and maximize the expected value of their payoffs. The firm value is
endogenous and depends on the “business plan” of the firm chosen by the claimants.
Consequently, the claimants need to decide both which business plan to implement (operational restructuring) and how to split the value (financial restructuring).

There are a finite number of possible business plans. The set of business plans is denoted by \( \{0, 1, \ldots, J\} \). In addition to depending on the business plan deterministically, the value of the firm also depends on the state of the world randomly. Let \( \tilde{R}^j \) denote the random firm value when business plan \( j \) is chosen. There are two equally likely states of the world, \( G \) and \( B \) and \( \tilde{R}^j \) takes the value \( R^j_\sigma \) when the state \( \sigma \) is realized. We refer to plan 0 as the liquidation plan and denote the liquidation value of firm in state \( \sigma \) by \( L_\sigma \), and so \( L_\sigma = R^0_\sigma \).

If an agreement cannot be reached and the firm is liquidated then proceeds are distributed according to the absolute priority rule. That is, the creditor is paid first up to the amount he is owed and the rest of the proceeds are paid to the equityholder. Since the liquidation value is random, the liquidation payoffs are also random. We let \( \tilde{L} = \tilde{R}^0 \) denote the random liquidation value and \( \tilde{\ell} \) denote the random payoff to the creditor in the event of liquidation. If the original claims are in place when liquidation takes place, then we have \( \tilde{\ell} = \min\{\tilde{L}, F\} \). Thus, the expected payoff of the creditor and the equityholder in the event of disagreement are given by \( E[\tilde{\ell}] \) and \( E[\tilde{L} - \tilde{\ell}] \) respectively.

We assume that both the reorganization and liquidation values are higher in the good state, i.e. \( R^j_G > R^j_B \) for all \( j \). If there exists plans \( j \) and \( j' \) such that \( R^j_G \leq R^{j'}_G \) and \( R^j_B \leq R^{j'}_B \), it is never efficient to implement plan \( j \) since plan \( j' \) results in a higher firm value regardless of the state of the world.\(^9\) We rule out such trivial cases and assume without loss of generality that the plans are enumerated in the order of increasing volatility. Therefore, we have \( R^1_G > R^{J-1}_G > \ldots > R^1_G > R^0_G = L_G > L_B = R^0_B > R^1_B > \ldots > R^{J-1}_B > R^J_B \). Furthermore, in order to rule out the case in which a plan

\(^9\)Specifically, we will focus on the most efficient equilibrium and such restructuring plans will never be part of such equilibria.
is never efficient to implement, we assume that for all \( j \in \{0, 1, \ldots, J\} \) there exists \( \Pr(G) \in [0, 1] \) such that
\[
\Pr(G) R^j_G + [1 - \Pr(G)] R^j_B > \Pr(G) R'^j_G + [1 - \Pr(G)] R'^j_B
\]
for all \( j' \neq j \).

At the beginning of the game, each claimant receives a private signal about the state of the world. Private signals are independently drawn from a common state-dependent distribution. The support of claimant \( i \)'s signal is denoted by \( S_i = [0, 1] \). Let \( \phi_i(s_i|\sigma) \) denote the density function for signal \( s_i \) conditional on the true state of the world being \( \sigma \). We assume that \( \phi_i(s_i|\sigma) \) is continuous and satisfies monotone likelihood ratio property (MLRP) so that \( \frac{\phi_i(s_i|G)}{\phi_i(s_i|B)} \) is strictly increasing in \( s_i \). In other words, higher signals are more likely conditional on the good state. A direct implication of these assumptions is that \( \Pr(G|s_c, s_e) \) is continuous and strictly increasing in \( s_c \) and \( s_e \). In addition, we assume that \( \frac{\phi_i(0|G)}{\phi_i(0|B)} = 0 \) and \( \frac{\phi_i(1|G)}{\phi_i(1|B)} = \infty \), i.e. extreme signals reveal the state of the world perfectly. This implies that with sufficiently low signals liquidation maximizes the expected value of the firm conditional on the signals, and with sufficiently high signals, another restructuring plan is optimal. Thus, our assumption that the extreme signals reveal the state of the world perfectly makes the problem non-trivial in the sense that the aggregation of private information matters in terms of maximizing the expected value of the firm.\(^{10}\)

An immediate implication of MLRP is that conditional on a pair of signals if plan \( j' \) yields a higher expected value than a less risky plan \( j \), then when the claimants receive more optimistic signals, it is still the case that the expected firm value under plan \( j' \) is higher than that of under plan \( j \). Consequently, as either \( s_c \) or \( s_e \) increases, more volatile business plans become more and more attractive in terms having a higher expected firm value.

**Lemma 1** If \( E[\tilde{R}^j|s_c, s_e] > E[\tilde{R}^j|s_c, s_e] \) for some \( j', j \) with \( j' > j \) and signal pair \( s_c, s_e \)

\(^{10}\)Note that MLRP alone does not imply that information aggregation is necessary for efficiency.
then $E[\tilde{R}'|s'_c, s'_e] > E[\tilde{R}|s'_c, s'_e]$ for all $s'_c \geq s_c$ and $s'_e \geq s_e$.

An important component of our analysis is that financial restructuring allows issuance of new securities in return for the cancelation of the old claims. In other words, unlike the standard bargaining models where proposals are restricted to the division of a cake using cash payments, here the proposals may specify the division of many cakes (one for each possible realization of the firm value) using payments in securities. Specifically, we assume that the set of feasible proposals is given by $\mathcal{P} = \{ h : R \rightarrow R | 0 \leq h(R) \leq R, \forall R \}$ with functions in $\mathcal{P}$ specifying the feasible securities that can be offered to the creditor. Thus the creditor’s payoff is a function of the realized firm value and the equityholder receives the difference between the firm value and the payoff to the creditor. The restrictions we impose on the set of feasible proposals, i.e. $0 \leq h(R)$ and $h(R) \leq R$, follow from the limited liability constraints for the creditor and the equityholder respectively. In all other aspects, they are general enough and includes various standard securities that are used in practice such as cash in the amount $C$: $h(R) = C$, debt with face value $D$: $h(R) = \min\{D, R\}$; and fraction $\alpha$ of the equity: $h(R) = \alpha R$ among other standard securities.

