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"Non-Structural and Structural Models in Productivity Analysis: Study of the British Isles during the 2007-2009 Financial Crisis"

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Non-Structural and Structural Models in Productivity Analysis: Study of the British Isles during the 2007-2009 Financial Crisis

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Abstract

The paper reviews and provides an extension of the benefits and drawbacks of the non-structural Stochastic Frontier Analysis (SFA) paradigm, as well as other number index-based procedures that we will utilize in the analysis. We then compare SFA with two structural models: the Pakes McGuire Model (PMM) (Pakes, Gowrisankaran and McGuire 1993) and the Midrigan and Xu Model (MXM) (Midrigan and Xu 2014). All three methods are used to estimate changes in firm-level productivity in the British Isles before and after the 2007-2009 financial crisis under the canonical single production assumption. The empirical results indicate that overall productivity was not impacted to any substantial degree by the financial crisis, according to both SFA and the PMM. However, the productivity loss estimated by MXM due to financial friction from the recession was substantial.

Key Words: Productivity, Non-Structural and Structural Models; Financial Friction.

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1. Introduction

The performance of a firm is usually measured by productivity, the ratio of a weighted average of outputs to a weighted average of inputs. Traditional productivity measurement includes labor productivity, capital productivity, and so on. However, these partial productivity measures may lead to a biased overall measure of productivity when firms use multiple inputs to produce outputs (Coelli, Rao, O'Donnell and Battese 2005). Total Factor Productivity (TFP) accounts for the effects on the total output that are not caused by inputs. TFP effectively evaluates overall productivity and has been widely used in literature (Rymes 1983; Norsworthy 1984; Mas, Maudos, Pérez and Uriel 1998; Islam 1999; Inklaar, O'Mahony and Timmer 2005; Cao, Ho, Jorgenson, Ren, Sun and Yue 2009).

In the production function, TFP is usually measured as the Solow residual. In a Cobb-Douglas production function, for example, the total output (Y) is a function of total factor productivity (A), labor input (L), and capital input (K) with the form $Y = AL^{\alpha}K^{1-\alpha}$. For a certain technology, the upper bound of TFP is the efficiency level of TFP, which determines the frontier of the production function. Many micro- and macro-level factors may create TFP loss, which keeps the actual production level below the frontier.

Non-structural approaches (Perelman 1995; Van Dijk and Szirmai 2011) have been widely used to estimate productivity and Stochastic Frontier Analysis (SFA) is a well-known approach to estimate the overall frontier function and technical efficiency. The panel stochastic frontier analysis employed in our analysis below uses a normalization in which the firm with the highest TFP has a relative efficiency that is the highest of all the firms in the sample at that particular point in time (Schmidt and Sickles 1984). Although this non-structural model can estimate the average efficiency level, and therefore predict the overall relative TFP loss for each unit, it cannot explain the sources of the inefficiency or TFP loss without a more detailed explanation of the inefficiency residual term.

Productivity defined by the Solow residual is a non-structural concept and cannot be given a structural interpretation without a more formal structural model. Our study complements non-structural SFA with two structural models from the literature and compares their explicit restricted reduced-form predictions with those from the unrestricted SFA model, using panel data from firms in the UK and Ireland.

The first structural model that considered is the Pakes-McGuire model (PMM) (Pakes, Gowrisankaran and McGuire 1993). This model computes the Markov Perfect Nash (MPN) Equilibria of an industry, where a firm's profit is a function of its own level of efficiency and a vector specifying the efficiency level of all its competitors. This approach can estimate an average efficiency level by solving both the MPN equilibria and social planner's problem.

The second structural model is the model of Midrigan and Xu (MXM) (Midrigan and Xu 2014), which decomposes the residual and estimates the partial TFP loss due to specific factors (financial friction or financial misallocation, in this case). This approach sets up an economic structure and calculates the efficient allocation (with highest attainable productivity) to satisfy the labor market clearing condition, the asset clearing condition, producer and worker optimization, and the no-arbitrage condition. The factors of interest are introduced into the model along with the borrowing constraints, which can lead to a lower possible TFP and therefore cause TFP loss.

We analyze the effect of financial frictions on productivity in the British Isles before and after the 2007-2009 financial crisis. The overall TFP loss estimate using SFA and PMM, as well as the finance-induced TFP loss implied by MXM, will be compared. The ratio of partial TFP to overall TFP reveals the relative magnitude of the inefficiency as a result of the misallocation of capital.

Our findings indicate that the overall TFP was not impacted to any substantial degree by the financial crisis, according to both Stochastic Frontier Analysis and the Pakes-McGuire Algorithm. However, the TFP loss due to financial friction estimated by Midrigan and Xu's Model as a result of the recession was substantial.

The remainder of the article is structured as follows. Section 2 introduces the Stochastic Frontier Analysis. Section 3 describes the Pakes-McGuire Algorithm, and Section 4 presents Midrigan and Xu's Model. Section 5 provides data descriptions. Empirical results derived from the three approaches are presented and compared in Section 6. Section 7 concludes.

2. Stochastic Frontier Analysis

Aigner, Lovell and Schmidt (1977) and Meeusen and Van den Broeck (1977) independently and simultaneously proposed the stochastic frontier production function model of the form:

$$\log(Y_i) = x_i{}'\beta + \nu_i - u_i, \qquad i = 1, \dots, N,$$

which equals the deterministic frontier production function plus a symmetric random error variable, ν_i, to account for measurement errors and other sources of non-systematic statistical noise. Y_i is the output of firm i, x_i is the vector of inputs typically in logarithms, and u_i is a non-negative random variable representing technical inefficiency (the distance to the frontier).

In most cases, ν_i is assumed to follow a normal distribution that is independent of each u_i. Both ν_i and u_i are often assumed to be uncorrelated with independent variables x_i. A variety of distributional assumptions are also imposed on u_i. Aigner, Lovell and Schmidt (1977) assumed u_i to be i.i.d. half-normal random variables and derived the Maximum Likelihood (ML) estimates. Stevenson (1980) introduced a normal truncated specification, while Greene (1990) considered the gamma specification.

The stochastic frontier literature in the early 1980s mainly consists of analyses of cross-sectional data. Schmidt and Sickles (1984) proposed three serious difficulties that the stochastic frontier model suffered at that time, including inconsistent firm-specific technical inefficiency estimations, strong assumptions about the distribution of technical inefficiency and statistical noise, and potentially incorrect assumptions that inefficiency is independent of the regressors. They provided a variety of estimators to solve these potential problems, given the availability of panel data. The panel stochastic frontier model is

$$\log(Y_{it}) = \alpha + x_{it}{}'\beta + \nu_{it} - u_i = \alpha_i + x_{it}{}'\beta + \nu_{it}, \qquad i = 1, \dots, N, t = 1, \dots, T. \tag{1}$$

Although the random noise differs for each firm over time, the inefficiency term is assumed to be fixed for each individual in the early panel data stochastic frontier analysis. If no time-invariant variables are included, then the fixed-effects method (FIX) guarantees that the consistency does not hinge on a lack of correlation between the regressors and the individual effects and all parameters are point identified. A necessary condition for consistency of the random-effects method (RND) is that u_i is uncorrelated with the regressors. For the RND methods, coefficients on time-invariant regressors are identified without resorting to additional orthogonality conditions (Hausman and Taylor 1981). Schmidt and Sickles (1984) developed such an estimator for the time-invariant version of the SFA.