As mentioned earlier, we are interested in comparing the outcomes when operational restructuring and financial restructuring are decided simultaneously, when operational restructuring is decided first, and when financial restructuring is decided first. Consequently, in our analysis we need to look at three different extensive forms. First consider the simultaneous game in which the operational and financial restructuring are decided at the same time. The extensive form of this game is as follows. After the signals are realized the creditor makes an announcement $\mu : S_c \rightarrow \mathcal{M}_c$. Without loss of generality, we take $\mathcal{M}_c$ to be equal to $S_c$. Having seen the creditor’s message, the equityholder proposes a business plan $j : \mathcal{M}_c \times S_e \rightarrow \{0, \ldots, J\}$ and a financial restructuring $\pi : \mathcal{M}_c \times S_e \rightarrow \mathcal{P}$. Next both claimants simultaneously vote on the proposal
bundle. If both claimants vote to accept, then both the business plan and the financial restructuring are implemented. Otherwise the firm is liquidated and the proceeds are distributed according to the absolute priority rule under the original claims, i.e. the creditor is paid the minimum of the proceeds and his claim $F$, and the equityholder received the rest, if any.

Next, consider the sequential game in which operational restructuring is decided first, and the financial restructuring is decided second. The extensive form of this game is as follows. First, the creditor sends his message $\mu : S_c \rightarrow \mathcal{M}_c$. Then, the equityholder proposes a business plan $j : \mathcal{M}_c \times S_e \rightarrow \{0, \ldots, J\}$. Both claimants simultaneously vote on the business plan. If it is accepted by both claimants, then it is implemented after the financial restructuring stage. Otherwise, the firm is liquidated after the financial restructuring stage.\footnote{This is inessential, it can be liquidated immediately.} Once the votes $v_i : \mathcal{M}_c \times S_i \times \{0, \ldots, J\} \rightarrow \{a, r\}$ are observed, the equityholder proposes a financial restructuring $\pi : \mathcal{M}_c \times S_e \times \{0, \ldots, J\} \times \{a, r\} \times \{a, r\} \rightarrow \mathcal{P}$. A second vote is taken, this time on the financial restructuring. If both claimants vote to accept the financial restructuring, then it is implemented. Otherwise, the old claims stay in place.

Finally, we consider the sequential game in which financial restructuring is decided first, and the operational restructuring is decided second. The extensive form of this game is as follows. First, the creditor sends his message $\mu : S_c \rightarrow \mathcal{M}_c$. Then, the equityholder proposes a financial restructuring $\pi : \mathcal{M}_c \times S_e \rightarrow \mathcal{P}$. Both claimants simultaneously vote on the financial restructuring. If it is accepted by both claimants, then it is implemented. Otherwise, the old claims stay in place. Once the votes $v_i : \mathcal{M}_c \times S_i \times \mathcal{P} \rightarrow \{a, r\}$ are observed, the equityholder proposes a business plan $j : \mathcal{M}_c \times S_e \times \mathcal{P} \times \{a, r\} \times \{a, r\} \rightarrow \{0, \ldots, J\}$. A second vote is taken, this time on the business plan. If both claimants vote for the business plan, then it is implemented.
Otherwise, the firm is liquidated.

We restrict attention to pure strategy profiles which consists of (i) a message strategy for the creditor, (ii) one or two proposal strategies for the equityholder depending on whether the issues are negotiated simultaneously or sequentially, and (iii) voting strategies for both claimants following each proposal. The equilibrium concept we use is Perfect Bayesian Equilibrium which consists of a strategy profile and a set of beliefs for both claimants which are obtained by Bayesian updating whenever possible, and the strategies are sequentially rational. As it is common for games of incomplete information there will be multiple equilibria and we will focus on efficient equilibrium. Consequently, our benchmark will be the first best. We say that an equilibrium is efficient (first best) if expected firm value is maximized for all possible signal pairs. Let \( j^*(s_c, s_e) \) denote the optimal business plan when the signal pair is \((s_c, s_e)\), that is, \( j^*(s_c, s_e) \) satisfies 

\[
E[\tilde{R}_j^*(s_c, s_e)|s_c, s_e] \geq E[\tilde{R}_j|s_c, s_e] \text{ for all } j, \text{ i.e.,}
\]

\[
\text{Pr}(G|s_c, s_e)R_{Gj}^*(s_c, s_e) + \text{Pr}(B|s_c, s_e)R_{Bj}^*(s_c, s_e) \geq \text{Pr}(G|s_c, s_e)R_{Gj} + \text{Pr}(B|s_c, s_e)R_{Bj}, \quad (1)
\]

for all \( j \in \{0, 1, \ldots, J\} \). Thus, an equilibrium is efficient if and only if \( j^*(s_c, s_e) \) is the business plan agreed upon in equilibrium for all \( s_c, s_e \).

## 3 Analysis

We start by establishing results that are applicable to all the extensive forms discussed. Given an equilibrium of any extensive form described above, let \( j(s_c, s_e) \) and \( \hat{\pi}(s_c, s_e) \) denote the business plan and financial restructuring agreed upon in equilibrium when the signal pair is \( s_c, s_e \). First, we establish that in order to have an efficient equilibrium, the creditor must be able to communicate his information and he should do so truthfully:
Lemma 2 In every efficient equilibrium the creditor reports his signal truthfully for all \( s_c > 0 \): If \( \hat{j}(m_c, s_e) = j^*(s_c, s_e) \) for all \( (s_c, s_e) \) then \( \mu(s_c) = s_c \) for all \( s_c > 0 \).

To see the intuition, notice that for any two distinct signals \( s_c, s'_c \) of the creditor, there exists a signal \( s_e \) such that when the equityholder’s signal is \( s_e \), the optimal restructuring plan is different under \( s_c \) and \( s'_c \), i.e., \( j^*(s_c, s_e) \neq j^*(s'_c, s_e) \). Therefore, for the equityholder to be able to propose the optimal restructuring plan, the creditor has to communicate his information to the equityholder. Since the creditor has only finite number of actions available to him without communication, but his signals can take a continuum of values, it is immediate that efficiency requires communication.

Corollary 1 In both simultaneous and sequential negotiations, communication is necessary for efficiency.

What is not immediate is whether or not communication itself is sufficient for existence of an efficient equilibrium. Recall that in a standard mechanism design approach, the mechanism designer commits to an outcome for any given vector of messages. Even then, efficiency is not always attained for the problem we study. As such it is natural that communication is not sufficient for existence of an efficient equilibrium when one restricts attention to a particular bargaining game. As we will see below, security design coupled with communication may, however, be sufficient for efficiency depending on the extensive form of the bargaining game.

As we have seen, the creditor must report his signal truthfully in an efficient equilibrium. Therefore, his equilibrium payoff function \( \hat{\pi}(s_c, s_e) \) must satisfy the following condition:

**Lemma 3** Every efficient equilibrium satisfies

\[
E[\hat{\pi}(s_c, s_e)(\tilde{R}j^*(s_c, s_e))|s_c] \geq E[\hat{\pi}(s'_c, s_e)(\tilde{R}j^*(s'_c, s_e))|s_c] \text{ for all } s_c. \tag{2}
\]

\(^{12}\)See for example, Fieseler, Kittsteiner and Moldovanu (2000).
We next analyze the equityholder’s decision. Efficient operational restructuring clearly requires that the equityholder has an incentive to propose and vote for the optimal business plan when \( j^* > 0 \). Similarly, she must not have an incentive to propose and vote for a business plan \( j > 0 \) when it is optimal to liquidate. The following lemma identifies a set of necessary conditions for these two to hold.

**Lemma 4** If \( j^*(s_c, s_e) \neq j^*(s_c', s'_e) = 0 \) for some \( s_c, s_e \) and \( s'_e \), and the equilibrium is efficient, then it must be the case that

\[
E[\tilde{R}^*_j(s_c, s_e) - \tilde{\pi}(s_c, s_e)(\tilde{R}^*_j(s_c, s_e))|s_c, s_e] \geq E[\tilde{L} - \tilde{\ell}|s_c, s_e] \tag{3}
\]

and

\[
E[\tilde{L} - \tilde{\ell}|s_c, s'_e] \geq E[\tilde{R}^*_j(s_c, s_e) - \tilde{\pi}(s_c, s_e)(\tilde{R}^*_j(s_c, s_e))|s_c, s'_e]. \tag{4}
\]

Note that although these two conditions are necessary, they are not sufficient for the equityholder to propose the optimal business plan. In particular, the conditions as to why the equityholder prefers \( j^* > 0 \) to another \( j > 0 \) is not stated above. When we characterize an efficient equilibrium, these additional conditions will need to be satisfied as well.

Next, we establish the necessity of financial restructuring in obtaining the most efficient outcome in equilibrium. In the absence of financial restructuring, the simultaneous and the sequential games are equivalent. Therefore, the following proposition holds for both.

**Proposition 1** Without financial restructuring, operational restructuring is efficient if and only if \( F \geq R^I_G \) or \( F \leq R^I_B \).

If \( F \) is larger than the total firm value independent of the state and the operational restructuring, i.e., \( F \geq R^I_G \), then essentially the creditor owns the entire firm. In contrast, if firm value is always larger than \( F \), i.e., \( F \leq R^I_B \), then the creditor can be
paid in full in every circumstance. In both of these cases, there is essentially no conflict of interest. Therefore, it is immediate that efficient operational restructuring can take place for very large or very low debt levels. The striking part of the above result is that for any intermediate level of debt, i.e., \( F \in (R_B^l, R_G^l) \), efficiency is not possible in the absence of financial restructuring. If \( L_G > F \) then the creditor is paid in full in the good state independent of the operational restructuring. Therefore, he has no incentive to accept a volatile reorganization and pushes for liquidation. Similarly, if \( F > L_B \) then the equityholder is never paid anything in bad state. Therefore, she prefers the most volatile operational restructuring. Consequently, the incentive compatibility conditions for an efficient equilibrium are most difficult to satisfy when \( L_G > F > L_B \). In what follows, in order to make our efficiency results more striking we will assume this is the case.

### 3.1 Simultaneous Game

In the simultaneous game, if liquidation is proposed, then it takes place regardless of how the claimants vote. However, if the creditor is offered a liquidation with payoff less than what he would have received under the existing financial structure, he will not approve the proposal and force liquidation under the original financial structure. Similarly, the equityholder will not propose liquidation together with a financial restructuring that decreases her payoff. Therefore, regardless of the offer, in equilibrium we have \( \ell_G = F \) and \( \ell_B = L_B \). Consequently, in order to minimize notation, without loss of generality, we assume that: (i) whenever liquidation is proposed, it allocates the claimants exactly their original liquidation payoffs; (ii) whenever liquidation is proposed, the claimants vote for it, (iii) whenever the equityholder is indifferent between proposing something else followed by voting against it and proposing liquidation, she does the latter. The equilibrium payoffs and outcomes do not depend on whether
these assumptions are satisfied or not because in either case the claimants receive their original liquidation payoffs. These assumptions effectively allow us to substitute the notation for the equilibrium proposals in place of the disagreement payoffs.

So far we have used $\pi$ to denote the creditor’s payoff function in the event of financial restructuring. However, in the simultaneous game financial restructuring is specified along with a specific business plan (operational restructuring). Consequently, $R$ and thus, $\pi(\cdot)(R)$ can take only two values. In this case, two securities, debt and equity, can fully specify the financial restructuring. Let $f_i(s_c, s_e)$ and $\alpha_i(s_c, s_e)$ denote the claimant $i$’s face value of debt and equity share when signal pair is $(s_c, s_e)$.\(^{13}\)

Our next result has two equally important components. First, in a simultaneous negotiation game an efficient equilibrium always exists. Second, in any efficient equilibrium all of the new debt is allocated to the creditor.

**Proposition 2** In equilibrium, simultaneous negotiation leads to efficient operational restructuring. In every efficient equilibrium $f_c(s_c, s_e) \in [R_B^j(s_c, s_e), R_G^j(s_c, s_e) - L_G + F]$ and $f_e(s_c, s_e) = 0$ for all $(s_c, s_e)$, i.e., the financial restructuring allocates all the debt to the creditor.

From Proposition 1, we know that if $F \in (R_G^L, R_B^L)$, then equilibrium is always inefficient absent financial restructuring. Therefore, existence of an efficient equilibrium in the presence of financial restructuring highlights the role of security design in conflict resolution. In particular, only the financial restructurings that preserve the relative seniority facilitates efficiency. To see the intuition, suppose that contrary to our result, an efficient restructuring can assign a fraction of the senior security to the equityholder when a (non-liquidation) restructuring is optimal. More specifically, consider a non-liquidation proposal that offers the equityholder a positive fraction of the

\(^{13}\)Note that since the creditor must report his signal truthfully in equilibrium we replaced his report with his signal for clarity.
senior security following creditor’s truthful message of \( s_c \). Now consider the decision problem of the equityholder with \( s_e = 0 \). Note that any efficient equilibrium requires the liquidation to take place. Furthermore, whenever liquidation takes place under simultaneous negotiations, any financial restructuring must preserve the original claims. Therefore, under liquidation the equityholder gets a non-zero payoff only in good state which has probability zero given \( s_e = 0 \). Consequently, she would be better off deviating and proposing a business plan together with a financial restructuring in which she receives some of the senior security. This contradicts efficiency since liquidation does not take place even though it is optimal.