Cornwell, Schmidt and Sickles (1990) introduced both the within estimator (CSSW), the generalized least squares estimator (CSSG), and a Hausman and Taylor (1981)-type estimator they labeled the Efficient IV estimator that allow for time-variant individual effects by replacing the firm effect with heterogeneous environmental variables whose effect on efficiency was firm specific. In their empirical example, they assumed that only the coefficients on the time and time-squared variables have such heterogeneous patterns, resulting in a parameterization of for the firm effects of α_i with $\alpha_{it} = \theta_{i1} + \theta_{i2}t + \theta_{i3}t^2$. Sickles (2005) later examined various specifications of the time-variant firm effect α_{it} modeled in other research, including $\alpha_{it} = \gamma(t)\alpha_i = [1 + \exp(bt + ct^2)]^{-1}\alpha_i$ (Kumbhakar 1990), $\alpha_{it} = \eta_{it}\alpha_i = \exp[-\eta(1 - T)]\alpha_i$ (Battese and Coelli 1992), $\alpha_{it} = \theta_t\alpha_i$ (Lee and Schmidt 1993), and the general factor model $\alpha_{it} = c_{i1}g_{1t} + c_{i2}g_{2t} + \cdots + c_{iL}g_{Lt}$ (Kneip 1994; Kneip, Sickles and Song 2003; Kneip, Sickles and Song 2012).

This study uses the Fixed Effects Estimator (FIX), Random Effect Estimator (RND), Kneip-Sickles-Song Estimator (KSS), and Battese-Coelli Estimator (BC). The first two are time-invariant estimators, and the last two are time-varying effects estimators. This study also applies the Cobb-Douglas stochastic frontier function with a constant returns to scale constraint, which restricts the sum of the coefficients of inputs to one, in large part to reduce issues of collinearity and in order to allow us to better compare our estimates with those from the more neoclassical structural methods we also explore that almost exclusively specify a constant returns to scale Cobb-Douglas production function.

Thus we estimate:

(2) $$\log(Y_{it}) = \alpha_t + \beta_1 \log(K_{it}) + \beta_2 \log(L_{it}) + \delta t + \nu_{it} - u_{it} \quad \text{s.t.}\, \beta_1 + \beta_2 = 1$$

$$\Rightarrow \log(Y_{it}) = \alpha_t + \beta_1 \log(K_{it}) + (1 - \beta_1) \log(L_{it}) + \delta t + \nu_{it} - u_{it}$$

$$\Rightarrow \log(Y_{it}) - \log(L_{it}) = \alpha_t + \beta_1((\log(K_{it}) - \log(L_{it})) + \delta t + \nu_{it} - u_{it}.$$

We include a linear time trend δt for FIX, RND, and BC but not for KSS, which specifies a general factor structure for the time-variant heterogeneity. α_t is constant over time for FIX and RND but time-variant for KSS and BC. v_{it} is the noise, and u_{it} is the nonnegative distance to the most efficient level of TFP. The equation $\exp(\alpha_{it}) = \exp(\alpha_t - u_{it})$ provides the individual TFP level for the i-th firm at time t. Assuming that the largest individual TFP is the efficient level of TFP, the average level of TFP loss can be derived, which indicates the average level of efficiency in the economy.

(3) $$\text{TFP loss ratio} = 1 - \frac{TFP}{TFP^e} = 1 - \left(\frac{1}{T}\sum_{t=1}^{T}\left(\frac{\frac{1}{N}\sum_{i=1}^{N}\exp(\alpha_t - u_{it})}{\exp(\alpha_t)}\right)\right)$$

$$= 1 - \left(\frac{1}{T}\frac{1}{N}\sum_{t=1}^{T}\sum_{i=1}^{N}\exp(-\mu_{it})\right) = 1 - \text{Average TE},$$

where TFP^e is the efficient level of total factor productivity and TFP is the average level of total factor productivity. The ratio $\frac{TFP}{TFP^e}$ equals the average technical efficiency (TE) that is typically used in the stochastic analysis. This study can thus estimate the average technical efficiency using the stochastic frontier analysis and then derive the TFP loss ratio.

3. Pakes-McGuire Model

This method computes the MPN equilibria (Maskin and Tirole 1988a; Maskin and Tirole 1988b) that are generated under the constraints of Ericson and Pakes (1992). This section employs the same notations and equations from the original work (see Pakes, Gowrisankaran and McGuire, 1993, pg. 7-36) in this section.

3.1 Model

The definition of “industry structure” is the range of efficiency levels among the various businesses in the model. A firm's profits arise from the industry structure as well as from the individual level of operational efficiency. Over the operating life of the firm, the efficiency of the business will change based on the stochastic environment of its operations. Decisions about investments, entries, and exits are made to achieve the highest level of future cash flow, based on expected discounted value (EDV), according to the current information set. Each business bases its decisions upon their prediction of the industry structure in the future. As a consequence, the true distribution of the future industry structure is a self-fulfilling prophecy. An MPN equilibrium is reached when the projected industry structure is the true distribution that results from the predictions. Thus, PMM calculates a subgame-perfect Nash equilibrium.

Industry Structure: $W = \{0, \ldots, \bar{w}\}$ is the set of efficiency values for each firm, where 0 is zero efficiency and $\bar{w}$ is the maximum level of efficiency. N is the maximum number of companies that can be simultaneously active in the industry. A state $[w, n]$ consists of a $w \in W^*, n \in N$, where $W^* = \{(w_1, \ldots, w_N) | w_j \in W, w_1 \geq w_2 \geq \cdots \geq w_N\}$. For any firm, w represents the economic environment, including the efficiency level of all firms, while n indicates which element of this vector is the efficiency of its own. W^* guarantees that the industry structure is represented as a weakly decreasing N-tuple to avoid having multiple w for the same industry structure.

Investment: A firm's efficiency level for the next period is determined by a Markov process that depends on its current efficiency level, current investment, and exogenous factors. This model denotes x as the current investment, $k \in W$ as the current efficiency level of a firm, and $k' \in W$ as its efficiency level in the next period. Let τ be the effect of firm-specific investment and v the effect of other firm-invariant exogenous variables that are the same for all firms. Then the controlled Markov process for the evolution of k' is

$$k' = k + \tau - v, \tag{4}$$

where

$$\mathrm{p}(\tau) = \begin{cases} (\alpha x)/(1 + \alpha x), & \text{if } \tau = 1 \\ 1/(1 + \alpha x), & \text{if } \tau = 0 \end{cases} \quad \text{and} \quad \mathrm{p}(v) = \begin{cases} \delta, & \text{if } v = 1 \\ 1 - \delta, & \text{if } v = 0. \end{cases}$$

Maximum Level of Efficiency. Eq. (4) ensures that efficiency can only improve with investment and shows that the incremental efficiency in a period is bounded with probability one. Hence, there exists an upper bound of efficiency $\bar{w}$. PMM computes $\bar{w}$ as the maximum efficiency level that a monopolist would ever reach by starting with a very large efficiency level and computing the monopolist's problem to see where the monopolist stops his or her investment.

Exit and Entry: Companies make the decision to exit if the future cash flow value drops below the stated scrap value of the business, which is denoted ϕ. The exiting business will only receive the current scrap value, not the current period profit. Business enters when the potential and expected future cash flow value is greater than the one-time cost of entry. The sunk cost of entry, X_E, is a random variable uniformly distributed between X_EL (lowest) and X_EH (highest). The potential entrants know the draw of X_E once they decide to enter. If they enter, they will not receive profit for that period, and enter with efficiency level W_E or $W\_E - 1$, depending on the value of ν.

3.2 The Algorithm

Matrices: Profit, Π, is an iteration-invariant matrix that calculates the one-shot game profit for each possible industry structure. Each iteration begins with the investment matrix, X, and the value function, V, from the output of the last iteration.

Iterative Procedure: The algorithm calculates Π and iterates on X and V until the maximum of the element-by-element difference between successive iterations in these two matrices is below a specified tolerance level. For each iteration, the calculation is done separately for each of the industry structures, using the previous values of X and V. Beginning with the most efficient company within the industry structure, its choice is updated using the most recent value of the iteration. The decision is renewed based on the value of the investment, exit, and entry. The value function is not included. These figures are used to calculate the policies for the firm with the next-highest efficiency. In turn, these updated choices are applied to the firm ranked third in efficiency, and so on.

Updating Exit and Entry: By comparing the value function of an incumbent competitor with the scrap value, a firm can predict if that incumbent competitor exits. The PMM defines the strategy set so that a firm must exit if it perceives that a competitor with a higher efficiency level than its own has exited. For any $[w, n]$, this model defines w' as the industry structure that results after exit has been accounted for, and m as the number of active firms in w'. After the decision of exit, this model iterates on whether there will be an entry if $m < N$. The value of future cash flows in state $[w', n]$ is compared with the one-time sunk cost in this process.

Updating Investment: Each firm chooses an optimal investment policy based on its perception of future competitors. The calculation is done separately for every $[w, n] \in (W^*, N)$. The value function at the i^{th} iteration is

(5) $$V^i(w,n) = \max\{\phi, sup_{x\geq 0}[\Pi(w',n) - cx + \tfrac{\beta\alpha x}{1+\alpha x}Cl(w'+e(n),n) + \tfrac{\beta}{1+\alpha x}Cl(w',n)]\},$$

where

$$Cl(w',n) = \lambda(w',n)\{\textstyle\sum_{\tau_1=0}^{1} \dots \ddot{\sum}_{\tau_n=0}^{1} \dots \sum_{\tau_N=0}^{1} \sum_{V=0}^{1} V^{i-1}[w' + W\_Ee(n_e) + \tau - iv, n]$$

$$\Pr\left[\tau_1 \middle| x_1^{i-1}, v\right] \dots \Pr[\tau_n | \ddot{x}, v] \dots \Pr\left[\tau_N \middle| x_N^{i-1}, v\right] \mathrm{p}(v)\} + [1 - \lambda(w',n)]\{\textstyle\sum_{\tau_1=0}^{1} \dots \ddot{\sum}_{\tau_n=0}^{1} \dots$$

$$\textstyle\sum_{\tau_N=0}^{1} \sum_{V=0}^{1} V^{i-1}[w' + \tau - iv, n] \Pr\left[\tau_1 \middle| x_N^{i-1}, v\right] \dots \Pr[\tau_n | \ddot{x}, v] \dots \Pr\left[\tau_N \middle| x_N^{i-1}, v\right] \mathrm{p}(v)\}.$$

In this value function, w' is the incumbent's efficiency level after updating for exit; $m(w')$ is the number of active competitors at $w = w'$; c is the cost in dollars of a dollar's worth of investment (equals 1 if no tax); $\lambda(w', n)$ is the probability of entry; $e(j)$ is a vector, all of whose elements are zero except for the j^{th} element, which is one; i is a vector, all of whose elements are one; τ is the vector containing the random τ of competitors; n_e is the position of the entrant for any industry structure; $n_e = m(w') + 1$ unless the permutation cycle has been reordered; $w_1', \dots, w_N'$ are the elements of the vector w'; and $x_1, \dots, x_N$ (except x_n) is the investment of the $N-1$ competitors at w'. A symbol ($\ddot{}$) in a summation indicates that the element is omitted. $Cl(\cdot)$ sums over the probability-weighted average of the possible states of future competitors, but not over the investing firm's own future states. It also indicates the firm's expected discounted value for each of the two possible realizations of the firm's own investment process, τ.

We denote $x^i[w', n]$ as the investment level that solves Eq. (5). To calculate it, the model first derives the optimal level of investment, $x[w', n]$, given that this investment is nonzero and that the firm does not exit. The actual level of investment, therefore, is either this number or zero, where zero is the solution if the optimal investment still leads to an exit decision or if $x[w', n]$ is negative. Let D_x denote the derivative with respect to x. The first-stage investment is thus

(6) $$x[w',n] = \mathrm{argsolv}_x\{c = \beta[D_x\left\{\tfrac{\alpha x}{1+\alpha x}\right\}Cl(w'+e(n),n) - D_x\left\{\tfrac{\alpha x}{1+\alpha x}\right\}Cl(w',n)]\}.$$

It is worth noting that $D_x\left\{\frac{1}{1+\alpha x}\right\} = \frac{\alpha}{(1+\alpha x)^2} = \alpha[1-\mathrm{p}(x)]^2$, where $\mathrm{p}(x) = \frac{\alpha x}{1+\alpha x}$. So, if $\mathrm{v1} = Cl(w' + e(n), n)$ and $\mathrm{v2} = Cl(w', n)$, the investment can be rewritten as

$$x = \mathrm{argsolv}_x\{c = \beta\alpha[1-\mathrm{p}(x)]^2(\mathrm{v1}-\mathrm{v2})\} \Rightarrow \mathrm{p}(x) = 1 - \sqrt{\frac{1}{\beta\alpha(\mathrm{v1}-\mathrm{v2})}}.$$

Taking the inverse of $\mathrm{p}(x)$, it can be seen that

$$x[w', n] = \frac{\mathrm{p}(x)}{\alpha - \alpha\mathrm{p}(x)}.$$

It is straightforward to derive the optimal value function by plugging the optimal investment into Eq. (5) and computing

$$V^i(w,n) = \max\{\phi, \Pi(w',n) - cx[w',n] + \frac{\beta\alpha x[w',n]}{1+\alpha x[w',n]}Cl(w'+e(n),n) + \frac{\beta}{1+\alpha x[w',n]}Cl(w',n)]\}.$$

If $V^i(w,n) = \phi$, then this model sets $x = 0$ with probability one. Hence, the actual investment is determined as

$$x^i[w',n] = I\{V^i(w,n) > \phi\}x[w',n],$$

where $I\{\cdot\}$ is the indicator function that takes the value of one if the condition is true, and the value of zero otherwise.

Calculating the Probability of Entry: After the exit decision is made, the value of entry is the value of an incumbent who realized that 1) there would be no other entry, and 2) if she enters, there would be no profits or investments in the current period for her. The expected discounted value of entering is

$$V^e(w') = \beta Cl[w' + W\_Ee[m(w')+1], m(w')+1;\ \lambda = 0].$$

A firm would like to enter if and only if $V^e > X\_E$, which is the random entry cost. Since the random cost is uniformly distributed between X_EL and X_EH, the probability of entry by an incumbent whose competitors are specified by w' and $m(w') < N$ is

$$\lambda(w',n) = \min\{\max[\frac{V^e(w\prime) - X\_EL}{X\_EH - X\_EL}, 0], 1\}.$$

Updating N: This model starts with the one-firm problem and solves for its value function and optimal policies. Then it proceeds to the two-firm problem, using the fixed values that is solved for in the one-firm problem as the starting values for X and V:

$$V^0[(w_1, w_2), 1] = V^\infty(w_1),\ \forall\ w_1, w_2 \in W,$$

$$V^0[(w_1, w_2), 2] = V^\infty(w_2),\ \forall\ w_1, w_2 \in W,$$

where $V^\infty(\cdot)$ is the fixed point for the one-firm problem. Analogously, for the N-firm problem with $N > 2$, the starting values are

$$V^0[w, n] = \begin{cases} V^\infty[(w_1, \ldots, w_{N-1}), n], & \text{if } n < N \\ V^\infty[(w_1, \ldots, w_{N-2}, w_N), n-1], & \text{if } n = N. \end{cases}$$

The elements of X are updated in the same way as V. This process is then repeated until $\lambda(w', n) = 0$ for all (w', n) with $m(w') \geq N - 1$.

3.3 Profit Function

The one-shot profit function that is utilized in this model is a homogenous products, Nash-in-quantities (Cournot) market[1] where differences in efficiencies among companies are reflected by differences in marginal costs. Let producers' different but constant marginal costs, $\theta(w_n)$, be a firm's specific efficiency index multiplied by a common factor price index. Accordingly, if $s\tau$ and sv are the logarithms of the firm's efficiency index and of the factor price index, respectively, then $w_n \equiv s\tau - sv$ and $\theta(w_n) = \gamma\exp(-w_n)$.