Our result on efficient financial restructurings has direct, testable implications on financial structure in firms exiting Chapter 11 proceedings. In particular, our analysis suggest that efficient operational restructurings are accompanied by financial restructurings in which the relative priority is preserved. That is senior securities must be allocated to senior claimants first, and junior claimants should receive their payments mostly in terms of junior securities. Although there is no study that has directly documented this implication, results presented in Franks and Torous (1989) is highly suggestive of preserving initial seniority in successful reorganizations. For example, of the payment to the banks, 49.70% is in the form of senior debt. In contrast, senior debt constitutes 0.86% of the payments to the trade creditors. Likewise, although preferred stockholders receive 29.94% of their recoveries in the form of warrants, secured debtholders never receive warrants.

Note that efficient financial restructuring is not unique in the sense that the creditor’s new debt has a face value that ranges from \( R_B^{s_c, s_e} \) to \( R_G^{s_c, s_e} - L_G + F \). Our next result is a direct implication of Proposition 2 showing what happens if we restrict total face value in a reorganized firm to the lowest possible level.

**Corollary 2** In equilibrium, a financial restructuring that minimizes \( f_c(s_c, s_e) + f_e(s_c, s_e) \)
for all \((s_c, s_e)\) achieves efficiency if and only if \(f_c(s_c, s_e) = R_B^*(s_c, s_e)\) and \(f_e(s_c, s_e) = 0\) for all \((s_c, s_e)\), i.e., the total new debt is lower for more volatile reorganizations.

The above corollary implies that firms that emerge from Chapter 11 proceedings with riskier business plans should have lower fraction of debt. Of course, many other models would generate such a prediction. Recall, however, that what we refer to as “debt” is the most senior security in the capital structure. More generally, our model predicts that firms with riskier business plans should have lower fractions of senior securities. Consequently, for an all equity firm that has common stock and warrants in its capital structure, we would expect to see larger fraction of warrants if the post-bankruptcy operations of the firm is riskier. Furthermore, proposing the issuance of a smaller amount of newly issued debt and thus compensating the senior claimants through a larger fraction of newly issued equity is a signal of good news and should be correlated with positive announcement effects in traded security prices. Senior creditors accept a smaller amount of newly issued debt in exchange for a larger fraction of upside potential only when the information received by the claimants point towards a larger likelihood of good state. Consistent with our prediction, the existing empirical literature has illustrated that market reacts positively to restructuring news in which senior claimants get most of their payments in the form of junior securities (see, e.g., Brown, James and Mooradian (1993)).

Another direct implication of Proposition 2 is on the division of the surplus. As we have seen, \(\hat{\pi}(s_c, s_e)(R_B^*) = R_B^{*(s_c, s_e)}\) so that the creditor receives the entire reorganization value in the bad state. This implies that in a reorganized firm the equityholder receives a positive payoff only in the good state. On the other hand, she receives \(L_G - F\) in good state and 0 in bad state under liquidation. She must have at least the same expected payoff for her to propose and accept a reorganization plan. Therefore, her payoff in good state must be at least \(L_G - F\). However, if she receives more in good
state under reorganization, then she will have an incentive to prevent liquidation even when liquidation is optimal. Consequently:

**Corollary 3** In any equilibrium of the simultaneous game, the creditor receives all the surplus.

### 3.2 Sequential Game

In a sequential game, the financial restructuring and operational restructuring are decided sequentially. We first analyze the case in which the financial restructuring takes place after operational restructuring.

In this case, our results from the simultaneous case changes remarkably. In particular, efficiency can no longer be achieved in equilibrium.

**Proposition 3** If operational restructuring is decided first in the sequential game, then the equilibrium is always inefficient.

Once the operational restructuring is decided, there is no financial restructuring that results in any wealth transfers in expected terms: any financial restructuring that is more desirable than the status quo (i.e. the original claims) to one of the claimants is less desirable than the status quo for the other party. Therefore, the creditor gets at most $\Pr(G|s_c)F + \Pr(B|s_c)R^*_B$ under an operational restructuring $j^* > 0$. Anticipating the incentive problem at the financial restructuring stage, the creditor never agrees to an operational restructuring in the first place. Demanding a liquidation results in strictly higher expected payoff, $\Pr(G|s_c)F + \Pr(B|s_c)L_B$.

Next we consider the game in which the financial restructuring takes place prior to the business restructuring.

Define $p^G_j(s_c) = \Pr(j^*(s_c, s_e) = j, \sigma|s_c)$, $p^G_j(s_c|m_c) = \Pr(j^*(m_c, s_e) = j, \sigma|s_c)$ and $q(s_c) = \Pr(\sigma = G|s_c) = \sum_{j=0}^{J} p^G_j$. Conditional on creditor’s signal $s_c$, define the ex-
pected surplus by

\[ \Delta(s_c) = \sum_{j=0}^{J} p^j_G(s_c)(R^j_G - L_G) - p^j_B(s_c)(L_B - R^j_B). \]

This expression represents the expected firm value under the optimal operational re-
structuring over and above the liquidation value conditional on the creditor’s informa-
tion. Note that the definition of optimal business plan \( j^* \) implies that

\[ p^j_G(s_c)(R^j_G - L_G) \geq p^j_B(s_c)(L_B - R^j_B) \]

for all \( s_c \). Since \( j = 0 \) denotes the liquidation alternative, this holds with equality if
and only if \( j = 0 \). It follows that \( \Delta(s_c) = 0 \) if \( s_c = 0 \), and \( \Delta(s_c) > 0 \) for all \( s_c > 0 \).

The assumptions that \( \phi_\sigma(s_i|\sigma) \) is continuous and satisfies MLRP imply that \( \Delta(s_c) \)
is a continuous function and is strictly increasing. We can also decompose the ex-
pected surplus into benefit and cost, \( \sum_{j=0}^{J} p^j_G(s_c)(R^j_G - L_G) \) and \( \sum_{j=0}^{J} p^j_B(s_c)(L_B - R^j_B) \),
respectively.

Finally, we define the expected surplus conditional on creditor’s signal \( s_c \) when the
creditor’s message is \( m_c \)

\[ \Delta(s_c|m_c) = \sum_{j=0}^{J} p^j_G(s_c|m_c)(R^j_G - L_G) - p^j_B(s_c|m_c)(L_B - R^j_B), \]

where 

\[ p^j_\sigma(s_c|m_c) = \Pr(j^*(m_c, s_c) = j, \sigma|s_c). \]

In our next proposition, we show that not only an efficient equilibrium exists but
also the equityholder’s surplus can be bounded away from zero. There are several
such equilibria, each associated with a different financial restructuring. Our proof is
constructive and we characterize one such equilibrium that involves a simple finan-
cial restructuring. From the construction of the equilibrium, it will be clear that the
equityholder’s surplus can be higher at the expense of a more complicated financial
restructuring. As we show in Lemma 5 in the appendix, there exists \( K \in (0, F - L_B) \)
that satisfy
\[
\sum_{j=1}^{J} p_{G}^{j}(s_c) K < \Delta(s_c) \text{ for all } s_c > 0.
\]

In the following proposition the left hand-side of this inequality will be the equityholder’s expected surplus.