Let q_n be firm n's output, $Q = \sum q_n$, f be the fixed cost of production, and D be the vertical intercept of the demand curve. The profits are given by

$$\pi_n = p(Q)q_n - \theta(w_n)q_n - f = (D - Q - \theta(w_n))q_n - f.$$

The unique Nash equilibrium for this problem has quantities and price as

$$q^*_{w_n} = \max\{0, p^* - \theta_i\},\ \text{ and }\ p^* = \frac{1}{n^*+1}\left[D + \textstyle\sum_{j=1}^{n^*}\theta\big(w_j\big)\right].$$

where n^* is the number of firms with positive q^*. Finally, the profit of the current period is

$$\pi(w, n) = \max\{-f, [\mathrm{p}^*(w, n) - \theta(w_n)]^2 - f\}$$

[1] Nash-in-quantities means the quantity is the strategic variable. In other words, the strategy space for each firm contains all the finite and non-negative levels of output. And each firm chooses the output to maximize profit, taking the output choice of its opponents as fixed. Nash-in-price is a similar game, but here it is supposed that the firms choose price instead of the output.

$$= \max\{-f, \left[\frac{1}{n^*+1}\left[D + \sum_{j=1}^{N} \theta(w_j) - \theta(w_n)\right]^2 - f\right\}.$$

3.4 Social Planner's Problem

We are interested in finding out how the social planner will respond when the technology is exactly the same as that faced by the MPN competitors. The result enables us to compare the average efficiency level (and TFP loss ratio) to the MPN case.

In the homogenous products, Nash-in-quantities profits model, the social planner will set $\mathrm{p} = \mathrm{mc}$ for the firm with the lowest marginal cost (with the highest efficiency level), and produce until supply equals demand. The methodology used to solve the social planner's issues is similar to the one used in the case of MPN competitors. The social planner's role is to maximize the stochastic profit function or the projected value of social surplus, i.e. the producer plus consumer surplus. The value function is then constructed using the value function, as defined in Eq. (5).

Since the social planner controls the entire economy, any industry structure results in only one state, not in N states. Moreover, one does not need to form perceptions about entry and exit or the behavior of cohorts, as the single agent (i.e., the social planner) controls all active firms at any time.

4. Midrigan and Xu's Model

Our study also applies the benchmark model introduced by Midrigan and Xu (2014). We outline below its set-up, decision rules, definition of equilibrium, TFP function, and first-best allocation of the economy. Our discussion uses the same notations and equations from Midrigan and Xu (2014).

4.1 Set up

The economy is populated by a measure N_t of producers and a measure one of workers. The labor productivity and producer's population grow at constant rates. Producers operate either in a traditional sector that uses only labor and an unproductive technology, or in a modern sector that uses capital and labor and a more productive technology. We will focus on financial misallocation in the modern sector. A one-time sunk entry cost is required for producers in the traditional sector who want to enter into the modern sector. Moreover, one-period noncontingent security and equity claims to producers' profits are the only two kinds of financial instruments in the model.

Traditional Sector Producers: A certain amount, $(\gamma - 1)N_t$, of new producers enter the economy at the end of period t, but only in the traditional sector. Producers in this sector face decreasing returns on technology $(\eta < 1)$ that produces output Y_t using labor L_t as the only factor of production:

(7) $$Y_t = \exp(z + e_t)^{1-\eta} L_t^{\eta}.$$

The model assumes that entrants draw the permanent productivity component z from some distribution $G(z)$, whose mean is normalized to unity. e_t is a transitory productivity component that evolves over time according to a finite-state Markov process of $E = (e_1, \dots, e_T)$ with transition probabilities $f_{i,j} = \Pr(e_{t+1} = e_j | e_t = e_i)$. Entrants draw their initial productivity component e_i from the stationary distribution associated with f, which we denote with $\overline{f_i}$.

All producers in the traditional sector aim to maximize their lifetime utility, which is $E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t)$. However, the budget constraints they face depend on whether remaining in the traditional sector or switching to the modern sector.

On the one hand, the budget constraint for those who stay in the traditional sector is

(8) $$C_t = Y_t - WL_t - (1 + r)D_t + D_{t+1},$$

where D_t denote the producer's debt position, which is non-positive since these producers are not allowed to borrow. All entering producers have no wealth, i.e. the initial D is equal to zero. Moreover, W and r are the equilibrium wage and interest rate.

On the other hand, traditional sector producers who enter the modern sector require an investment equal to $\exp(z)\kappa$ units of output, which is proportional to the permanent productivity component. Besides internal funds, both of the two financial instruments including one-period risk-free debt and equity claims to future profits are potential channels to finance the physical capital, K_{t+1}, and intangible capital, $\exp(z)\kappa$. In terms of debt, the borrowing constraint is

$$D_{t+1} \leq \theta(K_{t+1} + \exp(z)\kappa), \tag{9}$$

where $\theta \in [0,1]$ governs the strength of financial frictions in the economy, which requires the debt below a fraction of its capital stock. For equity claims, denote P_t as the price of the claim to the entire stream of profits, where profits are defined as $\Pi_t^m = Y_t - WL_t - (r+\delta)K_t$, where δ is the capital depreciation rate. The model assumes that producers can only issue claims to a fraction, $\theta\chi$, of their future profits, where $\chi \in [0,1]$. θ is characterizing the degree of financial development of the economy since it decides the producer's ability to both borrow and issue equity. The budget constraint of a producer that enters the modern sector is therefore

$$C_t + K_{t+1} + \exp(z)\,\kappa = Y_t - WL_t - (1+r)D_t + D_{t+1} + \theta\chi P_t. \tag{10}$$

Modern Sector Producers: The production function for the producers in the modern sector is

$$Y_t = \exp(z + e_t + \phi)^{1-\eta}(L_t^{\alpha} K_t^{1-\alpha}) \,,$$

where $\phi \geq 0$ determines the relative productivity of this sector, α controls the share of labor in production, and K_t is the amount of capital used in the previous period.

Producers in the modern sector can save and borrow at the risk-free rate, r, subject to the constraint (9). Their budget constraint is

$$C_t + K_{t+1} - (1-\delta)K_t = Y_t - WL_t - (1+r)D_t - \theta\chi\Pi_t^m + D_{t+1}. \tag{11}$$

The model assumes, as is standard in the investment literature, that output at date $t+1$ is produced with capital held in period t. The choice of how much to invest at the end of period t is, however, measurable with respect to e_{t+1}. This assumption of timing explains why the expected return to equity equals the risk free return.

Workers: A unit measure of workers is available in the economy, each of whom supplies $\gamma^t v_t$ efficiency units of labor, where v_t is the worker's idiosyncratic efficiency that evolves over time according to a finite-state Markov process. These workers have the same log preferences (utility function) as producers do. However, their budget constraint is

$$c + a_{t+1} + \int P_t^i \omega_{t+1}^i di = W\gamma^t v_t + (1+r)a_t + \int (P_t^i + \Pi_t^{m,i})\omega_t^i di,$$

where a_t denote a worker's holdings of risk-free assets and ω_t^i denote the number of shares he or she owns of producer i. The total asset holdings, $a_{t+1} + \int P_t^i \omega_{t+1}^i di$, are non-negative because the model assumes that workers cannot borrow.