**Proposition 4** If financial restructuring is decided first, then sequential negotiations are efficient. Furthermore, efficiency can be achieved by a financial restructuring in which equityholder’s expected surplus is strictly positive.

This result highlights the importance of the order in which issues are negotiated. Although an efficient equilibrium exists when financial restructuring is decided first, there is no efficient equilibrium when the order is reversed. This is because, once operational restructuring is approved, there is no incentive to transfer wealth between the claimants. Anticipating this lack of financial restructuring, the claimants do not have any incentive to share information and agree on the optimal reorganization at the first stage. On the other hand, changing the order and deciding first on financial restructuring enables the equityholder to “reward” the senior claimant by offering some of the upside potential whenever he reveals positive information. Thus, the senior claimant will have an incentive to reveal his information and approve a financial restructuring. In particular, conditional on his information (and his expectations as to which operational restructuring will take place), the senior claimant accepts a financial restructuring as long as he gets more than his expected payoff under liquidation. Moreover, it is possible to design a financial restructuring that would reward the claimants when the optimal restructuring plan is implemented at a later date, and punish them otherwise. Therefore, the junior claimant can commit to optimal restructuring plan.

In addition to its implications for efficiency, the order of negotiations also has distributional consequences. Note that under simultaneous negotiations, the equityholder
has to commit to a specific operational reorganization whenever she proposes a financial reorganization. Therefore, in simultaneous negotiations the creditor’s expected payoff under any operational reorganization must be at least as large as his expected payoff in liquidation under original claims. However, in sequential negotiations he has to decide on a financial restructuring when he does not know for sure which operational restructuring will be implemented. Conditional on his information, he approves a financial restructuring as long as his expected payoff exceeds that in liquidation under original claims. Therefore, he may agree to a financial restructuring that leaves some of the surplus to the equityholder. More specifically, at the operational restructuring stage he may end up with a payoff that is lower than his liquidation payoff under original claims since liquidation will take place under the new financial restructuring.

A financial restructuring that gives some of the surplus to the equityholder may be efficient under sequential negotiation, although it is always inefficient under simultaneous negotiations. The fact that the equityholder does not commit to a specific operational restructuring at the time of the financial restructuring implies that there is one less condition for efficient equilibrium payoffs. This leads to a larger set of financial restructurings that achieves efficiency. The following lemma shows that any financial restructuring that enables efficiency in the simultaneous game, can also lead to an efficient operational restructuring in a sequential game.

**Corollary 4** In the sequential game in which financial restructuring is decided first, efficiency can also be achieved by a financial restructuring that allocates all of the senior security and the expected surplus to the creditor.
4 Extensions

Our analysis focused on a single take-it-or-leave-it offer. It is possible to extend our analysis by allowing negotiations to continue several periods before liquidation takes place, and letting players discount the future payoffs. One implication of such an extension would be on the sharing of the surplus. In particular, the equityholder would receive some of the surplus, even when both issues are negotiated simultaneously. In order to see the intuition it suffices to consider a two period negotiation. The second period essentially becomes identical to a single period negotiation. Therefore, in the first period the equityholder can extract a surplus in the amount of the creditor’s loss due to one period delay. Furthermore, if a financial restructuring is not allowed under liquidation, then multiple period bargaining may lead to inefficient operational restructuring. In particular, when liquidation is optimal, the equityholder may offer a proposal that involves non-liquidation operational restructuring that gives her more than she would have received in liquidation and the creditor may accept such an offer as long as delay is more costly for him. Therefore, extended negotiations may lead to inefficient restructuring. In other words, our analysis suggests that the source of inefficiency in in Chapter 11 proceedings may have more to do with extended negotiations rather than the combination of two interdependent issues.

Our main results are robust to two other plausible modifications of the model. First, consider the case in which the creditor makes the offer following the equityholder’s message. Note that our arguments as to why the operational restructuring is inefficient in the absence of financial restructuring continue to hold in this game as well. In particular, the creditor will never propose and approve a non-liquidation restructuring if \( L_C > F \) whereas the equityholder will always misrepresent his signal and announce \( s_e = 1 \) as long as \( F > L_B \). It is also immediate from our existing analysis that once financial restructuring is allowed, operational efficiency can be achieved. However, it less clear as
to why the creditor must receive all of the newly issued senior security in every efficient 
equilibrium. First, note that an efficient operational restructuring the equityholder 
must communicate his information truthfully. Since otherwise proposing the optimal 
business plan may not be possible. Now suppose that in an efficient equilibrium the 
creditor offers the equityholder a positive fraction of the most senior security along 
with a (non-liquidation) operational restructuring. Then the equityholder will have an 
incentive to misrepresent his signal whenever \( s_e = 0 \), since reporting the truth results 
in zero expected payoff whereas misrepresentation leads to strictly positive expected 
payoff. Therefore, efficiency requires that the creditor must receive all of the newly 
issued senior security.

Second, note that in our analysis, we restricted attention to the case in which offers 
specify what to do with the firm and who gets what. It is also possible that the offer 
in the first stage specifies the identity of the proposer in the second stage. Our results 
would be unchanged even when we modify the strategy space to allow this.

To see this first consider the case in which operational restructuring is decided first. 
Now suppose that the offer in the first stage specifies what to do with the firm as well 
as who proposes the financial restructuring. As before, once operational restructuring 
is agreed upon, there is no financial restructuring that is acceptable to both claimants. 
Hence, regardless of the identity of the proposer, the original claims are preserved at 
the financial restructuring stage, and being able to propose at the second stage has no 
value. Given this, the creditor does not agree to any operational restructuring other 
than liquidation at the first stage.