Once again, there is no aggregate risk in this economy due to the assumption of timing. As a result, the lack of arbitrage implies that the return on the risk-free security is equal to the expected return on equity claims:

$$(1+r) = \frac{E_t[P_{t+1}^i + \Pi_{t+1}^{m,i}]}{P_t^i}.$$

4.2 Recursive Formulation and Decision Rules

Modern Sector Producers: The risk-free assumption on capital implies that producer profits are solely a function of its net worth, which is denoted as $A = K - D$. Moreover, profits, output, and the optimal choice of capital and labor are all homogeneous of degree one in $(A, \exp(z))$ so this model can rescale all variables by $\exp(z)$ including the rescaled net worth $a = A/\exp(z)$. Given the new notation, the Bellman equation is

(12) $$V^m(a, e_i) = \max_{a',c} \log(c) + \beta \sum_m f_{i,j} V^m(a', e_j).$$

Similarly, the budget constraint in Eq. (11) can be rewritten as

(13) $$\mathrm{c} + a' = (1 - \theta\chi)\pi^m(\alpha, e) + (1+r)a,$$

where

(14) $$\pi^m(\alpha, e) = \max_{k,l} \exp(e + \phi)^{1-\eta} (l^\alpha k^{1-\alpha})^\eta - Wl - (r+\delta)k.$$

Furthermore, the borrowing constraint in Eq. (9) reduces to

(15) $$k \leq \frac{1}{1-\theta} a + \frac{\theta}{1-\theta}\kappa.$$

This model characterizes the producer's net worth accumulation decision of the producer by

(16) $$\frac{1}{c(a,e_i)} = \beta \sum f_{i,j} [(1+r) + \frac{1}{1-\theta}\mu(a', e_j)] \frac{1}{c(a\prime, e_j)},$$

where $\mu(a, e)$ is the multiplier on the borrowing constraint (15). The producer's return to savings increases with the expectation that the borrowing constraint will be binding in future periods. Therefore, the producers have the incentive to accumulate net worth.

Accordingly, the decisions on the optimal level of capital and labor simplifies to

(17) $$\alpha\eta \frac{y(a,e)}{l(a,e)} = W$$

and

(18) $$(1-\alpha)\eta \frac{y(a,e)}{k(a,e)} = r + \delta + \mu(a,e).$$

Dispersion of net worth and productivity of businesses due to borrowing constraints causes dispersion in the marginal product of capital of individual producers. In turn, this causes TFP reductions due to misallocation. In the rescaled formulation of the problem, it is worth noting that the producer's permanent productivity component, z, has no independent effect on allocations.

Traditional Sector Producers: The next thing to consider is the problem of producers in the traditional sector. Since capital is not an input for these producers, their net worth is $a = -d$. This model also denotes x as their savings. The Bellman equation for such producers is

$$V^{\tau}(a, e_i) = \max_{a',c} \log(c) + \beta \max \ \{\textstyle\sum_j f_{i,j} V^{\tau}(a', e_j), \sum_m f_{i,j}\, V^{m}(a', e_j)\},$$

subject to

(19) $$c + x = \pi^{\tau}(e) + (1+r)a,$$

where

$$\pi^{\tau}(e) = \max_l \exp(e)^{1-\eta}\, l^{\eta} - Wl.$$

In each period, the producer's decision on whether to stay in the traditional sector or switch to the modern sector depends on the relative value of these two options. This decision also determines the evolution of its net worth. A producer who remains in the traditional sector simply inherits its past savings, $a' = x$, while a producer that enters the modern sector has

(20) $$a' = x - \kappa + \theta\chi p(a', e_i),$$

where $p(a', e_i)$ is the rescaled price of the equity claim to that satisfies

(21) $$p(a, e_i) = \frac{1}{1+r} \textstyle\sum_j f_{i,j}\, [p(a', e_j) + \pi^{m}(a', e_j)].$$

The producers in the modern sector may have negative net worth since they can borrow against the intangible capital. Besides the collateral constraint in Eq. (15), the natural borrowing constraint is

(22) $$a > a_{min} = -\frac{(1-\theta\chi)\pi^m(a_{min}, e_1)}{r},$$

which guarantees the producer's solvency even under the worst possible sequence of productivity shocks. This constraint may be more stringent than the collateral constraint and motivate producers to accumulate enough savings before entering the modern sector even in the absence of a collateral constraint.

4.3 Equilibrium

Denote $n_t^m(a,e)$ as the measure of modern-sector producers and $n_t^\tau(a,e)$ as the measure of traditional sector producers. The population of producers in these two sectors sum to $N_t = \gamma^t$: $\int_{A\times E} d\, n_t^m(a,e) + \int_{A\times E} d\, n_t^\tau(a,e) = N_t$.

On the one hand, the number of producers in the modern sector evolves according to

(23) $$n_{t+1}^m\big(A, e_j\big) = \int_A \sum_i f_{i,j}\, \mathrm{I}_{\{a^m(a,e_i)\in A\}} d\, n_t^m(a,e_i) + \int_A \sum_i f_{i,j}\, \mathrm{I}_{\{\xi(a,e_i)\in A\}} d\, n_t^\tau(a,e_i),$$

where $\xi(a,e)$ is an indicator of whether a producer in the traditional sector switches, $A = [\underline{a}, \overline{a}]$ is the compact set of values that a producer's net worth can take and A is a family of its subsets, $a^m(.)$ is the amount of net worth for a producer in the modern sector, and $a^{\tau,s}(.)$ is the savings decision of a producer who switches.

On the other hand, the measure of producers in the traditional sector is

(24) $$n_{t+1}^\tau\big(A, e_j\big) = \int_A \sum_i f_{i,j}\, \mathrm{I}_{\{\xi(a,e_i)=0, a^\tau(a,e_i)\in A\}} d\, n_t^\tau(a,e_i) + (\gamma-1)N_t \mathrm{I}_{\{0\in A\}}\overline{f_J},$$

where $\overline{f_J}$ is the stationary distribution of the transitory productivity and $a^\tau(.)$ is the net worth of a producer that stays in the traditional sector.

A balanced growth equilibrium must satisfy the following five conditions:

(I) the labor market clearing condition:

$$\int_{A\times E} l^\tau(e) d\, n_t^\tau(a,e) + \int_{A\times E} l^m(a,e) d\, n_t^m(a,e) = L_t = \gamma^t,$$

(II) the asset market clearing condition:

(25) $$A_{t+1}^w + \sum_{i=m,\tau} \int_{A\times E} \sum a_{t+1}^i(a,e)\, d\, n_{t+1}^i(a,e) = \int_{A\times E} k_{t+1}^m(a,e) d\, n_{t+1}^m(a,e),$$

or

$$C_t + K_{t+1} - (1-\delta)K_t + X_t = Y_t, \tag{26}$$

(III) producer and worker optimization,

(IV) the no-arbitrage condition in Eq. (21),

(V) the laws of motion for the measures in Eqs. (23) and (24).

All variables with time subscripts grow at a constant rate γ while all other variables are fixed. Solving the balanced growth equilibrium is equal to solving the stationary system where all the time-variant variables are rescaled by γ^t.

4.4 Efficient Allocations

The value of TFP in the economy is reduced by financial frictions, which occur in two ways: either by affecting a company's entry decision into the modern sector or by causing losses in the modern sector due to misallocation. The strength of these two paths is defined using two separate computations. The first computation determines the level of TFP losses in the modern sector as a result of capital misallocation when the number of modern producers (n^m) is given. The equilibrium of the model is taken as the stationary level. This calculation is similar to the one that was stated by Hsieh and Klenow (2009). The second computation calculates the optimal allocation of producers across the traditional and modern sectors by solving a planner's problem (i.e. n^m is not fixed). The broader question in this calculation is identifying the level of consumption in the economy and how it is limited by financial frictions that develop along the way at both intensive and extensive margins.

TFP Losses from Misallocation in the Modern Sector: Let i index producers and M be the set of all producers in the modern sector. Also, let L and K be the total amount of labor and capital used in that sector, respectively. Integrating the decision rules (17) and (18) across producers, the total amount of output produced by the modern sector is

$$(27) \qquad Y = \underbrace{\exp(\phi)^{1-\eta} \frac{\left(\int_{i\in M} \exp(e_i)(r+\delta+\mu_i)^{-\frac{(1-\alpha)\eta}{1-\eta}} di\right)^{1-\alpha\eta}}{\left(\int_{i\in M} \exp(e_i)(r+\delta+\mu_i)^{-\frac{\alpha\eta-1}{1-\eta}} di\right)^{(1-\alpha)\eta}}}_{=TFP} (L_t^{\alpha} K_t^{1-\alpha})^{\eta}.$$

This expression shows that TFP of the modern sector is determined by the exogenous productivity gap, ϕ, and an endogenous component that depends on the measure of producers, their efficiency, and the extent to which they are bind.

To calculate the efficient level of TFP given a measure of M producers, the model allocates capital and labor across producers so that the marginal products of capital and labor are the same across producers in order to maximize total output in the modern sector. Accordingly, the efficient level of output is given by

$$(28) \qquad Y^e = \underbrace{\exp(\phi)^{1-\eta} (\int_{i\in M} \exp(e_i)\, di)^{1-\eta}}_{=TFP^e} (L^{\alpha} K^{1-\alpha})^{\eta}.$$

Comparing Eqs. (27) and (28) and using the fact that the shadow cost of capital, $r+\delta+\mu$, is proportional to its average product, as in Eq. (18), the TFP losses from misallocation are

$$(29) \qquad \text{TFP losses} = \log(\int_{i\in M} \exp(e_i))^{1-\eta} - \log \frac{\left(\int_{i\in M} \exp(e_i)\left(\frac{y_i}{k_i}\right)^{-\frac{(1-\alpha)\eta}{1-\eta}}\right)^{1-\alpha\eta}}{\left(\int_{i\in M} \exp(e_i)\left(\frac{y_i}{k_i}\right)^{\frac{\alpha\eta-1}{1-\eta}}\right)^{(1-\alpha)\eta}}.$$

To clarify Eq. (29), suppose that the logarithm of y_i/k_i and e_i are jointly normally distributed. Eq. (29) then reduces to

$$(30) \qquad \text{TFP losses} = \frac{1}{2}\frac{(1-\alpha\eta)(1-\alpha)\eta}{1-\eta} \text{var}(\log(y_i/k_i)),$$

so that the TFP losses are proportional to the variance of the average product of capital. In other words, higher variability in the average product of capital across producers generates more TFP losses.

Efficient (First-Best) Allocations: To calculate the efficient allocation, this model must also derive the optimal number of producers across the two sectors. This can be done by solving the social planner's problem that is only constrained by the aggregate resource constraint in Eq. (26) and by the production technologies that we have assumed. Accordingly, their study chooses the amount of capital, K, the number of producers in the two sectors, n_i^τ and n_i^m, and the allocation of labor across those sectors, L^τ and L^m, to maximize

$$(31) \qquad \underbrace{\left(\sum_i \exp(e_i) n_i^\tau\right)^{1-\eta} (L^\tau)^\eta}_{output\ in\ traditional\ sector} + \underbrace{\left(\sum_i \exp(e_i+\phi) n_i^m\right)^{1-\eta} \left((L^m)^\alpha (K)^{1-\alpha}\right)^\eta}_{output\ in\ modern\ sector} - \underbrace{\left(\delta + \frac{\gamma}{\beta} - 1\right) K}_{cost\ of\ capital} - \underbrace{\frac{(\gamma-1)\kappa \sum_i n_i^m}{\beta}}_{sunk\ cost\ of\ entering},$$

subject to the restrictions on the measurements implied by Markov transition probabilities, $f_{i,j}$, and to the labor constraint, $L^\tau + L^m = 1$.

4.5 Summary

In short, the model has three kinds of players: workers, traditional producers, and modern producers. Traditional producers use only labor and unproductive technology, and cannot borrow money. Modern producers, on the other hand, use capital, labor, and more productive technology; they can also borrow. Traditional producers can become modern producers, but to do so they must incur a sunken entry fee, and they are allowed to borrow and issue claims to part of the future profit during that period of transformation. The amount that a producer can borrow is subject to collateral constraints. Workers face uninsurable idiosyncratic labor income risk and have access to financial markets. There are two types of financial instruments available: a one-period non-contingent security and equity claims to producers' profits.

These three kinds of players all try to maximize their lifetime utility. The equilibrium requires (I) a labor market clearing condition, (II) an asset market clearing condition, (III) producer and worker optimization, (IV) the no-arbitrage condition, and (V) the laws of motion. Eq. (3) is used to calculate the TFP loss due to financial misallocation. On the one hand, the actual TFP level in the equilibrium can be derived under this setup. On the other hand, the efficient level of TFP, TFP^e, is the solution to the planner's problem that is not restricted in any way concerning the allocation of labor and capital across firms.

5. Data

The data come from the *Fame* dataset[2] and cover the years 2005–2012. *Fame* contains comprehensive information, including balance sheets and profit and loss accounts, for approximately half a million companies in the UK and Ireland. Each firm's balance sheet provides total asset, liabilities, and shareholder funds information, while the profit and loss account provides operating profits, depreciation, amortization, impairment, remuneration, directors' remuneration, and the number of employees. This information allows us to construct a more accurate measure of the output (Y) and inputs (L and K).

The labor expenditure is the sum of remuneration and directors' remuneration. Remuneration includes wages and salaries, social security cost, pension costs, and other staff costs, while directors' remuneration includes director's fees, pension contributions, and other compensations. Labor quantity is defined as the number of employees.

Capital expenditure is defined as the sum of depreciation, amortization, and impairment. Capital services are based on "capital employed." Employed capital is the total assets less current liabilities. These are the values of the assets that contribute to a company's ability to generate revenues (liquidity), including both sunk cost (not used in production) and capital (used in production). We assume the same ratio between sunk cost and capital across firms. We use an 8 percent depreciation rate to adjust capital expenditures, which is recommended for the UK during our sample period by Chadha, Crystal, Pearlman, Smith and Wright (2016). Total sunk cost for all of the firms is total capital employed minus the total capital. This provides us with the average ratio between sunk cost and capital, which can be applied to estimate the capital in each firm.

In many productivity analyses (McGuckin, Nguyen, Taylor and Waite 1992; Timmer and Voskoboynikov 2014), the output is measured by value added. One way to calculate value added is to identify the net expenditures on intermediate inputs, while the other is to sum profit, depreciation cost and labor cost. Since *Fame* does not have intermediate input information, we use the second approach. We calculate value added as the sum of operating profit, labor

[2] http://www.bvdinfo.com/en-gb/our-products/company-information/national-products/fame

expenditure, and capital expenditure. Labor expenditure is the return to labor, while the "cash" profit (operating profit plus capital expenditure) is a return to capital.

All series are real prices in 2005 British Pounds (GBP). We deflate value-added, labor cost, and capital cost using the Producer Price Index (PPI) and Consumer Price Index (CPI): PPI and CPI of Ireland for firms in the Republic of Ireland and the PPI and CPI of the UK for firms in England, Scotland, Wales, North Ireland, and British Crown dependencies.

Missing observations, obvious reporting errors, and outliers (largest and smallest 0.5%) are excluded from our sample, leaving us with 17,873×8=142,984 firm-year observations over an eight-year period from 2005 to 2012.

Table 1 shows the sample statistics of the *Fame* data we utilize in our empirical analysis. Average output is £33 million before the crisis and £38 million after the crisis. Capital on average increases from £58 million before the crisis to £74 million after the crisis, while its expenditure increases from £5 million to £6 million. The average number of employees also increases from 671 to 737 between those two periods, while labor costs increase from £17 million to £19 million. These statistics point to an increase in output, capital, and labor of 14%, 28%, and 10%, respectively, indicating that firms in the British Isles were using more capital input in their portfolios after the financial crisis. During the financial crisis, we also see increasing labor productivity and decreasing capital productivity. Also, the debt-to-output ratio decreases from 2.3 to 2.1, but the equity-to-output increases from 1.2 to 1.4 after the financial slowdown. These two ratios imply a significant change in the debt/equity structure in firms after the recession. Equity occupies a much larger proportion of total assets after the slowdown.