Now, consider the case in which financial restructuring is decided first. For this 
case, we have seen that when the equityholder proposes in both stages, an efficient 
equilibrium exists. It can easily be seen that, in the modified game, there exists 
an equilibrium in which the first stage proposal specifies the equityholder to be the 
proposer for the second stage.
5 Conclusion

In this paper, we have analyzed a bargaining game between the senior and junior claimants of a firm in Chapter 11 bankruptcy. The claimants are asymmetrically informed about the desirability of various operational restructuring alternatives, and negotiate to decide both what to do with the firm (operational restructuring) and how to split the surplus (financial restructuring). We show that without communication or security design, it is not possible to have an efficient equilibrium. When both communication and security design is allowed, whether efficiency can be achieved or not depends on the order in which two issues (operational restructuring and financial restructuring) are negotiated. When the issues are negotiated simultaneously, there exists an efficient equilibrium. The financial restructuring that supports an efficient outcome requires the senior claimants to receive all of the newly issued senior securities. When the issues are negotiated sequentially, there does not exist an efficient equilibrium if operational restructuring is negotiated first. On the other hand, if financial restructuring is negotiated first, then the sequential negotiations are efficient. Our results highlight the sensitivity of the \textit{ex post} efficiency of Chapter 11 bankruptcy to the order in which the operational restructuring and financial restructuring decisions are made.
Appendix

Proof of Lemma 1: We have $R^*_G > R^*_j > R^*_j > R^*_B$ by definition. Let $\theta \equiv \Pr(G|s_c, s_e)$ and $\varphi \equiv \Pr(G|s'_c, s'_e)$ so that we have $\varphi \geq \theta$. Now $E[R|j^*, s_c, s_e] > E[R|j, s_c, s_e]$ is equivalent to $\theta R^*_G + (1 - \theta)R^*_j > \theta R^*_G + (1 - \theta)R^*_j$. Arranging terms results in $\theta(R^*_G - R^*_j) > (1 - \theta)(R^*_j - R^*_j)$. Given that $\varphi \geq \theta$ we must have $\varphi R^*_G + (1 - \varphi)R^*_j > \varphi R^*_G + (1 - \varphi)R^*_j$ must also hold.

Proof of Lemma 4: If an equilibrium is efficient, then for any signal pair $(s_c, s_e)$ such that $j^*(s_c, s_e) > 0$ the optimal restructuring plan must be proposed and voted for in that equilibrium. This requires that the equityholder must propose the optimal restructuring plan and vote for the plan regardless of the extensive form whenever the optimal restructuring plan is not liquidation. Therefore, her expected payoff in doing so, $E[\tilde{R}^*_j(s_c, s_e) - \hat{\pi}(s_c, s_e)(R^*_j)|s_c, s_e]$ must exceed the expected payoff under liquidation, $E[\tilde{L} - \tilde{l}|s_c, s_e]$. Thus, equation (3) must hold in an efficient equilibrium. Similarly, in an efficient equilibrium the equityholder must not have an incentive to implement $j > 0$ when liquidation is optimal. This implies equation (4).

Proof of Proposition 1: Note that if there is no financial restructuring then any financial restructuring proposal must leave the claims unchanged. Then it must be the case that $\hat{\pi}(s_c, s_e)(R) = \min\{R, F\}$ for all $s_c, s_e$.

We first show that if an efficient equilibrium exists without financial restructuring then $F \geq R^*_G$ or $F \leq R^*_B$. Suppose not. Then $\hat{\pi}(s_c, s_e)(R) = \min\{R, F\}$ and $R^*_B < F < R^*_G$. Since $R^*_B < F < R^*_G$, it follows that either $L_B \leq F < R^*_G$ or $L_G \geq F > R^*_B$ (or both).

First consider the case in which $L_B \leq F < R^*_G$. Recall that for sufficiently high signals $s_c$ and $s_e$, the optimal restructuring plan is $J$, i.e. $j^*(s_c, s_e) = J$. Consequently, under optimal restructuring plan, we have $\hat{\pi}(s_c, s_e)(R^*_G) = F$ and $\hat{\pi}(s_c, s_e)(R^*_B) = R^*_B$.

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Furthermore, $L_{\sigma} - \ell_{\sigma} = \max\{0, L_{\sigma} - F\}$. Therefore, equation (4) in Lemma 4 reduces to
\[
Pr(G|s_c, s_e') \max\{0, L_G - F\} \geq Pr(G|s_c, s_e')(R_G^I - F).
\]
This cannot hold since $R_G^I > L_G$ and $R_G^I > F$.

Next, consider the case in which $L_G \geq F > R_J G$. Note that for any $s_e$, there exists $s_c$ sufficiently high and $s_e'$ sufficiently low such that $j^*(s_c, s_e) \neq 0$ and $j^*(s_e', s_e) = 0$. Since the equilibrium is efficient and there is no financial restructuring, it must be the case that $\hat{\pi}(s_c, s_e)(R_G^I(s_c, s_e)) = F = \hat{\pi}(s_c, s_e)(R_G^I(s_c, s_e)) = \hat{\pi}(s_c, s_e)(L_G)$ and $\hat{\pi}(s_c, s_e)(R_B^I(s_c, s_e)) < \min\{L_B, F\} = \hat{\pi}(s_c, s_e)(R_B^I(s_c, s_e)) = \hat{\pi}(s_c, s_e)(L_B)$. Therefore, the creditor always prefers liquidation. Consequently, equation (2) in Lemma 3 cannot be satisfied.

To complete the proof, we need to show that if either $F \geq R_G^I$ or $F \leq R_B^I$ then an efficient equilibrium exists. In the former case, the creditor is always paid in full and has no incentive to misrepresent his signal or vote against the optimal restructuring proposal. Since the equityholder receives the full surplus she proposes the optimal restructuring plan. In the latter case, the equityholder would receive nothing in liquidation. Therefore, she is indifferent between the optimal restructuring plan and any other plan. Thus proposing the optimal restructuring plan and then voting for it is an equilibrium.

**Proof of Proposition 2:** Recall from Lemma 4 that if $j^*(s_c, s_e) \neq j^*(s_c, s_e') = 0$ for some $s_c, s_e$ and $s_e'$, and an efficient equilibrium exists, then it must be the case that
\[
E_\sigma[R_{\sigma}^I(s_c, s_e) - \hat{\pi}(s_c, s_e)(R_{\sigma}^I(s_c, s_e))|s_c, s_e] \geq E_\sigma[(L_{\sigma} - \ell_{\sigma})|s_c, s_e]
\]
and
\[
E_\sigma[(L_{\sigma} - \ell_{\sigma})|s_c, s_e'] \geq E_\sigma[R_{\sigma}^I(s_c, s_e) - \hat{\pi}(s_c, s_e)(R_{\sigma}^I(s_c, s_e))|s_c, s_e'].
\]
Using $\ell_G = F$ and $\ell_B = L_B$ yields

$$\Pr(G|s_c, s_e)[R_G^{j^*(s_c, s_e)} - \hat{\pi}(s_c, s_e)(R_G^{j^*(s_c, s_e)})] + \Pr(B|s_c, s_e)[R_B^{j^*(s_c, s_e)} - \hat{\pi}(s_c, s_e)(R_B^{j^*(s_c, s_e)})]$$

$$\geq \Pr(G|s_c, s_e)(L_G - F)$$

and

$$\Pr(G|s_c, s'_e)(L_G - F) \geq \Pr(G|s_c, s'_e)[R_G^{j^*(s_c, s_e)} - \hat{\pi}(s_c, s_e)(R_G^{j^*(s_c, s_e)})]$$

$$+ \Pr(B|s_c, s'_e)[R_B^{j^*(s_c, s_e)} - \hat{\pi}(s_c, s_e)(R_B^{j^*(s_c, s_e)})].$$