(Place Table 1 Here)

6. Quantitative Analysis

6.1 Stochastic Frontier Analysis Based Overall TFP Loss

Table 2 provides estimation results of the Stochastic Frontier Analysis using the different models (FIX, RND, KSS and BC) discussed in Section 2 for both the pre-crisis period (2005-2007) and the post-crisis period (2008-2012). The first two rows in Table 2 give the estimated coefficient and standard error of the simple time trend, respectively. All estimators show that the technology change increases TFP in the first period, but decreases TFP after the financial crisis. The second two rows are the coefficient estimation and standard error of capital input, respectively.

(Place Table 2 Here)

The estimations in Table 2 help to predict average technical efficiency. Table 2 also shows the TFP loss by different models for the pre- and post-crisis periods, which can be directly derived after we estimate the average technical efficiency using Eq. (3). The estimated TFP losses based on estimates from the four models are robust, and the average is 64.5% (equal to an average 35.5% technical efficiency) before 2008. These TFP losses by different models are all dropped after the recession and decrease by an average 1.3%, to 63.2% (equal to 36.8% average efficiency). Figure 1 provides the average efficiencies from different estimators in the two periods. The KSS estimator indicates that the average efficiency was at a low level in 2007 and 2008 but rebounded very soon above the pre-recession level.

(Place Figure 1 Here)

6.2 Pakes-McGuire Algorithm Based on Overall TFP Loss

Pakes, Gowrisankaran and McGuire (1993) defined several constants in their algorithm. The same parameters as the original paper, except for three in the profit function, are utilized. We also allow the profit function to change before and after the financial crisis. PMM provides the outputs, including the efficiency level of all the active firms in every period, the average lifespan of firms, the average investment and profit in one period, and so on. To this end, we calibrate the three parameters in the profit function (D, f, and γ) by requiring that the model provides similar statistics with *Fame* data, including the average lifespan of firms and the investment-to-profit ratio. Then, the average efficiency level can be estimated.

We follow Pakes, Gowrisankaran and McGuire (1993) by assuming an economy that starts with one firm and that the efficiency level of companies can range from 0 to 19. Pakes et al. assume new firms enter with an efficiency level of 4. In our analysis we calculate the average efficiency of the newly established firms in the Fame data, which is 0.45 using SFA and therefore assume new firms in the UK enter with efficiency levels of 9 (i.e. 19*0.45). We also looked at the stability of our results to the assumption that the economy begins with one firms by starting with multiple firms and found that our results to be quite robust to varying this initial condition.[3]

All of these statistics are computed by simulating such an economy. We also use the value function, investment, and entry/exit decisions to evaluate the optimal policies and update the industry structure. We have separate but similar programs to evaluate the statistics for the MPN equilibrium and for the social planner's problem before and after the crisis, respectively. The industry is simulated 10,000 times and the average efficiency level is the mean of all the active firms in those 10,000 periods.

Based on *Fame* data, the average lifespan of firms is 17.4 years before and 16.1 years after the crisis. Goodridge, Haskel and Wallis (2012) provide the total annual investment in the UK from 2005 to 2011. The annual Gross Value Added (GVA) of the UK is available in the Regional Gross Value Added report from the Office for National Statistics (ONS). These two datasets provide the investment-to-value added ratio. *Fame* data provides the value added-to-profit ratio. Therefore, the average investment-to-profit ratio can be derived, which calculates to 0.634 before and 0.525 after the recession in 2008.

Table 3 presents the parameters used in the PMM, as well as the outputs. The average efficiency level before and after the crisis is 6.86 and 7.03, respectively. Since the highest efficiency level witnessed is 13 in both periods, this study predicts a 47.2% and 45.9% TFP loss before and after the crisis, respectively. Therefore, the overall TFP was not impacted to any substantial degree by the financial crisis.

(Place Table 3 Here)

6.3 Midrigan and Xu's Model Based TFP Loss

This model groups the parameters into two categories. The first category includes parameters that determine the process for entrepreneurial productivity, as well as the size of the financing frictions. We calibrate these parameters by requiring that the model accounts for the

[3] The SFA approaches in section 6.1 also give an average efficiency of 44.6% for the newly established firm in 2005 (the entrants) in the British Isles. Therefore, this paper sets a corresponding efficiency level of 9 (out of 19) in the PMM approach for new companies.

salient features (Part A in Table 4) of the *Fame* data. The second category includes preference and technology parameters that are difficult to directly identify using the *Fame* data. We assign these parameter values as follows: 1) labor elasticity (α) is estimated using the *Fame* data; 2) span of control (η), discount factor ($\beta(1+\mu)^{-1}$), persistence of workers in the unemployed state (λ_0)[4], and relative efficiency in modern sector ($(1+\eta)\phi$) are similar to the ones in Midrigan and Xu (2014); 3) capital depreciation (δ) follow UK's average level as discussed in Section 5; 4) growth rate (γ) follows the UK's average level discussed in the first category (Part A in Table 4), which is calculated using the *Fame* data; and 5) persistence of workers in the employed state (λ_1)[5] so that λ_0 and λ_1 guarantee that the fraction of workers who supply labor before the crisis is 60 percent, a number consistent with UK's employment to population and with a 1.75 percentage point decrease after the crisis[6]. Table 4 summarizes the parameter values that we used in our experiments, as well as the results.

(Place Table 4 Here)

Data in Part A of Table 4 represent average levels from the *Fame* sample from 2005 to 2007 with the exception of the autocorrelation coefficient and intangibles investment-to-output ratio. The autocorrelation coefficient is calculated using the entire dataset from 2005 to 2012. Intangible investment information is the average level for the UK during the relevant period. We assign the parameters in Part B for the column market "Benchmark" to make the environment as close as possible to the data. Then, the parameters in Part C are calibrated to ensure that the outcome statistics in Part A of the benchmark correspond to the statistics in the column market "Data." At the same time, the calibrated parameters in Part C of the benchmark also guarantee all the conditions of equilibrium. As a result, the TFP loss due to financial misallocation is 0.5% before the recession. After the financial crisis, a significant decrease in debt is witnessed in the *Fame* data. The average Debt-to-Output ratio decreases from 2.3 to 2.1. The calibration result shows that the borrowing constraint θ falls from 0.62 to 0.45. We find that the TFP loss due to financial friction increases from 0.5% to 2.1%.

Another method for estimating the TFP loss due to misallocation is based on the use of Eqs. (29) and (30). Using the variance of the average product of capital derived from *Fame*, the

[4] The probability of unemployed workers staying unemployed.
[5] The probability of employed workers remaining employed.
[6] Data are from Federal Reserve Bank of St. Louis at https://fred.stlouisfed.org/series/GBREPRNA

TFP losses (in Eq. (30)) are 0.626 and 0.893 for the pre- and post-crisis period, respectively. We can thus derive the TFP loss due to financial friction via Eq. (3), which yields a figure of 46.5% before and 59.1% after the recession. This method suggests a much larger TFP loss than the first structural method we adopted using the Midrigan and Xu model (46.5% and 59.1% vs. 0.5% and 2.1%.). The major difference between the two approaches for estimating TFP loss due to financial friction can be traced to Midrigan and Xu (2014). They found that the TFP loss due to financial friction in Korea was 0.3% for the benchmark model, but 16.2% for the data model. A fairly large TFP loss was also found using Data from China and Colombia, as well as the findings of Hsieh and Klenow (2009). Midrigan and Xu emphasized that Eq. (29) may overstate the TFP losses from misallocation due to financial frictions, since differences in average capital production may reflect differences in technologies, technological barriers to capital reallocation, or other inefficiencies, such as taxes or markups. Nevertheless, they found Eq. (29) to be useful, as it provides an upper bound for the losses from capital misallocation. However, both approaches suggest significantly increased TFP loss due to financial misallocation after the financial crisis.