We divide the first inequality by $\Pr(G|s_c, s_e)$ and the second by $\Pr(G|s_c, s'_e)$:

$$[R_G^{j^*(s_c, s_e)} - \hat{\pi}(s_c, s_e)(R_G^{j^*(s_c, s_e)})] + \frac{\Pr(B|s_c, s_e)}{\Pr(G|s_c, s_e)}[R_B^{j^*(s_c, s_e)} - \hat{\pi}(s_c, s_e)(R_B^{j^*(s_c, s_e)})] \geq (L_G - F),$$

(5)

and

$$(L_G - F) \geq [R_B^{j^*(s_c, s_e)} - \hat{\pi}(s_c, s_e)(R_B^{j^*(s_c, s_e)})] + \frac{\Pr(B|s_c, s'_e)}{\Pr(G|s_c, s'_e)}[R_B^{j^*(s_c, s_e)} - \hat{\pi}(s_c, s_e)(R_B^{j^*(s_c, s_e)})].$$

(6)

Note that $j^*(s_c, s_e) > j^*(s_c, s'_e)$ implies that $s_e > s'_e$, and hence $\frac{\Pr(B|s_c, s'_e)}{\Pr(G|s_c, s'_e)} > \frac{\Pr(B|s_c, s_e)}{\Pr(G|s_c, s_e)}$. Therefore, we must have $R_B^{j^*(s_c, s_e)} = \hat{\pi}(s_c, s_e)(R_B^{j^*(s_c, s_e)})$. Together with the inequalities (5) and (6) this implies that $f_c^{j^*(s_c, s_e)} \in [R_B^{j^*(s_c, s_e)}, R_G^{j^*(s_c, s_e)} - L_G + F]$ and $f_c^{j^*(s_c, s_e)} = 0$ when the new securities are restricted to only debt and equity. Considering the fact that for all $j^*(s_c, s_e) \neq 0$, there exists $s'_e$ such that $j^*(s_c, s'_e) = 0$, we must have $f_c^j \geq R_B^j$ and $f_c^j = 0$ for all $j \in \{1, 2, \ldots, J\}$ as a necessary condition for an efficient outcome.

Next we construct an efficient perfect Bayesian equilibrium, and show that conditions $f_c^{j^*(s_c, s_e)} \in [R_B^{j^*(s_c, s_e)}, R_G^{j^*(s_c, s_e)} - L_G + F]$ and $f_c^{j^*(s_c, s_e)} = 0$ are not only necessary but also sufficient. Let $\alpha_{c}^{j^*(s_c, s_e)}(R_G^{j^*(s_c, s_e)} - f_c^{j^*(s_c, s_e)}) = L_G - F$. In equilibrium, the creditor reveals his signal, $s_c$ correctly, and the equity holder proposes $j^*(s_c, s_e)$, $f_c^{j^*(s_c, s_e)} \in [R_B^{j^*(s_c, s_e)}, R_G^{j^*(s_c, s_e)} - L_G + F]$, $f_c^{j^*(s_c, s_e)} = 0$ and $\alpha_{c}^{j^*(s_c, s_e)} = \frac{L_G - F}{R_G^{j^*(s_c, s_e)} - f_c^{j^*(s_c, s_e)}}$ and $\alpha_{c}^{j^*(s_c, s_e)} = 1 - \alpha_{c}^{j^*(s_c, s_e)}$. If the equityholder proposes a business plan $j$ that is not
accompanied by \( f^j_c \in [R^j_B, R^j_G - L_G + F] \), \( f^j_e = 0 \) and \( \alpha^j_e = \frac{L_G - F}{R^j_G - f^j_c} \) and \( \alpha^j_c = 1 - \alpha^j_e \), the creditor believes that \( s_e = 0 \). Note that the inequalities (5) and (6) imply that the expected payoff to the equityholder under restructuring is equal to that under liquidation. Therefore, given the equilibrium beliefs, the equityholder is indifferent and does not deviate. The creditor on the other hand captures the entire profit in equilibrium and thus cannot profitably deviate. ■

**Proof of Proposition 3:** Take any \( s_c, s_e \) such that \( j^*(s_c, s_e) > 0 \), and suppose that the optimal plan \( j^*(s_c, s_e) \) is proposed and accepted prior to any financial restructuring. Then the equityholder will neither offer nor accept a financial restructuring that makes her worse off than the existing financial structure:

\[
\Pr(G|s_c, s_e)[R^j_G(s_c, s_e) - \hat{\pi}(s_c, s_e)(R^j_G(s_c, s_e))] + \Pr(B|s_c, s_e)[R^j_B(s_c, s_e) - \hat{\pi}(s_c, s_e)(R^j_B(s_c, s_e))]
\geq \Pr(G|s_c, s_e)[R^j_G(s_c, s_e) - F].
\]

Therefore, the creditor can get at most

\[
\Pr(G|s_c, s_e)F + \Pr(B|s_c, s_e)R^j_B(s_c, s_e).
\]

But this is strictly less than

\[
\Pr(G|s_c, s_e)F + \Pr(B|s_c, s_e)L_B,
\]

what he can get in liquidation. Therefore, the creditor never votes for a reorganization plan. ■

**Lemma 5** There exists \( K \in (0, F - L_B) \) such that

\[
\sum_{j=1}^{J} p^j_G(s_c)K < \Delta(s_c) \text{ for all } s_c > 0.
\]
**Proof:** For a given \( s_e \), let \( s_e(s_e) \) satisfy

\[(R_G^1 - L_G) \Pr(\sigma = G | s_e, s_e(s_e)) = (L_B - R_B^1) \Pr(\sigma = B | s_e, s_e(s_e)).\]