7. Conclusion

This paper has estimated the overall TFP loss, as well as the loss specifically caused by financial friction, in the British Isles both before and after the Great Recession. Stochastic Frontier Analysis estimates an average 64.5% and 63.2% overall TFP loss before and after the crisis, respectively. The overall TFP losses predicted by the Pakes-McGuire Model are 47.2% and 45.9% before and after the crisis, respectively. Both of these estimates show that the TFP loss does not change significantly after the financial crisis. The loss even decreased marginally after the recession.

However, the TFP loss due to financial friction estimated by Midrigan and Xu's Model increases more than three times after the recession, which shows the negative effect of financial constraints on productivity. Consequently, the ratio of finance-caused TFP loss increases significantly as a part of the overall TFP loss.

This study provides an example on how to use both non-structural and structural models to estimate overall productivity performance, as well as the proportion of loss caused by different factors. The paper can serve as a template for future studies that rely on various approaches to model TFP growth, both highly structured models and more robust but possibly less insightful non-structural alternatives.

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Table 1 Summary Statistics

Variable	Pre-Financial Crisis (2005-2007)				Post-Financial Crisis (2008-2012)			
	Mean	S.D.	Min	Max	Mean	S.D.	Min	Max
Output (Y) (£ million)	33.04	446.3	0.021	38800	37.55	519.8	0.019	48500
Capital (K) (£ million)	58.00	971.3	0.006	82700	74.26	1307.1	0.004	94700
Labor (L)	671.4	6914.2	1	507480	737.0	8208	1	648254
Capital Expenditure (δK)	4.94	161.1	0.0003	28200	5.94	137.7	0.0003	12500
Labor Expenditure (WL)	17.47	139.2	0.009	6259.3	19.46	163.7	0.007	8256.1
Debt to Output	2.33	40.64	0	6507	2.15	17.52	0	2988
Equity to Output	1.20	2.11	0	59.7	1.43	2.41	0	68.1

Table 2 Stochastic Frontier Estimators

		Pre-Financial Crisis (2005-2007)				Post-Financial Crisis (2008-2012)			
		FIX	RND	KSS	BC	FIX	RND	KSS	BC
Time Trend	$\hat{\delta}$	0.017	0.012	--	0.007	-0.015	-0.016	--	-0.030
	S.E.	0.001	0.001	--	0.006	0.001	0.001	--	0.003
Capital	$\hat{\beta}_1$	0.199	0.282	0.217	0.282	0.224	0.280	0.265	0.280
	S.E.	0.004	0.003	0.005	0.003	0.003	0.002	0.004	0.002
Average TE		0.341	0.365	0.339	0.376	0.357	0.376	0.355	0.385
TFP Loss Ratio		0.659	0.635	0.661	0.624	0.643	0.624	0.645	0.615

Table 3 Calibration and Result of PMM

		Pre-Financial Crisis 2005-2007			Post-Financial Crisis 2008-2012		
		Data	MPNE	Social Planner	Data	MPNE	Social Planner
A. Used to calibrate model							
Average lifespan of firms		17.4	17.7		16.1	15.6	
Average investment-to-profit ratio		0.634	0.644		0.525	0.548	
B. Assigned parameters							
Constant used in investment fn.	α			3			
Cost for a GBP investment	c			1			
Maximum Number of firms	N			3			
Highest efficiency level attainable	$\overline{w}$			19			
Efficiency level for entrants	W_E			9			
Sunk cost of entry	X_E			0.2			
Lowest sunk entry cost	X_EL			0.15			
Highest sunk entry cost	X_EH			0.25			
Discount factor	β			0.92			
Prob. of outside alternative rising	δ			0.7			
Scrap value at exit	ϕ			0.1			
C. Calibrated parameters							
Vertical intercept of demand	D		4			4.95	
Fixed cost of production	f		0.2			0.8	
Capital-to-cost parameter	γ		1			1.1	
D. Result							
Average efficiency level			6.86	8.71		7.03	8.9
Max. efficiency level appeared			13	--		13	--

Table 4 Calibration and Result of MXM

		Pre-Financial Crisis 2005-2007		Post-Financial Crisis 2008-2012	
		Data	Benchmark	Data	Benchmark
A. Used to calibrate model					
S.D. output growth		0.27	0.35	0.28	0.25
S.D. output		1.39	1.35	1.43	1.28
1-year autocorrelation		0.98	0.97	0.98	0.98
3-year autocorrelation		0.96	0.92	0.96	0.96
5-year autocorrelation		0.93	0.88	0.93	0.94
Intangibles investment-to-output, %		12.0	11.2	12.0	3.6
Output growth rate, %		5.0	5.0	1.43	1.43
Debt-to-output		2.3	2.3	2.1	2.1
Equity-to-output		1.2	1.2	1.4	1.2
$\mathrm{var}(\log(y_i/k_i))$		1.0227		0.9885	
B. Assigned parameters					
labor elasticity	α		0.6		0.6
span of control	η		0.85		0.85
capital depreciation	δ		8.0		8.0
discount factor	$\beta(1+\mu)^{-1}$		0.92		0.92
growth rate	γ		1.05		1.0143
persistence unit worker state	λ_1		0.667		0.646
persistence zero worker state	λ_0		0.5		0.5
relative efficiency in modern sector	$(1+\eta)\phi$		0.2		0.2
C. Calibrated parameters					
collateral constraint	θ		0.62		0.45
equity issuance constraint	χ		0.705		0.562
standard deviation transitory shocks	σ_ε		0.37		0.482
persistence transitory shocks	ρ		0.815		0.745
cost of entering modern sector	κ		16		9
variance exog. permanent component	$\mathrm{var}(z)$				
wage	W		0.94		0.94
D. Result					
Loss misallocation, %		**46.5**	**0.5**	**59.1**	**2.1**

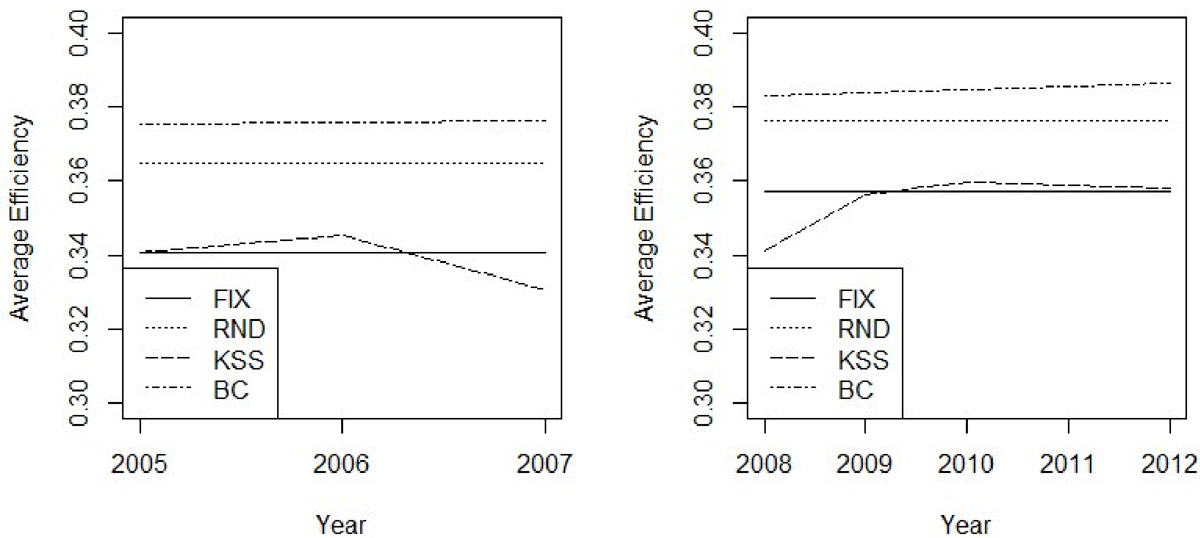

Figure 1 Average efficiencies from different estimators in the two periods