In other words, for a given \( s_e \), \( s_e(s_e) \) is the signal that makes the expected firm value equal under liquidation and business plan \( j = 1 \). Note that

\[
\frac{\Pr(G|s_e,s_e)}{\Pr(B|s_e,s_e)} = \frac{\phi_e(s_e|G)\phi_e(s_e|G)}{\phi_e(s_e|B)\phi_e(s_e|B)}.
\]

Therefore, \( s_e(s_e) \) must satisfy

\[
\phi_e(s_e|G)\phi_e(s_e|G) = \frac{L_B - R_B^1}{R_G^1 - L_G} \quad \text{for all } s_e > 0.
\]

Given that \( \frac{\phi_e(s_e|G)}{\phi_e(s_e|B)} \) is strictly increasing we must have

\[
\frac{\phi_e(s_e|G) \int_{s_e(s_e)}^1 \phi_e(s|G) ds}{\phi_e(s_e|B) \int_{s_e(s_e)}^1 \phi_e(s|B) ds} > \frac{L_B - R_B^1}{R_G^1 - L_G} \quad \text{for all } s_e > 0.
\]

Note that left hand side of the above inequality is \( \sum_{j=1}^J \frac{L^j_c}{L^j_B} \). Arranging the terms, we have

\[
R_G^1 - L_G - (L_B - R_B^1) \sum_{j=1}^J \frac{L^j_c}{L^j_B} > 0 \quad \text{for all } s_e > 0.
\]

Furthermore, from the definition of \( \Delta(s_e) \), the left hand side of the above inequality is strictly less than \( \frac{\Delta(s_e)}{\sum_{j=1}^J \frac{L^j_c}{L^j_B}} \) for all \( s_e > 0 \). Therefore, \( \frac{\Delta(s_e)}{\sum_{j=1}^J \frac{L^j_c}{L^j_B}} \) is strictly positive for all \( s_e > 0 \). Thus, \( K \in (0, 1) \) exists for all \( s_e > 0 \).

**Proof of Proposition 4:** We construct an efficient equilibrium. The equilibrium we construct is as follows: In period (i), the creditor’s message is truthful, \( m_c = \mu(s_e) = s_e \).

In period (ii), if \( m_c = 0 \), then the equityholder proposes a financial restructuring that preserves the original claims, i.e., \( \pi(m_c,s_e)(R) = \min\{F,R\} \) for all \( m_c, s_e \) with \( m_c = 0 \).

Otherwise, she proposes \( \pi : \mathcal{M}_c \times S_e \to \mathcal{P} \) where for all \( m_c \) and \( s_e \)

\[
\pi(m_c,s_e)(R) = \begin{cases} R & \text{if } R < F - K \\ F - K & \text{if } F - K \leq R \leq L_G \\ F - K + R - L_G & \text{if } R > L_G \end{cases} \quad (7)
\]
In period (iii) both claimants approve the new financial restructuring. In period (iv) the equityholder proposes \( j^*(m_c, s_e) \) and in period (v) both claimants approve. For simplicity we fix the off the beliefs equilibrium path such that creditor (equityholder) believes that \( s_c = 0 \) \( (s_c = 0) \).

We now verify that these beliefs support the above equilibrium outcome. We first consider the case in which \( s_c \neq 0 \). If no deviations are detected until the last period, then the equityholder is indifferent between liquidation and \( j^* \) as his expected payoff is \( Pr(G|s_c, s_e)(L_G - F + K) \) under the new claims. Since the equityholder is indifferent, the creditor captures the entire surplus by voting for \( j^* \). Consequently, the creditor’s expected payoff goes down by \( \Delta(s_c) \) if he votes against so he approves as well. Note that in period (iv), the creditor cannot detect a deviation as long as any operational restructuring other than liquidation is proposed. If on the other hand liquidation is proposed, the vote is immaterial as liquidation will take place whether or not it is voted for.

As in period (v), the equityholder’s expected payoff is constant in period (iv) over all possible deviations and equilibrium strategies she may have as long as there has been no prior deviation. Thus, she has no incentive to deviate as long as no deviation has occurred. If there was a deviation in period (iii), at least one of the claimants must have voted against the financial restructuring and hence the claims are not restructured. This is possible only when at least one of the claimants believes that liquidation is optimal. There are three possibilities: either \( s_c = 0 < s_e \), or \( s_e = 0 < s_c \), or \( s_c = s_e = 0 \). If \( s_c = 0 \), then regardless of what the equityholder believes, there will be liquidation, since the creditor’s expected payoff is maximized under liquidation with the original financial structure. If on the other hand, \( s_c > 0 \) and \( s_e = 0 \), the equityholder may propose a restructuring other than liquidation. But since claims are not restructured, the creditor’s expected payoff when he votes against financial restructuring in period (iii) is \( qF + (1 - q)L_B \). In contrast, his expected payoff if he votes for the financial
Restructuring in period (iii) is

\[ q[F - K + \frac{\Delta(s_c)}{q}] + (1 - q)L_B \]

if he had not deviated. Since \( L_B \leq F - K \) and \( K < \frac{\Delta(s_c)}{q} \), the creditor does not deviate in period (iii) as long as there has been no prior deviation. The equityholder approves in period (iii) as well, given that, if he deviates, he gets \( Pr(G|s_c, s_e)(L_G - F) \) in comparison to \( Pr(G|s_c, s_e)(L_G - F + K) \).

In period (ii) the equityholder does not deviate for the same reason: if she deviates, her expected payoff is \( Pr(G|s_c, s_e)(L_G - F + K) \) in comparison to \( Pr(G|s_c, s_e)(L_G - F) \). Note that at this stage she cannot detect a deviation.

Finally, in period (i) the creditor’s message is truthful. To see this first note that reporting \( m_c = 0 \) results in original financial structure being preserved followed by liquidation. Thus, when \( m_c = 0 \), the expected payoff of the creditor is \( qF + (1 - q)L_B \) which is strictly less than \( q[F - K + \frac{\Delta(s_c)}{q}] + (1 - q)L_B \) as we argued above. Reporting another untruthful message \( m_c \neq s_c \) will result in an expected payoff of \( q[F - K + \frac{\Delta(s_c|m_c)}{q}] + (1 - q)L_B \). Note that \( \Delta(s_c|m_c) \) is maximized at \( m_c = \mu(s_c) = s_c \). Therefore, this deviation is not profitable either.

For the case with \( s_c = 0 \), the expected surplus \( \Delta(s_c) = 0 \) and thus deviations are not profitable for the creditor. Consequently, the equityholder proposes liquidation and the firm is liquidated under original claims. □

**Proof of Corollary 4:** The proof follows immediately from that of Proposition 4 once \( K \) is replaced with zero. □
References


